Lecture Notes in Microeconomic Theory

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This file is a revision of the book up to Chapter 3 and including the review problems.

A fully revised version of the book will be available before September 2007.

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Preface

This short book contains my lecture notes for the first quarter of a microeconomics course for PhD or Master's degree economics students. The lecture notes were developed over a period of almost 15 years during which I taught the course, or parts of it, at Tel Aviv, Princeton, and New York universities.

I am publishing the lecture notes with some hesitation. Several superb books are already on the shelves. I most admire Kreps (1990), which pioneered the transformation of the game theoretic revolution in economic theory from research papers into textbooks. His book covers the material in depth and includes many ideas for future research. Mas-Colell, Whinston, and Green (1995) continued this trend with a very comprehensive and detailed textbook. There are three other books on my short list: Bowles (2003), which brings economics back to its authentic, political economics roots; Jehle and Reny (1997), with its very precise style; and the classic Varian (1984). These five books constitute an impressive collection of textbooks for the standard advanced microeconomics course.

My book covers only the first quarter of the standard course. It does not aim to compete with these books, but to supplement them. I had it published only because I think that some of the didactic ideas in the book might be beneficial to students and teachers, and it is to this end that I insisted on retaining its lecture notes style.

A special feature of this book is that it is also posted on the Internet and access is entirely free. My intention is to update the book annually (or at least in years when I teach the course). To access the latest electronic version of the book, visit: http://arielrubinstein.tau.ac.il/micro1/.

Throughout the book I use only male pronouns. This is my deliberate choice and does not reflect the policy of the editors or the publishers. I believe that continuous reminders of the he/she issue simply divert readers' attention. Language is of course very important in shaping our thinking and I don't dispute the importance of the type of language we use. But I feel it is more effective to raise the issue of discrimination against women in the discussion of gender-related issues, rather than raising flags on every page of a book on economic theory.

I would like to thank all my teaching assistants, who contributed comments during the many years I taught the course: Rani Spiegler, Kfir Eliaz, Yoram Hamo, Gabi Gayer and Tamir Tshuva at Tel Aviv University; Bilge Yilmaz, Ronny Razin, Wojciech Olszewski, Attila Ambrus, Andrea Wilson, Haluk Ergin and Daisuke Nakajima at Princeton; and Sophie Bade and Anna Ingster at NYU. Special thanks are due to Sharon Simmer and Rafi Aviav who helped me with the English editing and to Gabi Gayer and Daniel Wasserteil who prepared the figures.

Introduction

As a new graduate student, you are at the beginning of a new stage of your life. In a few months you will be overloaded with definitions, concepts, and models. Your teachers will be guiding you into the wonders of economics and will rarely have the time to stop to raise fundamental questions about what these models are supposed to mean. It is not unlikely that you will be brainwashed by the professional-sounding language and hidden assumptions. I am afraid I am about to initiate you into this inevitable process. Still, I want to use this opportunity to pause for a moment and alert you to the fact that many economists have strong and conflicting views about what economic theory is. Some see it as a *set of theories* that can (or should) be tested. Others see it as a *bag of tools* to be used by economic agents, and yet others see it as a *framework* through which professional and academic economists view the world.

My own view may disappoint those of you who have come to this course with practical motivations. In my view, economic theory is no more than an arena for the *investigation of concepts* we use in thinking about economics in real life. What makes a theoretical model "economics" is that the concepts we are analyzing are taken from real-life reasoning about economic issues. Through the investigation of these concepts we indeed try to understand reality better, and the models provide a language that enables us to think about economic interactions in a systematic way. But I do not view economic models as an attempt to describe the world or to provide tools for predicting the future. I object to looking for an ultimate truth in economic theory, and I do not expect it to be the foundation for any policy recommendation. Nothing is "holy" in economic theory and everything is the creation of people like yourself.

Basically, this course is about a certain class of economic *concepts* and *models*. Although we will be studying formal concepts and models, they will always be given an interpretation. An economic model differs substantially from a purely mathematical model in that it is a *combination* of a mathematical model and its *interpretation*. The names of the mathematical objects are an integral part of an economic model. When mathematicians use terms such as "field" or "ring" which are in everyday use, it is only for the sake of convenience. When they name a collection of sets a "filter," they are doing so in an associative manner; in principle, they could call it "ice cream cone." When they use the term "good ordering" they are not making an ethical judgment. In contrast to mathematics, interpretation is an essential ingredient of any economic model.

The word "model" sounds more scientific than "fable" or "fairy tale" but I don't see much difference between them. The author of a fable draws a parallel to a situation in real life and has some moral he wishes to impart to the reader. The fable is an imaginary situation which is somewhere between fantasy and reality. Any fable can be dismissed as being unrealistic or simplistic but this is also the fable's advantage. Being something between fantasy and reality, a fable is free of extraneous details and annoying diversions. In this unencumbered state, we can clearly discern what cannot always be seen from the real world. On our return to reality, we are in possession of some sound advice or a relevant argument that can be used in the real world. We do exactly the same thing in economic theory. Thus, a good model in economic theory, like a good fable, identifies a number of themes and elucidates them. We perform thought exercises which are only loosely connected to reality and which have been stripped of most of their real-life characteristics. However, in a good model, as in a good fable, something significant remains. One can think about this book as an attempt to introduce the characters that inhabit economic fables. Here, we observe the characters in isolation. In models of markets and games, we further investigate the interactions between the characters.

It is my hope that some of you will react and attempt to change what is currently called economic theory, and that you will acquire alternative ways of thinking about economic and social interactions. At the very least, this course should teach you to ask hard questions about economic models and the sense in which they are relevant to real life economics. I hope that you walk away from this course with the recognition that the answers are not as obvious as they might appear.

Microeconomics

In this course we deal only with microeconomics, a collection of models in which the primitives are details about the behavior of units called *economic agents*. Microeconomic models investigate assumptions about economic agents' activities and about interactions between these agents. An economic agent is the basic unit operating in the model. Most often, we do have in mind that the economic agent is an individual, a person with one head, one heart, two eyes, and two ears. However, in some economic models, an economic agent is taken to be a nation, a family, or a parliament. At other times, the "individual" is broken down into a collection of economic agents, each operating in distinct circumstances and each regarded as an economic agent. When we construct a model with a particular economic scenario in mind, we might have some degree of freedom regarding whom we take to be the economic agents.

We should not be too cheerful about the statement that an economic agent in microeconomics is not constrained to being an individual. The facade of generality in economic theory might be misleading. We have to be careful and aware that when we take an economic agent to be a group of individuals, the reasonable assumptions we might impose on it are distinct from those we might want to impose on a single individual.

An economic agent is described in our models as a unit that responds to a scenario called a *choice problem*, where the agent must make a choice from a set of available alternatives. The economic agent appears in the microeconomic model with a specified deliberation process he uses to make a decision. In most of current economic theory, the deliberation process is what is called *rational* choice. The agent decides what action to take through a process in which he

1. asks himself "What is desirable?"
2. asks himself "What is feasible?"
3. chooses the most desirable from among the feasible alternatives.

Rationality in economics does not contain judgments about desires. A rational agent can have preferences which the entire world views as being against the agent's interest. Furthermore, economists are fully aware that almost all people, almost all the time, do not practice this kind of deliberation.

Nevertheless, we find the investigation of economic agents who follow the rational process to be important, since we often refer to rational decision making in life as an ideal process. It is meaningful to talk about the concept of "being good" even in a society where all people are evil; similarly, it is meaningful to talk about the concept of a "rational man" and about the interactions between rational economic agents even if all people systematically behave in a nonrational manner.

Bibliographic Notes

For an extended discussion of my views about economic theory see Rubinstein (2006).

Lecture Notes in Microeconomic Theory

LECTURE 1

Preferences

Preferences

Although we are on our way to constructing a model of rational choice, we begin the course with an "exercise," formulating the notion of "preferences" independently of the concept of choice. In this lecture we view preferences as the mental attitude of an individual (economic agent) toward alternatives independent of any actual choice. We seek to develop a "proper" formalization of this concept, which plays such a central role in economics.

Note that naturally we don't think about preferences only in the context of choice. For example, we often talk about an individual's tastes over the paintings of the masters even if he never makes a decision based on those preferences. We refer to the preferences of an agent were he to arrive tomorrow on Mars or travel back in time and become King David even if he does not believe in the supernatural.

Imagine that you want to fully describe the preferences of an agent toward the elements in a given set X. For example, imagine that you want to describe your own attitude toward the universities you apply to before finding out to which of them you have been admitted. What must the description include? What conditions must the description fulfill?

We take the approach that a description of preferences should fully specify the attitude of the agent toward each pair of elements in X. For each pair of alternatives, it should provide an answer to the question of how the agent compares the two alternatives. We present two versions of this question. For each version we formulate the consistency requirements necessary to make the responses "preferences" and examine the connection between the two formalizations.

The Questionnaire Q

Let us think about the preferences on a set X as *answers* to a long questionnaire Q which consists of all quiz questions of the type:

Q(x,y) (for all distinct x and y in X):
How do you compare x and y? Tick one and only one of the following three options:

☐ I prefer x to y (this answer is denoted as $x \succ y$).
☐ I prefer y to x (this answer is denoted by $y \succ x$).
☐ I am indifferent (this answer is denoted by I).

A "legal" answer to the questionnaire is a response in which exactly one of the boxes is ticked in each question. We do not allow refraining from answering a question or ticking more than one answer. Furthermore, by allowing only the above three options we exclude responses that demonstrate:

a lack of ability to compare, such as

☐ They are incomparable.
☐ I don't know what x is.
☐ I have no opinion.

a dependence on other factors, such as

☐ It depends on what my parents think.
☐ It depends on the circumstances (sometimes I prefer x but usually I prefer y).

intensity of preferences, such as

☐ I somewhat prefer x.
☐ I love x and I hate y.

confusion, such as

☐ I both prefer x over y and y over x.
☐ I can't concentrate right now.

The constraints that we place on the legal responses of the agents constitute our implicit assumptions. Particularly important are the assumption that the elements in the set X are all comparable, and the fact that we ignore the intensity of preferences.

A legal answer to the questionnaire can be formulated as a function f which assigns to any pair (x, y) of distinct elements in X exactly one of the three "values": $x \succ y$ or $y \succ x$ or I, with the interpretation that $f(x, y)$ is the answer to the question $Q(x, y)$. (Alternatively, we can use the notation of the soccer betting industry and say that $f(x, y)$ must be 1, 2, or $\times$ with the interpretation that $f(x, y) = 1$ means that x is preferred to y, $f(x, y) = 2$ means that y is preferred to x and $f(x, y) = \times$ means indifference.)

Not all legal answers to the questionnaire Q qualify as *preferences over the set* X. We will adopt two "consistency" restrictions:

First, the answer to $Q(x, y)$ must be identical to the answer to $Q(y, x)$. In other words, we want to exclude the common "framing effect" by which people who are asked to compare two alternatives tend to prefer the first one.

Second, we require that the answers to $Q(x, y)$ and $Q(y, z)$ are consistent with the answer to $Q(x, z)$ in the following sense: If the answers to the two questions $Q(x, y)$ and $Q(y, z)$ are "x is preferred to y" and "y is preferred to z" then the answer to $Q(x, z)$ must be "x is preferred to z," and if the answers to the two questions $Q(x, y)$ and $Q(y, z)$ are "indifference" then so is the answer to $Q(x, z)$.

To summarize, here is my favorite formalization of the notion of preferences:

Definition 1

Preferences on a set X are a function f that assigns to any pair (x, y) of distinct elements in X exactly one of the three "values" $x \succ y$, $y \succ x$ or I so that for any three different elements x, y and z in X, the following two properties hold:

- *No order effect*: $f(x, y) = f(y, x)$.
- *Transitivity*:
  if $f(x, y) = x \succ y$ and $f(y, z) = y \succ z$ then $f(x, z) = x \succ z$ and
  if $f(x, y) = I$ and $f(y, z) = I$ then $f(x, z) = I$.

Note again that I, $x \succ y$, and $y \succ x$ are merely symbols representing verbal answers. Needless to say, the choice of symbols is not an arbitrary one. (Why do I use the notation I and not $x \sim y$?)

A Discussion of Transitivity

Transitivity is an appealing property of preferences. How would you react if somebody told you he prefers x to y, y to z and z to x? You would probably feel that his answers are "confused." Furthermore, it seems that, when confronted with an intransitivity in their responses, people are embarrassed and want to change their answers.

On one occasion before giving this lecture, I asked students in Tel Aviv university to fill out a questionnaire similar to Q regarding a set X that contains nine alternatives, each specifying the following four characteristics of a travel package: location (Paris or Rome), price, quality of the food, and quality of the lodgings. The questionnaire included only thirty six questions since for each pair of alternatives x and y, only one of the questions, $Q(x, y)$ or $Q(y, x)$, was randomly selected to appear in the questionnaire (thus the dependence on order of an individual's response was not checked within the experimental framework). Out of eighteen MA students, only two had no intransitivities in their answers, and the average number of triples in which intransitivity existed was almost nine. Many of the violations of transitivity involved two alternatives that were actually the same, but differed in the order in which the characteristics appeared in the description: "A weekend in Paris at a four-star hotel with food quality Zagat 17 for \$574," and "A weekend in Paris for \$574 with food quality Zagat 17 at a four-star hotel." All students expressed indifference between the two alternatives, but in a comparison of these two alternatives to a third alternative—"A weekend in Rome at a five-star hotel with food quality Zagat 18 for \$612"—half of the students gave responses that violated transitivity.

In spite of the appeal of the transitivity requirement, note that when we assume that the attitude of an individual toward pairs of alternatives is transitive, we are excluding individuals who base their judgments on procedures that cause systematic violations of transitivity. The following are two such examples.

1. *Aggregation of considerations as a source of intransitivity.* In some cases, an individual's attitude is derived from the aggregation of more basic considerations. Consider, for example, a case where $X = \{a, b, c\}$ and the individual has three primitive considerations in mind. The individual finds an alternative x better than an alternative y if a majority of considerations support x. This aggregation process can yield intransitivities. For example, if the three considerations rank

the alternatives as follows: $a \succ_1 b \succ_1 c$, $b \succ_2 c \succ_2 a$ and $c \succ_3 a \succ_3 b$, then the individual determines a to be preferred over b, b over c, and c over a, thus violating transitivity.

2. *The use of similarities as an obstacle to transitivity.* In some cases, an individual may express indifference in a comparison between two elements that are too "close" to be distinguishable. For example, let $X = \Re$ (the set of real numbers). Consider an individual whose attitude toward the alternatives is "the larger the better"; however, he finds it impossible to determine whether a is greater than b unless the difference is at least 1. He will assign $f(x, y) = x \succ y$ if $x \geq y + 1$ and $f(x, y) = I$ if $|x - y| < 1$. This is not a preference relation since $1.5 \sim 0.8$ and $0.8 \sim 0.3$, but it is not true that $1.5 \sim 0.3$.

Did we require too little? Another potential criticism of our definition is that our assumptions might have been too weak and that we did not impose some reasonable further restrictions on the concept of preferences. That is, there are other similar consistency requirements we may want to impose on a legal response to qualify it as a description of preferences. For example, if $f(x, y) = x \succ y$ and $f(y, z) = I$, we would naturally expect that $f(x, z) = x \succ z$. However, this additional consistency condition was not included in the above definition since it follows from the other conditions: If $f(x, z) = I$, then by the assumption that $f(y, z) = I$ and by the no order effect, $f(z, y) = I$, and thus by transitivity $f(x, y) = I$ (a contradiction). Alternatively, if $f(x, z) = z \succ x$, then by no order effect $f(z, x) = z \succ x$, and by $f(x, y) = x \succ y$ and transitivity $f(z, y) = z \succ y$ (a contradiction).

Similarly, note that for any preferences f, we have that if $f(x, y) = I$ and $f(y, z) = y \succ z$, then $f(x, z) = x \succ z$.

The Questionnaire R

A second way to think about preferences is through an imaginary questionnaire R consisting of all questions of the type:

R(x,y) (for all $x, y \in X$, not necessarily distinct):
Is x at least as preferred as y? Tick one and only one of the following two options:

☐ Yes
☐ No

By a "legal" response we mean that the respondent ticks exactly one of the boxes in each question. To qualify as preferences a legal response must also satisfy two conditions:

1. The answer to at least one of the questions $R(x, y)$ and $R(y, x)$ must be Yes. (In particular, the "silly" question $R(x, x)$ which appears in the questionnaire must get a Yes response.)
2. For every $x, y, z \in X$, if the answers to the questions $R(x, y)$ and $R(y, z)$ are Yes, then so is the answer to the question $R(x, z)$.

We identify a response to this questionnaire with the binary relation $\succsim$ on the set X defined by $x \succsim y$ if the answer to the question $R(x, y)$ is Yes.

(*Reminder*: An *n-ary relation* on X is a subset of X^n. Examples: "Being a parent of" is a binary relation on the set of human beings; "being a hat" is an unary relation on the set of objects; "$x + y = z$" is a 3-nary relation on the set of numbers; "x is better than y more than x' is better than y'" is 4-nary relation on a set of alternatives, etc. An *n-ary relation* on X can be thought of as a response to a questionnaire regarding all n-tuples of elements of X where each question can get only a Yes/No answer.)

This brings us to the traditional definition of preferences:

Definition 2

Preferences on a set X is a binary relation $\succsim$ on X satisfying:

- *Completeness*: For any $x, y \in X$, $x \succsim y$ or $y \succsim x$.
- *Transitivity*: For any $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

Table 1.1

A response to $Q(x,y)$ and $Q(y,x)$	A response to $R(x,y)$ and $R(y,x)$
$x \succ y$	Yes, No
I	Yes, Yes
$y \succ x$	No, Yes

The Equivalence of the Two Definitions

We now discuss the sense in which the two definitions of preferences on the set X are equivalent. There are many ways to construct a one-to-one correspondence between the objects satisfying the two definitions. But, when we think about the equivalence of two definitions in economics we are thinking about much more than the existence of a one-to-one correspondence (i.e., a one-to-one and onto function): the correspondence also has to *preserve the interpretation*. Note the similarity to the notion of an isomorphism in mathematics where a correspondence has the preserve "structure". For example, an isomorphism between two topological spaces X and Y is a one-to-one function from X onto Y that is required to preserve the open sets. In economics, the analogue to "structure" is the less formal notion of interpretation.

We will now construct a one-to-one and onto function, named $Translation$, between answers to Q that qualify as preferences by the first definition and answers to R that qualify as preferences by the second definition, such that the correspondence preserves the meaning of the responses to the two questionnaires.

To illustrate, imagine that you have two books. Each page in the first book is a response to the questionnaire Q which qualifies as preferences by the first definition. Each page in the second book is a response to the questionnaire R which qualifies as preferences by the second definition. The correspondence matches each page in the first book with a unique page in the second book, so that a reasonable person will recognize that the different responses to the two questionnaires reflect the same mental attitudes toward the alternatives.

Since we assume that the answers to all questions of the type $R(x,x)$ are "Yes," the classification of a response to R as a preference only requires the specification of the answers to questions $R(x,y)$, where $x \neq y$. Table 1.1 presents the translation of responses.

This translation preserves the interpretation we have given to the responses, that is, "I prefer x to y" has the same meaning as the statement

"I find x to be at least as good as y, but I don't find y to be at least as good as x."

The following observations provide the proof that *Translation* is indeed a one-to-one correspondence between the set of preferences, as given by definition 1, and the set of preferences as given by definition 2.

By the assumption on Q of a no order effect, for any two alternatives x and y, one and only one of the following three answers could have been received for both $Q(x, y)$ and $Q(y, x)$: $x \succ y$, I and $y \succ x$. Thus, the responses to $R(x, y)$ and $R(y, x)$ are well defined.

Next we verify that the response to R that we have constructed with the table is indeed a preference relation (by the second definition).

Completeness: In each of the three rows, the answers to at least one of the questions $R(x, y)$ and $R(y, x)$ is affirmative.

Transitivity: Assume that the answers to $R(x, y)$ and $R(y, z)$ are affirmative. This implies that the answer to $Q(x, y)$ is either $x \succ y$ or I, and the answer to $Q(y, z)$ is either $y \succ z$ or I. Transitivity of Q implies that the answer to $Q(x, z)$ must be $x \succ z$ or I, and therefore the answer to $R(x, z)$ must be affirmative.

To see that *Translation* is indeed a one-to-one function, note that for any two different responses to the questionnaire Q there must be a question $Q(x, y)$ for which the responses differ; therefore, the corresponding responses to either $R(x, y)$ or $R(y, x)$ must differ.

It remains to be shown that the range of the *Translation* function includes all possible preferences as defined by the second definition. Let $\succsim$ be preferences in the traditional sense (a response to R). We have to specify a function f, a response to Q, which is converted by *Translation* to $\succsim$. Read from right to left, the table provides us with such a function f.

By the completeness of $\succsim$, for any two elements x and y, one of the entries in the right-hand column is applicable (the fourth option, that the two answers to $R(x, y)$ and $R(y, x)$ are "No," is excluded), and thus the response to Q is well defined and by definition satisfies no order effect.

We still have to check that f satisfies the transitivity condition. If $f(x, y) = x \succ y$ and $f(y, z) = y \succ z$, then $x \succsim y$ and not $y \succsim x$ and $y \succsim z$ and not $z \succsim y$. By transitivity of $\succsim$, $x \succsim z$. In addition, not $z \succsim x$ since if $z \succsim x$, then the transitivity of $\succsim$ would imply $z \succsim y$. If $f(x, y) = I$ and $f(y, z) = I$, then $x \succsim y$, $y \succsim x$, $y \succsim z$ and $z \succsim y$. By transitivity of $\succsim$, both $x \succsim z$ and $z \succsim x$, and thus $f(x, z) = I$.

Summary

I could have replaced the entire lecture with the following two sentences: "Preferences on X are a binary relation $\succsim$ on a set X satisfying completeness and transitivity. Notate $x \succ y$ when both $x \succsim y$ and not $y \succsim x$, and $x \sim y$ when $x \succsim y$ and $y \succsim x$." However, the role of this chapter was not just to introduce a formal definition of preferences, but also to conduct a modeling exercise and to make two methodological points:

1. When we introduce two formalizations of the same verbal concept, we have to make sure that they indeed carry the same meaning.
2. When we construct a formal concept, we make assumptions beyond those explicitly mentioned. Being aware of the implicit assumptions is important or understanding the concept and is useful in coming up with ideas for alternative formalizations.

Bibliographic Notes

Recommended readings. Kreps 1990, 17–24; Mas-Colell et al. 1995, chapter 1, A–B.

Fishburn (1970) contains a comprehensive treatment of preference relations.

Problem Set 1

Problem 1. (*Easy*)
Let $\succsim$ be a preference relation on a set X. Define $I(x)$ to be the set of all $y \in X$ for which $y \sim x$.

Show that the set (of sets!) $\{I(x)|x \in X\}$ is a partition of X, i.e.,

- For all x and y, either $I(x) = I(y)$ or $I(x) \cap I(y) = \emptyset$.
- For every $x \in X$, there is $y \in X$ such that $x \in I(y)$.

Problem 2. (*Standard*)
Kreps (1990) introduces another formal definition for preferences. His primitive is a binary relation P interpreted as "strictly preferred." He requires P to satisfy:

- *Asymmetry*: For no x and y do we have both xPy and yPx.
- *Negative-Transitivity*: For all x, y, and $z \in X$, if xPy, then for any z either xPz or zPy (or both).

Explain the sense in which Kreps' formalization is equivalent to the traditional definition.

Problem 3. (*Standard*)
In economic theory we are often interested in other types of binary relations, for example, the relation xSy: "x and y are almost the same." Suggest properties that would correspond to your intuition about such a concept.

Problem 4. (*Difficult.* Based on Kannai and Peleg 1984.)
Let Z be a finite set and let X be the set of all nonempty subsets of Z. Let $\succsim$ be a preference relation on X (not Z).

Consider the following two properties of preference relations on X:

a. If $A \succsim B$ and C is a set disjoint to both A and B, then $A \cup C \succsim B \cup C$, and
if $A \succ B$ and C is a set disjoint to both A and B, then $A \cup C \succ B \cup C$.

b. If $x \in Z$ and $\{x\} \succ \{y\}$ for all $y \in A$, then $A \cup \{x\} \succ A$, and
if $x \in Z$ and $\{y\} \succ \{x\}$ for all $y \in A$, then $A \succ A \cup \{x\}$.

a. Discuss the plausibility of the properties in the context of interpreting $\succsim$ as the attitude of the individual toward sets from which he will have to make a choice at a "second stage."

b. Provide an example of a preference relation that (i) Satisfies the two properties. (ii) Satisfies the first but not the second property. (iii) Satisfies the second but not the first property.

c. Show that if there are x, y, and $z \in Z$ such that $\{x\} \succ \{y\} \succ \{z\}$, then there is no preference relation satisfying both properties.

Problem 5. (*Fun*)

Listen to the illusion called the Shepard Scale. (You can find it on the internet. Currently, it is available at http://asa.aip.org/demo27.html.)

Can you think of any economic analogies?

LECTURE 2

Utility

The Concept of Utility Representation

Think of examples of preferences. In the case of a small number of alternatives, we often describe a preference relation as a list arranged from best to worst. In some cases, the alternatives are grouped into a small number of categories and we describe the preferences on X by specifying the preferences on the set of categories. But, in my experience, most of the examples that come to mind are similar to: "I prefer the taller basketball player," "I prefer the more expensive present," "I prefer a teacher who gives higher grades," "I prefer the person who weighs less."

Common to all these examples is that they can naturally be specified by a statement of the form "$x \succsim y$ if $V(x) \geq V(y)$" (or $V(x) \leq V(y)$), where $V : X \to \Re$ is a function that attaches a real number to each element in the set of alternatives X. For example, the preferences stated by "I prefer the taller basketball player" can be expressed formally by: X is the set of all conceivable basketball players, and $V(x)$ is the height of player x.

Note that the statement $x \succsim y$ if $V(x) \geq V(y)$ always defines a preference relation since the relation $\geq$ on $\Re$ satisfies completeness and transitivity.

Even when the description of a preference relation does not involve a numerical evaluation, we are interested in an equivalent numerical representation. We say that *the function* $U : X \to \Re$ *represents the preference* $\succsim$ if for all x and $y \in X$, $x \succsim y$ if and only if $U(x) \geq U(y)$. If the function U represents the preference relation $\succsim$, we refer to it as a *utility function* and we say that $\succsim$ has a *utility representation*.

It is possible to avoid the notion of a utility representation and to "do economics" with the notion of preferences. Nevertheless, we usually use utility functions rather than preferences as a means of describing an economic agent's attitude toward alternatives, probably because we find it more convenient to talk about the maximization of a numerical function than of a preference relation.

Note that when defining a preference relation using a utility function, the function has an intuitive meaning that carries with it additional information. In contrast, when the utility function is formed in order to represent an existing preference relation, the utility function has no meaning other than that of representing a preference relation. Absolute numbers are meaningless in the latter case; only relative order has meaning. Indeed, if a preference relation has a utility representation, then it has an infinite number of such representations, as the following simple claim shows:

Claim:
If U represents $\succsim$, then for any strictly increasing function $f : \Re \rightarrow \Re$, the function $V(x) = f(U(x))$ represents $\succsim$ as well.

Proof:
$a \succsim b$
iff $U(a) \geq U(b)$ (since U represents $\succsim$)
iff $f(U(a)) \geq f(U(b))$ (since f is strictly increasing)
iff $V(a) \geq V(b)$.

Existence of a Utility Representation

If any preference relation could be represented by a utility function, then it would "grant a license" to use utility functions rather than preference relations with no loss of generality. Utility theory investigates the possibility of using a numerical function to represent a preference relation and the possibility of numerical representations carrying additional meanings (such as, a is preferred to b more than c is preferred to d).

We will now examine the basic question of "utility theory": Under what assumptions do utility representations exist?

Our first observation is quite trivial. When the set X is finite, there is always a utility representation. The detailed proof is presented here mainly to get into the habit of analytical precision. We start with a lemma regarding the existence of minimal elements (an element $a \in X$ is *minimal* if $a \precsim x$ for any $x \in X$).

Lemma:

In any finite set $A \subseteq X$ there is a minimal element (similarly, there is also a maximal element).

Proof:

By induction on the size of A. If A is a singleton, then by completeness its only element is minimal. For the inductive step, let A be of cardinality $n+1$ and let $x \in A$. The set $A-\{x\}$ is of cardinality n and by the inductive assumption has a minimal element denoted by y. If $x \succsim y$, then y is minimal in A. If $y \succsim x$, then by transitivity $z \succsim x$ for all $z \in A-\{x\}$ and thus x is minimal.

Claim:

If $\succsim$ is a preference relation on a finite set X, then $\succsim$ has a utility representation with values being natural numbers.

Proof:

We will construct a sequence of sets inductively. Let X_1 be the subset of elements that are minimal in X. By the above lemma, X_1 is not empty. Assume we have constructed the sets $X_1, \ldots, X_k$. If $X = X_1 \cup X_2 \cup \ldots \cup X_k$ we are done. If not, define X_{k+1} to be the set of minimal elements in $X - X_1 - X_2 - \cdots - X_k$. By the lemma $X_{k+1} \neq \emptyset$. Since X is finite we must be done after at most $|X|$ steps. Define $U(x) = k$ if $x \in X_k$. Thus, $U(x)$ is the step number at which x is "eliminated." To verify that U represents $\succsim$, let $a \succsim b$. Then $b \notin X - X_1 - X_2 - \cdots - X_{U(a)}$ and thus $U(a) \geq U(b)$.

Without any further assumptions on the preferences, the existence of a utility representation is guaranteed when the set X is countable (recall that X is countable and infinite if there is a one-to-one function from the natural numbers to X, namely, it is possible to specify an enumeration of all its members $\{x_n\}_{n=1,2,\ldots}$).

Claim:

If X is countable, then any preference relation on X has a utility representation with a range $(-1, 1)$.

Proof:

Let $\{x_n\}$ be an enumeration of all elements in X. We will construct the utility function inductively. Set $U(x_1) = 0$. Assume that you have completed the definition of the values $U(x_1), \ldots, U(x_{n-1})$ so that $x_k \succsim x_l$ iff $U(x_k) \geq U(x_l)$. If x_n is indifferent to x_k for some $k < n$, then assign $U(x_n) = U(x_k)$. If not, by transitivity, all numbers in the set $\{U(x_k) |\ x_k \prec x_n\} \cup \{-1\}$ are below all numbers in the set $\{U(x_k) |\ x_n \prec x_k\} \cup \{1\}$. Choose $U(x_n)$ to be between the two sets. This guarantees that for any $k < n$ we have $x_n \succsim x_k$ iff $U(x_n) \geq U(x_k)$. Thus, the function we defined on $\{x_1, \ldots, x_n\}$ represents the preference on those elements.

To complete the proof that U represents $\succsim$, take any two elements, x and $y \in X$. For some k and l we have $x = x_k$ and $y = x_l$. The above applied to $n = \max\{k, l\}$ yields $x_k \succsim x_l$ iff $U(x_k) \geq U(x_l)$.

Lexicographic Preferences

Lexicographic preferences are the outcome of applying the following procedure for determining the ranking of any two elements in a set X. The individual has in mind a sequence of criteria that could be used to compare pairs of elements in X. The criteria are applied in a fixed order until a criterion is reached that succeeds in distinguishing between the two elements, in that it determines the preferred alternative. Formally, let $(\succsim_k)_{k=1,\ldots,K}$ be a K-tuple of orderings over the set X. The lexicographic ordering induced by those orderings is defined by $x \succsim_L y$ if (1) there is k^* such that for all $k < k^*$ we have $x \sim_k y$ and $x \succ_{k*} y$ or (2) $x \sim_k y$ for all k. Verify that $\succsim_L$ is a preference relation.

Example:

Let X be the unit square, i.e., $X = [0, 1] \times [0, 1]$. Let $x \succsim_k y$ if $x_k \geq y_k$. The lexicographic ordering $\succsim_L$ induced from $\succsim_1$ and $\succsim_2$ is: $(a_1, a_2) \succsim_L (b_1, b_2)$ if $a_1 > b_1$ or both $a_1 = b_1$ and $a_2 \geq b_2$. (Thus, in this example, the left component is the primary criterion while the right component is the secondary criterion.)

We will now show that the preferences $\succsim_L$ do *not* have a utility representation. The lack of a utility representation excludes lexicographic preferences from the scope of standard economic models although they are derived from a simple and commonly used procedure.

Claim:

The lexicographic preference relation $\succsim_L$ on $[0,1] \times [0,1]$, induced from the relations $x \succsim_k y$ if $x_k \geq y_k$ $(k = 1, 2)$, does not have a utility representation.

Proof:

Assume by contradiction that the function $u : X \to \Re$ represents $\succsim_L$. For any $a \in [0,1]$, $(a,1) \succ_L (a,0)$ we thus have $u(a,1) > u(a,0)$. Let $q(a)$ be a rational number in the nonempty interval $I_a = (u(a,0), u(a,1))$. The function q is a function from X into the set of rational numbers. It is a one-to-one function since if $b > a$ then $(b,0) \succ_L (a,1)$ and therefore $u(b,0) > u(a,1)$. It follows that the intervals I_a and I_b are disjoint and thus $q(a) \neq q(b)$. But the cardinality of the rational numbers is lower than that of the continuum, a contradiction.

Continuity of Preferences

In economics we often take the set X to be an infinite subset of a Euclidean space. The following is a condition that will guarantee the existence of a utility representation in such a case. The basic intuition, captured by the notion of a continuous preference relation, is that if a is preferred to b, then "small" deviations from a or from b will not reverse the ordering.

Definition C1:

A preference relation $\succsim$ on X is *continuous* if whenever $a \succ b$ (namely, it is not true that $b \succsim a$), there are neighborhoods (balls) B_a and B_b

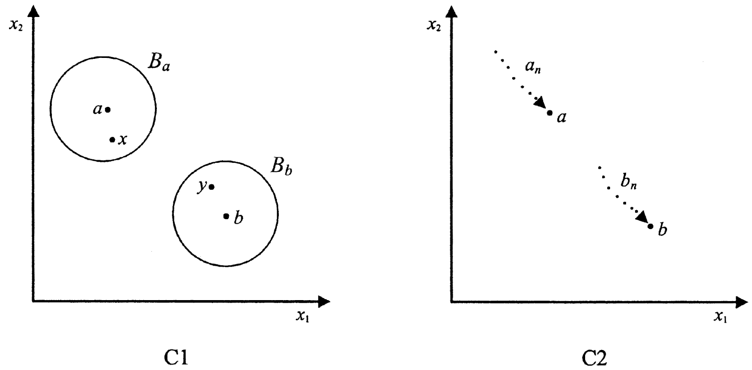

Figure 2.1
Two definitions of continuity of preferences.

around a and b, respectively, such that for all $x \in B_a$ and $y \in B_b$, $x \succ y$ (namely, it is not true that $y \succsim x$). (See fig. 2.1.)

Definition C2:

A preference relation $\succsim$ on X is *continuous* if the graph of $\succsim$ (that is, the set $\{(x, y)|x \succsim y\} \subseteq X \times X$) is a closed set (with the product topology); that is, if $\{(a_n, b_n)\}$ is a sequence of pairs of elements in X satisfying $a_n \succsim b_n$ for all n and $a_n \to a$ and $b_n \to b$, then $a \succsim b$. (See fig. 2.1.)

Claim:

The preference relation $\succsim$ on X satisfies C1 if and only if it satisfies C2.

Proof:

Assume that $\succsim$ on X is continuous according to C1. Let $\{(a_n, b_n)\}$ be a sequence of pairs satisfying $a_n \succsim b_n$ for all n and $a_n \to a$ and $b_n \to b$. If it is not true that $a \succsim b$ (that is, $b \succ a$), then there exist two balls B_a and B_b around a and b, respectively, such that for all $y \in B_b$ and $x \in B_a$, $y \succ x$. There is an N large enough such that for all $n > N$, both $b_n \in B_b$ and $a_n \in B_a$. Therefore, for all $n > N$, we have $b_n \succ a_n$, which is a contradiction.

Assume that $\succsim$ is continuous according to C2. Let $a \succ b$. Denote by $B(x, r)$ the set of all elements in X distanced less than r from x. Assume by contradiction that for all n there exist $a_n \in B(a, 1/n)$ and

$b_n \in B(b, 1/n)$ such that $b_n \succsim a_n$. The sequence (b_n, a_n) converges to (b, a); by the second definition (b, a) is within the graph of $\succsim$, that is, $b \succsim a$, which is a contradiction.

Remarks

1. If $\succsim$ on X is represented by a *continuous* function U, then $\succsim$ is continuous. To see this, note that if $a \succ b$ then $U(a) > U(b)$. Let $\varepsilon = (U(a) - U(b))/2$. By the continuity of U, there is a $\delta > 0$ such that for all x distanced less than δ from a, $U(x) > U(a) - \varepsilon$, and for all y distanced less than δ from b, $U(y) < U(b) + \varepsilon$. Thus, for x and y within the balls of radius δ around a and b, respectively, $x \succ y$.
2. The lexicographic preferences which were used in the counterexample to the existence of a utility representation are not continuous. This is because $(1, 1) \succ (1, 0)$, but in any ball around $(1, 1)$ there are points inferior to $(1, 0)$.
3. Note that the second definition of continuity can be applied to any binary relation over a topological space, not just to a preference relation. For example, the relation $=$ on the real numbers ($\Re^1$) is continuous while the relation $\neq$ is not.

Debreu's Theorem

Debreu's theorem, which states that continuous preferences have a *continuous* utility representation, is one of the classic results in economic theory. For a complete proof of Debreu's theorem see Debreu 1954, 1960. Here we prove only that continuity guarantees the existence of a utility representation.

Lemma:

If $\succsim$ is a continuous preference relation on a convex set $X \subseteq \Re^n$, and if $x \succ y$, then there exists z in X such that $x \succ z \succ y$.

Proof:

Assume not. Construct inductively two sequences of points, $\{x_t\}$ and $\{y_t\}$, in the interval that connects the points x and y in the following way. First define $x_0 = x$ and $y_0 = y$. Assume that the two points, x_t and y_t are defined, belong to the interval that connects the points x and y and satisfy $x_t \succsim x$ and $y \succsim y_t$. Consider the middle point between x_t and y_t and denote it by m. According to the assumption, either $m \succsim x$ or $y \succsim m$. In the former case define $x_{t+1} = m$ and $y_{t+1} = y_t$, and in the latter case define $x_{t+1} = x_t$ and $y_{t+1} = m$. The sequences $\{x_t\}$ and $\{y_t\}$ are converging, and they must converge to the same point z since the distance between x_t and y_t converges to zero. By the continuity of $\succsim$ we have $z \succsim x$ and $y \succsim z$ and thus, by transitivity, $y \succsim x$, contradicting the assumption that $x \succ y$.

Comment on the Proof:

Another proof could be given for the more general case, in which the assumption that the set X is convex is replaced by the assumption that it is a connected subset of $\Re^n$. (Remember that a connected set cannot be covered by two disjoint open sets.) If there is no z such that $x \succ z \succ y$, then X is the union of two disjoint sets $\{a|a \succ y\}$ and $\{a|x \succ a\}$, which are open by the continuity of the preference relation, contradicting the connectedness of X.

Recall that a set $Y \subseteq X$ is *dense* in X if in every open subset of X there is an element in Y. For example, the set $Y = \{x \in \Re^n|\ x_k$ is a rational number for $k = 1, .., n\}$ is a countable dense set in $\Re^n$.

Proposition:

Assume that X is a convex subset of $\Re^n$ that has a countable dense subset Y. If $\succsim$ is a continuous preference relation, then $\succsim$ has a (continuous) utility representation.

Proof:

By a previous claim we know that there exists a function $v : Y \to [-1, 1]$, which is a utility representation of the preference relation $\succsim$ restricted to Y. For every $x \in X$, define $U(x) = sup\{v(z)|z \in Y$ and $x \succ z\}$. Define $U(x) = -1$ if there is no $z \in Y$ such that $x \succ z$, which means that x is the minimal element in X. (Note that it could be that $U(z) < v(z)$ for some $z \in Y$.)

If $x \sim y$, then $x \succ z$ iff $y \succ z$. Thus, the sets on which the supremum is taken are the same and $U(x) = U(y)$.

If $x \succ y$, then by the lemma there exists z in X such that $x \succ z \succ y$. By the continuity of the preferences $\succsim$ there is a ball around z such that all the elements in that ball are inferior to x and superior to y. Since Y is dense, there exists $z_1 \in Y$ such that $x \succ z_1 \succ y$. Similarly, there exists $z_2 \in Y$ such that $z_1 \succ z_2 \succ y$. Finally,
$U(x) \geq v(z_1)$ (by the definition of U and $x \succ z_1$),
$v(z_1) > v(z_2)$ (since v represents $\succsim$ on Y and $z_1 \succ z_2$), and
$v(z_2) \geq U(y)$ (by the definition of U and $z_2 \succ y$).

Bibliographic Notes

Recommended readings. Kreps 1990, 30–32; Mas-Colell et al. 1995, chapter 3, C.

Fishburn (1970) covers the material in this lecture very well. The example of lexicographic preferences originated in Debreu (1959) (see also Debreu 1960, in particular Chapter 2, which is available online at http://cowles.econ.yale.edu/P/cp/p00b/p0097.pdf.)

Problem Set 2

Problem 1. (*Easy*)
The purpose of this problem is to make sure that you fully understand the basic concepts of utility representation and continuous preferences.

a. Is the statement "if both U and V represent $\succsim$ then there is a *strictly* monotonic function $f : \Re \to \Re$ such that $V(x) = f(U(x))$" correct?
b. Can a continuous preference be represented by a discontinuous function?
c. Show that in the case of $X = \Re$, the preference relation that is represented by the discontinuous utility function $u(x) = [x]$ (the largest integer n such that $x \geq n$) is not a continuous relation.
d. Show that the two definitions of a continuous preference relation (C1 and C2) are equivalent to

Definition C3: For any $x \in X$, the upper and lower contours $\{y| \;\; y \succsim x\}$ and $\{y| \;\; x \succsim y\}$ are closed sets in X,

and to

Definition C4: For any $x \in X$, the sets $\{y| \;\; y \succ x\}$ and $\{y| \;\; x \succ y\}$ are open sets in X.

Problem 2. (*Moderate*)
Give an example of preferences over a countable set in which the preferences cannot be represented by a utility function that returns only integers as values.

Problem 3. (*Moderate*)
Consider the sequence of preference relations $(\succsim^n)_{n=1,2,..}$, defined on $\Re^2_+$ where $\succsim^n$ is represented by the utility function $u_n(x_1, x_2) = x_1^n + x_2^n$. We will say that the sequence $\succsim^n$ converges to the preferences $\succsim^*$ if for every x and y, such that $x \succ^* y$, there is an N such that for every $n > N$ we have $x \succ^n y$. Show that the sequence of preference relations $\succsim^n$ converges to the preferences $\succ^*$ which are represented by the function $max\{x_1, x_2\}$.

Problem 4. (*Moderate*)
The following is a typical example of a utility representation theorem:

Let $X = \Re^2_+$. Assume that a preference relation $\succsim$ satisfies the following three properties:
ADD: $(a_1, a_2) \succsim (b_1, b_2)$ implies that $(a_1 + t, a_2 + s) \succsim (b_1 + t, b_2 + s)$ for all t and s.
MON: If $a_1 \geq b_1$ and $a_2 \geq b_2$, then $(a_1, a_2) \succsim (b_1, b_2)$; in addition, if either $a_1 > b_1$ or $a_2 > b_2$, then $(a_1, a_2) \succ (b_1, b_2)$.
CON: Continuity.

a. Show that if $\succsim$ has a linear representation (that is, $\succsim$ is represented by a utility function $u(x_1, x_2) = \alpha x_1 + \beta x_2$ with $\alpha > 0$ and $\beta > 0$), then $\succsim$ satisfies ADD, MON and CON.
b. Suggest circumstances in which ADD makes sense.
c. Show that the three properties are necessary for $\succsim$ to have a linear representation. Namely, show that for any pair of the three properties there is a preference relation that does not satisfy the third property.
d. (*This part is difficult*) Show that if $\succsim$ satisfies the three properties, then it has a linear representation.

Problem 5. (*Moderate*)
Utility is a numerical representation of preferences. One can think about the numerical representation of other abstract concepts. Here, you will try to come up with a possible numerical representation of the concept "approximately the same" (see Luce (1956) and Rubinstein (1988)). For simplicity, let X be the interval $[0, 1]$.

Consider the following six properties of the binary relation S:

(S-1) For any $a \in X$, aSa.
(S-2) For all $a, b \in X$, if aSb then bSa.
(S-3) Continuity (the graph of the relation S in $X \times X$ is a closed set).
(S-4) Betweenness: If $d \geq c \geq b \geq a$ and dSa then also cSb.
(S-5) For any $a \in X$ there is an open interval around a such that xSa for every x in the interval.
(S-6) Denote $M(a) = max\{x|xSa\}$ and $m(a) = min\{x|aSx\}$. Then, M and m are (weakly) increasing functions and are strictly increasing whenever they do not have the values 0 or 1.

a. Do these assumptions capture your intuition about the concept "approximately the same"?
b. Show that the relation S_ε, defined by $aS_\varepsilon b$ if $|b - a| \leq \varepsilon$ (for positive ε), satisfies all assumptions.
c. (*Difficult*) Let S be a binary relation that satisfies the above six properties and let ε be a strictly positive number. Show that there is a strictly

increasing and continuous function $H : X \to \Re$ such that aSb if and only if $|H(a) - H(b)| \leq \varepsilon$.

Problem 6. (*Reading*)
Read Kahneman (2000) and discuss his distinction between the different types of "psychological utilities."

LECTURE 3

Choice

Choice Functions

Until now we have avoided any reference to behavior. We have talked about preferences as a summary of the decision maker's mental attitude toward a set of alternatives. But economics is about behavior, and therefore we now move on to modeling "agent behavior". The term "agent behavior" refers not only to an agent's actual choices, made when he confronts a certain choice problem, it contains a full description of his behavior in all scenarios we imagine he might confront.

Consider a *grand set* X of possible alternatives. We view a *choice problem* as a nonempty subset of X, and we refer to a choice from $A \subseteq X$ as specifying one of A's members.

Modeling a choice scenario as a set of alternatives implies assumptions of rationality according to which the agent's choice does not depend on the way the alternatives are presented. For example, if the alternatives appear in a list, he ignores the order in which they are presented and the number of times an alternative appears in the list. If there is an alternative with a default status, he ignores that as well. As a rational agent he considers only the set of alternatives available to him.

In some contexts, not all choice problems are relevant. Therefore we allow that the agent's behavior be defined only on a set D of subsets of X. We will refer to a pair (X, D) as a *context*.

Example:

Imagine that we are interested in a student's behavior regarding his selection from the set of universities to which he has been admitted. Let $X = \{x_1, \ldots, x_N\}$ be the set of all universities with which the student is familiar. A choice problem A is interpreted as the set of universities to which he has been admitted. If the fact that the student was admitted to some subset of universities does not imply his admission outcome for other universities, then D contains the $2^N - 1$ nonempty subsets of X. But if, for example, the universities are listed according to difficulty in being admitted (x_1 being the most difficult) and if the fact that the student is admitted to x_k means that he is admitted to all less "prestigious" universities, that is, to all x_l with $l > k$, then D will consist of the N sets $A_1, \ldots, A_N$ where $A_k = \{x_k, \ldots, x_N\}$.

We think about an agent's behavior as a hypothetical response to a questionnaire that contains questions of the following type, one for each $A \in D$:

Q(A): Assume you must choose from a set of alternatives A. Which alternative do you choose?

A permissible response to this questionnaire requires that the agent select a unique element in A for every question Q(A). We implicitly assume that the agent cannot give any other answer such as "I choose either a or b"; "the probability of my choosing $a \in A$ is $p(a)$"; "I don't know", etc.

Formally, given a context (X, D), a *choice function* C assigns to each set $A \in D$ a unique element of A with the interpretation that $C(A)$ is the chosen element from the set A.

Our understanding is that a decision maker behaving in accordance with the function C will choose $C(A)$ if he has to make a choice from a set A. This does not mean that we can actually observe the choice function. At most we might observe some particular choices made by the decision maker in some instances. Thus, a choice function is a description of hypothetical behavior.

Rational Choice Functions

It is typically assumed in economics that choice is an outcome of "rational deliberation." Namely, the decision maker has in mind a preference relation $\succsim$ on the set X and, given any choice problem A in D, he chooses an element in A which is $\succsim$ optimal. Assuming that it is well defined, we define the *induced choice function* $C_{\succsim}$ as the function that assigns to every nonempty set $A \in D$ the $\succsim$-best element of A. Note that the preference relation is fixed, that is, it is independent of the choice set being considered.

Dutch Book Arguments

Some of the justifications for the assumption that choice is determined by "rational deliberation" are normative, that is, they reflect a perception that people should be rational in this sense and, if they are not, they should convert to reasoning of this type. One interesting class of arguments supporting this approach is referred to in the literature as "Dutch book arguments." The claim is that an economic agent who behaves according to a choice function that is not induced from maximization of a preference relation will not survive.

The following is a "sad" story about a monkey in a forest with three trees, a , b, and c. The monkey is about to pick a tree to sleep in. Assume that the monkey can only assess two alternatives at a time and that his choice function is $C(\{a, b\}) = b$, $C(\{b, c\}) = c$, $C(\{a, c\}) = a$. Obviously, his choice function cannot be derived from a preference relation over the set of trees. Assume that whenever he is on tree x it comes to his mind occasionally to jump to one of the other trees, namely, he makes a choice from a set $\{x, y\}$ where y is one of the two other trees. This induces the monkey to perpetually jump from one tree to another - not a particularly desirable mode of behavior in the jungle.

Another argument – which is more appropriate to human beings – is called the "money pump" argument. Assume that a decision maker behaves like the monkey with respect to three alternatives a, b, and c. Assume that, for all x and y, the choice $C(x, y) = y$ is strong enough so that whenever he is about to choose alternative x and somebody gives him the option to also choose y, he is ready to pay one cent for the opportunity to do so. Now, imagine a manipulator who presents the agent with the choice problem $\{a, b, c\}$. Whenever the decision maker is about to make the choice a, the manipulator allows him to revise his choice to b for one cent. Similarly, every time he is about to choose b or c, the manipulator sells him for one cent the opportunity to choose c or a accordingly. The decision maker will cycle through the intentions to choose a, b and c until his pockets are emptied or until he learns his lesson and changes his behavior.

The above arguments are open to criticism. In particular, the elimination of patters of behavior which are inconsistent with rationality require an environment in which the economic agent is indeed confronted with the above sequence of choice problems. The arguments are presented here as interesting ideas and not necessarily as convincing arguments for rationality.

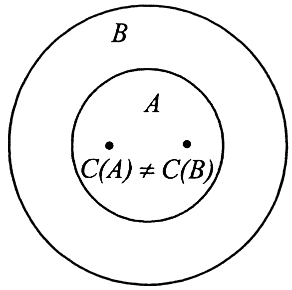

Figure 3.1
Violation of condition $*$.

Rationalizing

Economists were often criticized for making the assumption that decision makers maximize a preference relation. The most common response to this criticism is that we don't really need this assumption. All we need to assume is that the decision maker's behavior can be described *as if* he were maximizing some preference relation.

Let us state this "economic defense" more precisely. We will say that a choice function C *can be rationalized* if there is a preference relation $\succsim$ on X so that $C = C_{\succsim}$ (that is, $C(A) = C_{\succsim}(A)$ for any A in the domain of C).

We will now identify a condition under which a choice function can indeed be presented as if derived from some preference relation (i.e., can be rationalized).

Condition $*$:
We say that C satisfies condition $*$ if for any two problems A,$B \in D$, if $A \subset B$ and $C(B) \in A$ then $C(A) = C(B)$. (See fig. 3.1.)

Note that if $\succsim$ is a preference relation on X, then $C_{\succsim}$ (defined on a set of subsets of X that have a single most preferred element) satisfies $*$.

An example of a choice procedure which does not satisfy condition $*$., consider the *second-best procedure*: the decision maker has in mind an ordering $\succsim$ of X and for any given choice problem set A chooses the element from A, which is the $\succsim$-maximal from the nonoptimal alternatives. If A contains all the elements in B besides the $\succsim$-maximal, then $C(B) \in A \subset B$ but $C(A) \neq C(B)$.

We will now show that condition $*$ is a sufficient condition for a choice function to be formulated *as if* the decision maker is maximizing some preference relation.

Proposition:

Assume that C is a choice function with a domain containing at least all subsets of X of size 2 or 3. If C satisfies $*$, then there is a preference $\succsim$ on X so that $C = C_{\succsim}$.

Proof:

Define $\succsim$ by $x \succsim y$ if $x = C(\{x, y\})$.

Let us first verify that the relation $\succsim$ is a preference relation.

Completeness: Follows from the fact that $C(\{x, y\})$ is always well defined.

Transitivity: If $x \succsim y$ and $y \succsim z$, then $C(\{x, y\}) = x$ and $C(\{y, z\}) = y$. If $C(\{x, z\}) \neq x$ then $C(\{x, z\}) = z$. By $*$ and $C(\{x, z\}) = z$, $C(\{x, y, z\}) \neq x$. By $*$ and $C(\{x, y\}) = x$, $C(\{x, y, z\}) \neq y$, and by $*$ and $C(\{y, z\}) = y$, $C(\{x, y, z\}) \neq z$. A contradiction to $C(\{x, y, z\}) \in \{x, y, z\}$.

We still have to show that $C(B) = C_{\succsim}(B)$. Assume that $C(B) = x$ and $C_{\succsim}(B) \neq x$. That is, there is $y \in B$ so that $y \succ x$. By definition of $\succsim$, this means $C(\{x, y\}) = y$, contradicting $*$.

What Is an Alternative

Some of the cases where rationality is violated can be attributed to the incorrect specification of the space of alternatives. Consider the following example taken from Luce and Raiffa (1957): A diner in a restaurant chooses *chicken* from the menu {*steak tartare*, *chicken*} but chooses *steak tartare* from the menu {*steak tartare*, *chicken*, *frog legs*}. At first glance it seems that he is not rational (since his choice conflicts with $*$). Assume that the motivation for the choice is that the existence of *frog legs* is an indication of the quality of the chef. If the dish *frog legs* is on the menu, the cook must then be a real expert, and the decision maker is happy ordering *steak tartare*, which requires expertise to make. If the menu lacks *frog legs*, the decision maker does not want to take the risk of choosing *steak tartare*.

Rationality is "restored" if we make the distinction between "*steak tartare* served in a restaurant where *frog legs* are also on the menu (and the cook must then be a real chef)" and "*steak tartare* in a restaurant where *frog legs* are not served (and the cook is likely a novice)." Such a distinction makes sense since the *steak tartare* is not the same in the two choice sets.

Note that if we define an alternative to be (a, A), where a is a physical description and A is the choice problem, any choice function C can be rationalized by a preference relation satisfying $(C(A), A) \succsim (a, A)$ for every $a \in A$.

The lesson to be learned from the above discussion is that care must be taken in specifying the term "alternative." An alternative a must have the same meaning for every A which contains a.

Choice Functions as Internal Equilibria

The choice function definition we have been using requires that a single element be assigned to each choice problem. If the decision maker follows the rational-man procedure using a preference relation with indifferences, the previously defined induced choice function $C_{\succsim}(A)$ might be undefined because for some choice problems there would be more than one optimal element. This is one of the reasons that in some cases we use the alternative following concept to model behavior.

A *choice correspondence* C is required to assign to every nonempty $A \subseteq X$ a nonempty *subset* of A, that is, $\emptyset \neq C(A) \subseteq A$. According to our interpretation of a choice problem, a decision maker has to select a unique element from every choice set. Thus, $C(A)$ cannot be interpreted as the choice made by the decision maker when he has to make a choice from A. The revised interpretation of $C(A)$ is the *set* of all elements in A that are satisfactory in the sense that if the decision maker is about to make a decision and choose $a \in C(A)$, he has no desire to move away from it. In other words, a choice correspondence reflects an "internal equilibrium": If the decision maker facing A considers an alternative outside $C(A)$, he will continue searching for another alternative. If he happens to consider an alternative inside $C(A)$, he will take it.

Given a preference relation $\succsim$ we define the induced choice function (assuming it is never empty) as $C_{\succsim}(A) = \{x \in A \mid x \succsim y \; for \; all \; y \in A\}$.

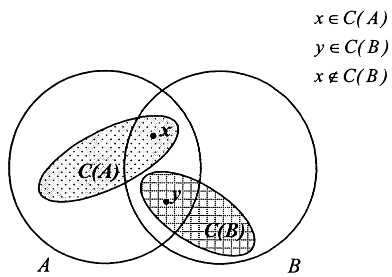

Figure 3.2
Violation of the weak axiom.

When $x, y \in A$ and $x \in C(A)$ we say that x is revealed to be at least as good as y. If, in addition, $y \notin C(A)$ we say that x is revealed to be strictly better than y. Condition $*$ is now replaced by the condition WA which requires that if x is revealed to be at least as good as y then y is not revealed to be strictly better than x.

The Weak Axiom of Revealed Preference (WA):
We say that C satisfies WA if whenever $x, y \in A \cap B$, $x \in C(A)$ and $y \in C(B)$, it is also true that $x \in C(B)$ (fig. 3.2).

Note that conditions $*$ and WA are equivalent for choice functions. Note also that for the next proposition, we could make do with a weaker version of WA which makes the same requirement only for any two sets $A \subset B$ where A is a set of two elements.

Proposition:
Assume that C is a choice correspondence with a domain that includes at least all subsets of size 2 or 3. Assume that C satisfies WA. Then, there is a preference $\succsim$ so that $C = C_{\succsim}$.

Proof:
Define $x \succsim y$ if $x \in C(\{x, y\})$. We will now show that the relation is a preference:

Completeness: Follows from $C(\{x, y\}) \neq \emptyset$.

Transitivity: If $x \succsim y$ and $y \succsim z$ then $x \in C(\{x,y\})$ and $y \in C(\{y,z\})$. If $x \notin C(\{x,z\})$, then $C(\{x,z\}) = \{z\}$. By WA, $x \notin C(\{x,y,z\})$ (if it were, x would be revealed to be as good as z while z is revealed to be strictly preferred to x.) Similarly, $y \notin C(\{x,y,z\})$ and $z \notin C(\{x,y,z\})$, contradicting the nonemptiness of $C(\{x,y,z\})$.

It remains to be shown that $C(B) = C_{\succsim}(B)$.

Assume that $x \in C(B)$ and $x \notin C_{\succsim}(B)$. That is, there is $y \in B$ so that it is not true that $x \succsim y$, or in other words, $C(\{x,y\}) = \{y\}$, thus contradicting WA.

Assume that $x \in C_{\succsim}(B)$ and $x \notin C(B)$. Let $y \in C(B)$. By WA $x \notin C(\{x,y\})$ and thus $C(\{x,y\}) = \{y\}$. Therefore $y \succ x$, contradicting $x \in C_{\succsim}(B)$.

The Satisficing Procedure

The fact that we can present any choice function satisfying condition $*$ (or WA) as an outcome of the optimization of some preference relation provides support for the view that the scope of microeconomic models is wider than simply models in which agents carry out explicit optimization. But, have we indeed expanded the scope of economic models?

Consider the following "decision scheme," named *satisficing* by Herbert Simon. Let $v : X \to \Re$ be a valuation of the elements in X, and let $v^* \in \Re$ be a threshold of satisfaction. Let O be an ordering of the alternatives in X. Given a set A, the decision maker arranges the elements of this set in a list $L(A,O)$ according to the ordering O. He then chooses the first element in $L(A,O)$ that has a v-value at least as large as v^*. If there is no such element in A, the decision maker chooses the last element in $L(A,O)$.

Let us show that the choice function induced by this procedure satisfies condition $*$. Assume that a is chosen from B and is also a member of $A \subset B$. The list $L(A,O)$ is obtained from $L(B,O)$ by eliminating all elements in $B - A$. If $v(a) \geq v^*$ then a is the first satisfactory element in $L(B,O)$, and is also the first satisfactory element in $L(A,O)$. Thus, a is chosen from A. If all elements in B are unsatisfactory, then a must be the last element in $L(B,O)$. Since A is a subset of B, all elements in A are unsatisfactory and a is the last element in $L(A,O)$. Thus, a is chosen from A.

Note, however, that even a "small" variation in this scheme can lead to a variation of the procedure such that it no longer satisfies $*$. For example:

Satisficing using two orderings: Let X be a population of university graduates who are potential candidates for a job. Given a set of actual candidates, count their number. If the number is smaller than 5, order them alphabetically. If the number of candidates is above 5, order them by their social security number. Whatever ordering is used, choose the first candidate whose undergraduate average is above 85. If there are none, choose the last student on the list.

Condition $*$ is not satisfied. It may be that a is the first candidate with a satisfactory grade in a long list of students ordered by their social security numbers. Still, a might not be the first candidate with a satisfactory grade on a list of only three of the candidates appearing on the original list when they are ordered alphabetically.

To summarize, the satisficing procedure, though it is stated in a way that seems unrelated to the maximization of a preference relation or utility function, can be described as if the decision maker maximizes a preference relation. I know of no other examples of interesting general schemes for choice procedures that satisfy $*$ other than the "rational man" and the satisficing procedures. However, later on, when we discuss consumer theory, we will come across several other appealing examples of demand functions that can be rationalized though they appear to be unrelated to the maximization of a preference relation.

Psychological Motives Not Included within the Framework

The more modern attack on the standard approach to modeling economic agents comes from psychologists, notably from Amos Tversky and Daniel Kahneman. They have provided us with beautiful examples demonstrating not only that rationality is often violated, but that there are systematic reasons for the violation resulting from certain elements within our decision procedures. Here are a few examples of this kind that I find particularly relevant.

Framing

The following experiment (conducted by Tversky and Kahneman 1986) demonstrates that the way in which alternatives are framed may affect decision makers' choices. Subjects were asked to imagine being confronted by the following choice problem:

An outbreak of disease is expected to cause 600 deaths in the US. Two mutually exclusive programs are expected to yield the following results:

a. 400 people will die.
b. With probability 1/3, 0 people will die and with probability 2/3, 600 people will die.

In the original experiment, a different group of subjects was given the same background information and asked to choose from the following alternatives:

c. 200 people will be saved.
d. With probability 1/3, all 600 will be saved and with probability 2/3, none will be saved.

While 78% of the first group chose b, only 28% of the second group chose d. These are "problematic" results since by any reasonable criterion a and c are identical alternatives, as are b and d. Thus, the choice from $\{a, b\}$ should be consistent with the choice from $\{c, d\}$.

Both questions were presented in the above order to $1,200$ students taking Game Theory courses with the result that 74% chose b and 49% chose d. It seems plausible that many students kept in mind their answer to the first question while responding to the second one and therefore the level of incosistency was reduced. Nonetheless, a large proportion of students gave different answers to the two problems, which makes the findings even more problematic.

Overall, the results expose the sensitivity of choice to the framing of the alternatives. What is more basic to rational decision making than taking the same choice when only the manner in which the problems are stated is different?

Simplifying the Choice Problem and the Use of Similarities

The following experiment was also conducted by Tversky and Kahneman. One group of subjects was presented with the following choice problem:

Choose one of the two roulette games a or b. Your prize is the one corresponding to the outcome of the chosen roulette game as specified in the following tables:

(a)	Color	White	Red	Green	Yellow
	Chance %	90	6	1	3
	Prize \$	0	45	30	−15

(b)	Color	White	Red	Green	Yellow
	Chance %	90	7	1	2
	Prize \$	0	45	−10	−15

A different group of subjects was presented the same background information and asked to choose between:

(c)	Color	White	Red	Green	Blue	Yellow
	Chance %	90	6	1	1	2
	Prize \$	0	45	30	−15	−15

and

(d)	Color	White	Red	Green	Blue	Yellow
	Chance %	90	6	1	1	2
	Prize \$	0	45	45	−10	−15

In the original experiment, 58% of the subjects in the first group chose a, while nobody in the second group chose c. When the two problems were presented, one after the other, to about 1,000 students, 49% chose a and 5% chose c. Interestingly, the median response time among the students who answered a was 55 seconds, whereas the median response time of the students who answered b was 91 seconds.

The results demonstrate a common procedure people practice when confronted with a complicated choice problem. We often transfer the complicated problem into a simpler one by "canceling" similar elements. While d clearly dominates c, the comparison between a and b is not as easy. Many subjects "cancel" the probabilities of Yellow and Red and are left with comparing the prizes of Green, a process that leads them to choose a.

Incidentally, several times in the past, when I presented these choice problems in class, I have had students (some of the best students, in fact) who chose c. They explained that they identified the second problem with the first and used the procedural rule: "I chose a from $\{a, b\}$. The alternatives c and d are identical to the alternatives a and b, respectively. It is only natural then, that I choose c from $\{c, d\}$." This observation brings to our attention a hidden facet of the rational-man model. The model does not allow a decision maker to employ a rule such as: "In the past I chose x from B. The choice problems A and B are similar. Therefore, I shall choose x from A."

Reason-Based Choice

Making choices sometimes involves finding reasons to pick one alternative over the others. When the deliberation involves the use of reasons strongly associated with the problem at hand ("internal reasons"), we often find it difficult to reconcile the choice with the rational man paradigm.

Imagine, for example, a European student who would choose *Princeton* if allowed to choose from $\{Princeton,\ LSE\}$ and would choose *LSE* if he had to choose from $\{Princeton,\ Chicago,\ LSE\}$. His explanation is that he prefers an American university so long as he does not have to choose between American schools—a choice he deems harder. Having to choose from $\{Princeton,\ Chicago,\ LSE\}$, he finds it difficult deciding between *Princeton* and *Chicago* and therefore chooses not to cross the Atlantic. His choice does not satisfy $*$, not because of a careless specification of the alternatives (as in the restaurant's menu example discussed previously), but because his reasoning involves an attempt to avoid the difficulty of making a decision.

Another example follows Huber, Payne, and Puto (1982):

Let $a = (a_1, a_2)$ be "a holiday package of a_1 days in Paris and a_2 days in London." Choose one of the four vectors $a = (7, 4)$, $b = (4, 7)$, $c = (6, 3)$, and $d = (3, 6)$.

All subjects in the experiment agreed that a day in Paris and a day in London are desirable goods. Some of the subjects were requested to choose between the three alternatives a, b, and c; others had to choose between a, b, and d. The subjects exhibited a clear tendency toward choosing a out of the set $\{a, b, c\}$ and choosing b out of the set $\{a, b, d\}$.

A related experiment is reported in Shafir, Simonson and Tversky (1993). A group of subjects was asked to imagine having to choose between a camera priced \$170 and a better camera, by the same producer, which costs \$240. Another group of subjects was asked to imagine having to choose between three cameras - the two described above and a third, much more sophisticated camera, priced at \$470. The addition of the third alternative significantly increased the proportion of subjects who chose the \$240 camera. The common sense explanation for this choice is that subjects faced a conflict between two desires, to buy a better camera and to pay less. They resolved the conflict by choosing the "compromise alternative."

To conclude, decision makers look for reasons to prefer one alternative over the other. Typically, making decisions by using "external reasons" (which do not refer to the properties of the choice set) will not cause violations of rationality. However, applying "internal reasons" such as "I prefer the alternative a over the alternative b since a clearly dominates the other alternative c while b does not" might cause conflicts with condition $*$.

Mental Accounting

The following intuitive example is taken from Kahneman and Tversky (1984). Members of one group of subjects were presented with the following question:

1. Imagine that you have decided to see a play and paid the admission price of \$10 per ticket. As you enter the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay \$10 for another ticket?

Members of another group were asked to answer the following question:

2. Imagine that you have decided to see a play where the admission is \$10 per ticket. As you arrive at the theater, you discover that you have lost a \$10 bill. Would you still pay \$10 for a ticket for the play?

If the rational man only cares about seeing the play and his wealth, he should realize that there is no difference between the consequence of replying "Yes" to question 1 and replying "Yes" to question 2 (in both cases he will own a ticket and will be poorer by \$20). Similarly, there is no difference between the consequence of replying "No" to question 1 and replying "No" to question 2. Thus, the rational man should give the same answer to both questions. Nonetheless, only 46% said they would buy another ticket after they had lost the first one while 88% said they would buy a ticket after losing the banknote. It is likely that in this case subjects have conudcted a calculation where they compared the "mental price" of a ticket to its subjective value. Many of those who decided not to buy another ticket after losing the first one attributed a price of $20 to the ticket rather than $10. This example demonstrates that deicison makers may conduct "mental calculations" which are inconsistent with rationality.

Bibliographic Notes

Recommended readings. Kreps 1990, 24–30; Mas-Colell et al. 1995, chapter 1 C,D.

An excellent book on the lecture's subject is Kreps (1988). For the sources of consistency in choice and revealed preference assumptions, see Samuelson (1948), Houthakker (1950), and Richter (1966). Simon (1955) is the source of the discussion of satisficing. For a discussion of the bounded rationality approach to choice, see Rubinstein (1998). Sen (1993) provides a more philosophical discussion of the subject of this chapter. An excellent introduction to the Dutch Books arguments is Yaari (1985). Kahneman and Tversky (2000) is a definitive textbook on the psychological criticism of the economic approach to rationality. Rabin (1998) surveys the modern economics and psychology approach.

Problem Set 3

Problem 1. (*Easy*)
The following are descriptions of decision making procedures. Discuss whether the procedures can be described in the framework of the choice model discussed in this lecture and whether they are compatible with the "rational man" paradigm.

a. The decision maker has in mind a ranking of all alternatives and chooses the alternative that is the worst according to this ranking.
b. The decision maker chooses an alternative with the intention that another person will suffer the most.
c. The decision maker asks his two children to rank the alternatives and then chooses the alternative that has the best average ranking.
d. The decision maker has an ideal point in mind and chooses the alternative that is closest to the ideal point.
e. The decision maker looks for the alternative that appears most often in the choice set.
f. The decision maker always selects the first alternative that comes to his attention.
g. The decision maker searches for someone he knows who will choose an action that is feasible for him.
h. The decision maker orders all alternatives from left to right and selects the median.

Problem 2. (*Moderately difficult*)
Let us say that you have to make a choice from a set A. Consider the following two paths which lead to a choice: (a) you make a choice from the entire set or (b) you first partition A into the subsets A_1 and A_2, then make a selection from each of these sets and finally make a choice from the two selected elements.

a. Formulate a "path independence" property.
b. Show that the rational decision maker satisfies the property.
c. Find examples of choice procedures that do not satisfy this property.
d. Show that if a (single-valued) choice function satisfies path independence, then it is consistent with rationality.
e. Assume that C is a (multivalued) choice correspondence satisfying path independence. Can it be rationalized by a preference relation?

Problem 3. (*Easy*)
Check whether the following two choice correspondences satisfy WA:

$C(A) = \{x \in A|$ *the number of* $y \in X$ *for which* $V(x) \geq V(y)$ *is at least* $|X|/2\}$, and if the set is empty then $C(A) = A$.

$D(A) = \{x \in A|$ *the number of* $y \in A$ *for which* $V(x) \geq V(y)$ *is at least* $|A|/2$ $\}$.

Problem 4. (*Moderately difficult*)
Consider the following choice procedure. A decision maker has a strict ordering $\succsim$ over the set X and, separately, he assigns to each $x \in X$ a natural number $class(x)$ interpreted as the "class" of x. Given a choice problem A he chooses the element in A that is the best among those elements in A, that belong to the most common class in A (that is, the class that appears in A most often). If there is more than one most common class, he picks the best element from the members of A that belong to a most common class with the highest class number.

a. Is the procedure consistent with the "rational man" paradigm?
b. Can every choice function be "explained" as an outcome of such a procedure?

Problem 5. (*Moderately difficult.* Based on Kalai, Rubinstein, and Spiegler 2002)
Consider the following two choice procedures. Explain the procedures and try to persuade a skeptic that they "make sense." Determine for each of them whether they are consistent with the rational-man model.

a. The primitives of the procedure are two numerical (one-to-one) functions u and v defined on X and a number v^*. For any given choice problem A, let $a^* \in A$ be the maximizer of u over A, and let b^* be the maximizer of v over A. The decision maker chooses a^* if $v(a^*) \geq v^*$ and chooses b^* if $v(a^*) < v^*$.
b. The primitives of the procedure are two numerical (one-to-one) functions u and v defined on X and a number u^*. For any given choice problem A, the decision maker chooses the element $a^* \in A$ that maximizes u if $u(a^*) \geq u^*$, and v if $u(a^*) < u^*$.

Problem 6. (*Moderately difficult,* See Rubinstein and Salant 2006)
The standard economic choice model assumes that choice is made from a *set*. Let us construct a model where the choice is assumed to be from a *list*. (Note that the list $< a, b >$ is distinct from $< a, a, b >$ and $< b, a >$).

Let X be a finite grand set. A *list* is a nonempty finite vector of elements in X. In this problem, consider a *choice function* C to be a function that assigns to each vector $L =< a_1, \ldots, a_K >$ a single element from $\{a_1, \ldots, a_K\}$. Let $< L_1, \ldots, L_m >$ be the concatenation of the m lists $L_1, \ldots, L_m$. (Note that if the length of L_i is k_i , the length of the concatenation is $\Sigma_{i=1,\ldots,m} k_i$). We say that L' *extends* the list L if there is a list M such that $L' =< L, M >$.

We say that a choice function C satisfies property I if for all $L_1, \ldots, L_m$ $C(< L_1, \ldots, L_m >) = C(< C(L_1), \ldots, C(L_m) >)$.

a. Interpret property I. Give two examples of choice functions that satisfy I and two examples of choice functions which do not.
b. Define formally the following two properties of a choice function:
   Order Invariance: A change in the order of the elements of the list does not alter the choice.
   Duplication Invariance: Deleting an element that appears elsewhere in the list does not change the choice.
c. Characterize the choice functions that satisfy the following three properties together: Order Invariance, Duplication Invariance, and condition I.
d. Assume now that in the back of the decision maker's mind is a value function u defined on the set X (such that $u(x) \neq u(y)$ for all $x \neq y$). For any choice function C define $v_C(L) = u(C(L))$.

We say that C *accommodates a longer list* if whenever L' extends L, $v_C(L') \geq v_C(L)$ and there is a list L' which extends a list L for which $v_C(L') > v_C(L)$.

e. Give two interesting examples of choice functions that accommodate a longer list.
f. Give two interesting examples of choice functions which satisfy property I but which do not accommodate a longer list.

Review Problems

The following is a collection of questions I have given in exams during the last few years.

Problem 1 (Princeton 2002)

Consider a consumer with a preference relation in a world with two goods, X (an aggregated consumption good) and M ("membership in a club," for example), which can be consumed or not. In other words, the consumption of X can be any nonnegative real number, while the consumption of M must be either 0 or 1.

Assume that the consumer's preferences are strictly monotonic, continuous, and satisfy the following property:

> *Property E*: For every x there is y such that $(y, 0) \succ (x, 1)$ (that is, there is always some amount the aggregated consumption good that can compensate for the loss of membership).

1. Show that any consumer's preference relation can be represented by a utility function of the type
$$u(x, m) = \begin{cases} x & if \quad m = 0 \\ x + g(x) & if \quad m = 1 \end{cases} .$$
2. (Less easy) Show that the consumer's preference relation can also be represented by a utility function of the type
$$u(x, m) = \begin{cases} f(x) & if \quad m = 0 \\ f(x) + v & if \quad m = 1 \end{cases} .$$
3. Explain why continuity and strong monotonicity (without property E) are not sufficient for (1).
4. Calculate the consumer's demand function.
5. Taking the utility function to be of the form described in (1), derive the consumer's indirect utility function. For the case where the function g is differentiable, verify the Roy equality with respect to commodity M.

Problem 2 (Princeton 2001)

A consumer has to make his decision *before* he is informed whether a certain event, which is expected with probability α, happened or not. He assigns a vNM utility $v(x)$ to the consumption of the bundle x in case the event occurs, and a vNM utility $w(x)$ to the consumption of x should the event not occur. The consumer maximizes his expected utility. Both v and w satisfy the standard assumptions about the consumer. Assume also that v and w are concave.

1. Show that the consumer's preference relation is convex.
2. Find a connection between the consumer's indirect utility function and the indirect utility functions derived from v and w.
3. A new commodity appears on the market: "A discrete piece of information that tells the consumer whether the event occurred or not." The commodity can be purchased prior to the consumption decision. Use the indirect utility functions to characterize the demand function for the new commodity.

Problem 3 (Princeton 2001)

1. Define a formal concept for "$\succsim_1$ is closer to $\succsim_0$ than $\succsim_2$."
2. Apply your definition to the class of preference relations represented by $U_1 = tU_2 + (1-t)U_0$, where the function U_i represents $\succsim_i$ $(i = 0, 1, 2)$.
3. Consider the above definition in the consumer context. Denote by $x_k^i(p, w)$ the demand function of $\succsim_i$ for good k. Is it true that if $\succsim_1$ is closer to $\succsim_0$ than $\succsim_2$, then $|x_k^1(p, w) - x_k^0(p, w)| \le |x_k^2(p, w) - x_k^0(p, w)|$ for any commodity k and for every price vector p and wealth level w?

Problem 4 (Princeton 1997)

A decision maker forms preferences over the set X of all possible distributions of a population over two categories (like living in two locations). An element in X is a vector (x_1, x_2) where $x_i \ge 0$ and $x_1 + x_2 = 1$. The decision maker has two considerations in mind:

- He thinks that if $x \succsim y$, then for any z, the mixture of $\alpha \in [0,1]$ of x with $(1-\alpha)$ of z should be at least as good as the mixture of α of y with $(1-\alpha)$ of z.
- He is indifferent between a distribution that is fully concentrated in location 1 and one which is fully concentrated in location 2.

1. Show that the only preference relation that is consistent with the two principles is the degenerate indifference relation ($x \sim y$ for any $x, y \in X$).
2. The decision maker claims that you are wrong as his preference relation is represented by a utility function $|x_1 - 1/2|$. Why is he wrong?

Problem 5 (Princeton 2000. Based on Fishburn and Rubinstein 1982.)

Let $X = \Re^+ \times \{0, 1, 2, \ldots\}$, where (x, t) is interpreted as receiving \$$x$ at time t. A preference relation on X has the following properties:

- There is indifference between receiving \$0 at time 0 and receiving \$0 at any other time.
- For any positive amount of money, it is better to receive it as soon as possible.
- Money is desirable.
- The preference between (x, t) and $(y, t+1)$ is independent of t.
- Continuity.

1. Define formally the continuity assumption for this context.
2. Show that the preference relation has a utility representation.
3. Verify that the preference relation represented by the utility function $u(x)\delta^t$ (with $\delta < 1$ and u continuous and increasing) satisfies the above properties.
4. Formulize a concept "one preference relation is more impatient than another."
5. Discuss the claim that preferences represented by $u_1(x)\delta_1^t$ are more impatient than preferences represented by $u_2(x)\delta_2^t$ if and only if $\delta_1 < \delta_2$.

Problem 6 (Tel Aviv 2003)

Consider the following consumer problem. There are two goods, 1 and 2. The consumer has a certain endowment. Before the consumer are two "exchange functions": he can exchange x units of good 1 for $f(x)$ units of good 2, or he can exchange y units of good 2 for $g(y)$ units of good 1. Assume the consumer can only make one exchange.

1. Show that if the exchange functions are continuous and the consumer's preference relation satisfies monotonicity and continuity, then a solution to the consumer problem exists.
2. Explain why strong convexity of the preference relation is not sufficient to guarantee a unique solution if the functions f and g are increasing and convex.
3. Interpret the statement "the function f is increasing and convex"?
4. Suppose both functions f and g are differentiable and concave and that the product of their derivatives at point 0 is 1. Suppose also that the preference relation is strongly convex. Show that under these conditions, the agent will not find two different exchanges, one exchanging good 1 for good 2, and one exchanging good 2 for good 1, optimal.
5. Now assume $f(x) = ax$ and $g(y) = by$. Explain this assumption. Find a condition that will ensure it is not profitable for the consumer to make more than one exchange.

Problem 7 (Tel Aviv 1999)

Consider a consumer in a world with K goods and preferences satisfying the standard assumptions regarding the consumer. At the start of trade, the consumer is endowed with a bundle of goods e and he chooses the best bundle from the budget set $B(p, e) = \{x|px = pe\}$. The consumer's preference over bundles of goods can be represented by a utility function u. Define $V(p, e) = max\{u(x)|px = pe\}$.

1. Explain the meaning of the function V and show that $V(tp, e) = V(p, e)$ where t is any positive number.
2. Show that for every bundle e, the set of vectors p, such that $V(p, e) \le V(p^*, e)$, is convex.
3. Fix all prices but p_i, and all quantities in the initial bundle but w_i. Consider the two-dimensional space where the parameters on the axes are p_i and w_i. Show that the slope of the indifference

curve of V is $(x_i(p,w) - w_i)/p_i$ where $x(p,w)$ is the solution to the consumer's problem $B(p,w)$.

Problem 8 (Tel Aviv 1998)

A consumer with wealth $w = 10$ "must" obtain a book from one of three stores. Denote the prices at each store as p_1, p_2, p_3. All prices are below w in the relevant range. The consumer has devised a strategy: he compares the prices at the first two stores and obtains the book from the first store if its price is not greater than the price at the second store. If $p_1 > p_2$, he compares the prices of the second and third stores and obtains the book from the second store if its price is not greater than the price at the third store. He uses the remainder of his wealth to purchase other goods.

1. What is this consumer's "demand function"?
2. Does this consumer satisfy "rational man" assumptions?
3. Consider the function $v(p_1, p_2, p_3) = w - p_{i^*}$, where i^* is the store from which the consumer purchases the book if the prices are (p_1, p_2, p_3). What does this function represent?
4. Explain why $v(\cdot)$ is not monotonically decreasing in p_i. Compare with the indirect utility function of the classic consumer model.

Problem 9 (Tel Aviv 1999)

Tversky and Kahneman (1986) report the following experiment: each participant receives a questionnaire asking him to make two choices, one from $\{a, b\}$ and the second from $\{c, d\}$:

a. A sure profit of \$240.
b. A lottery between a profit of \$1000 with probability 25% and 0 with probability 75%.

c. A sure loss of \$750.
d. A lottery between a loss of \$1000 with probability 75% and 0 with probability 25%.

The participant will receive the sum of the outcomes of the two lotteries he chooses. Out of the participants 73% chose the combination a and d. What do you make of this result?

Problem 10 (Princeton 2000)

Consider the following social choice problem: a group has n members (n is odd) who must choose from a set containing 3 elements $\{A, B, L\}$, where A and B are prizes and L is the lottery which yields each of the prizes A and B with equal probability. Each member has a strict preference over the three alternatives that satisfies vNM assumptions. Show that there is a non-dictatorial social welfare function which satisfies the independence of irrelevant alternatives axiom (even the strict version I^*) and the Pareto axiom (Par). Reconcile this fact with Arrow's Impossibility Theorem.

Problem 11 (Tel Aviv 2003. Based on Gilboa and Schmeidler 1995.)

An agent must decide whether to do something, Y, or not to do it, N.

A history is a sequence of results for past events in which the agent chose Y; each result is either a success S or a failure F. For example, (S, S, F, F, S) is a history with five events in which the action was carried out. Two of them (events 3 and 4) ended in failure while the rest were successful.

The decision rule D is a function that assigns the decision Y or N to every possible history.

Consider the following properties of decision rules:

- $A1$ After every history that contains only successes, the decision rule will dictate Y, and after every history that contains only failures, the decision rule will dictate N.
- $A2$ If the decision rule dictates a certain action following some history, it will dictate the same action following any history that is derived from the first history by reordering its members. For example, $D(S, F, S, F, S) = D(S, S, F, F, S)$.
- $A3$ If $D(h) = D(h')$, then this will also be the decision following the concatenation of h and h'. (Reminder: The concatenation of $h = (F, S)$ and $h' = (S, S, F)$ is (F, S, S, S, F)).

1. For every $i = 1, 2, 3$, give an example of a decision rule that does not fulfill property Ai but does fulfill the other two properties.
2. Give an example of a decision rule that fulfills all three properties.
3. (Difficult) Characterize the decision rules that fulfill the three properties.

Problem 12 (NYU 2005, inspired by Chen, M.K., V. Lakshminarayanan and L. Santos (2005))

In an experiment, a monkey is given $m = 12$ coins which he can exchange for apples or bananas. The monkey faces m consecutive choices in which he gives a coin either to an experimenter holding a apples or another experimenter holding b bananas.

1. Assume that the experiment is repeated with different values of a and b and that each time the monkey trades the first 4 coins for apples and the next 8 coins for bananas.
   Show that the monkey's behavior is consistent with the classical assumptions of consumer behavior (namely, that his behavior can be explained as the maximization of a montonic, continuous and convex preference relation on the space of bundles).
2. Assume that it was later observed that when the monkey holds an arbitrary number m of coins, then, irrespective of the values of a and b, he exchanges the first 4 coins for apples and the remaining $m - 4$ coins for bananas. Is this behavior consistent with the rational consumer model?

Problem 13 (NYU 2005)

A consumer has classical preferences in a world of K goods. The goods are split into two categories, 1 and 2, of K_1 and K_2 goods respectively ($K_1 + K_2 = K$). The consumer receives two types of money: w_1 units of money which can only be exchanged for goods in the first category and w_2 units of money which can only be exchanged for goods in the second category.

Define the induced preference relation over the two-dimensional space (w_1, w_2). Show that these preferences are monotonic, continuous and convex.

Problem 12 (NYU 2005. Inspired by Chen, M.K., V.Lakshminarayanan and L.Santos 2005

In an experiment a monkey was given $m = 12$ coins. The monkey faces m consecutive choice. In each instance it gives one coin to one of two experimenters, one of whom is holding a apples and the other is holding b bananas.

1. Assume that the experiment is repeated with different values of a and b and that every time the monkey trades the first 4 coins for apples and then trades the next 8 coins for bananas. The experimenter claims that the monkey's choices confirm consumer theory. Show that the above monkey's behavior is indeed consistent with the classical assumptions of consumer behavior (namely, that his behavior can be explained as the maximization of a monotonic, continuous, convext preference relation on the space of bundles.)
2. Assume that later it was observed that when the monkey holds an arbitrary number m of coins, then independently of a and b, he exchanges the first 4 coins for apples and then exchanges the remaining $m - 4$ coins for bananas. Is this behavior consistent with the consumer model?

Problem 13 (NYU 2005)

A consumer lives in a world of K commodities. He holds classical preferences over those commodities. The goods are split into two categories, 1 and 2, of K_1 and K_2 goods, respectively ($K_1 + K_2 = K$.) The consumer receives two types of money: w_1 units of wealth which can be exchanged for goods in the first category only and w_2 units of wealth which can only be exchanged for goods in the second category.

Define the induced preference relation over the two-dimensional space (w_1, w_2). Show that those preferences are monotonic, continuous and convex.

Problem 14 (NYU 2005)

Let X be a finite set containing at least three elements. Let C be a choice correspondence. Consider the following axiom:

If $A, B \subseteq X$, $B \subseteq A$ and $C(A) \cap B \neq \emptyset$, then $C(B) = C(A) \cap B$.

1. Show that the axiom is equivalent to the existence of a preference relation $\succsim$ such that $C(A) = \{x \in A | x \succ a \text{forall} a \in A\}$.
2. Consider a weaker axiom:
   If $A, B \subseteq X$, $B \subseteq A$ and $C(A) \cap B \neq \emptyset$, then $C(B) \subseteq C(A) \cap B$.
   Is it sufficient for the above equivalence?

Problem 15 (NYU 2006)

Consider a consumer in a world of 2 commodities, who has to make choices from budget sets parametrized by (p, w), with the additional constraint that the consumption of good 1 is limited by some external bound $c \geq 0$. That is, in his world, a choice problem is a set for the form $B(p, w, c) = \{x | px \leq w \text{and} x_1 \leq c\}$. Denote by $x(p, w, c)$ the choice of the consumer from $B(p, w, c)$.

1. Assume $px(p, w, c) = w$ and that $x_1 = \min\{0.5w/p_1, c\}$. Show that this behavior is consistent with the assumption that the demand is derived from a maximization of some preference relation.
2. Assume $px(p, w, c) = w$ and that $x_1(p, w, c) = \min\{0.5c, w/p_1\}$. Show that this consumer's behavior is inconsistent with preference maximization.
3. Assume that the consumer chooses his demand by maximizing the utility function $u(x)$. Denote the indirect utility by $V(p, w, c) = u(x(p, w, c))$. Assume V is "well-behaved." Sketch the idea of how one can derive the demand function from the function V. Separate between the case that $\partial V/\partial c(p, w, c) > 0$ and the case that $\partial V/\partial c(p, w, c) = 0$.

Problem 16 (NYU 2006. Based on Rubinstein and Salant 2006)

Let X be a grand finite set. Consider a model where a choice problem is a pair (A, a) where A is a subset of X and $a \in A$ is interpreted as a default alternative.

A decision maker's behavior can depend on the default alternative and thus is described by a function $c^*(A, a)$ which assigns to each choice problem (A, a) an element in A.

Assume that c^* satisfies the following two properties:

A default bias: If $c^*(A, a) = x$ then $c^*(A, x) = x$.

Extended IIA: If $c^*(A, a) = x$ and $x \in B \subseteq A$, then $c^*(B, a) = x$.

1. Give two examples of functions c^* which satisfy the two properties.

   Define a relation $x \succ y$ if $c^*(\{x, y\}, y) = x$.

2. Show that the relation is asymmetric and transitive.
3. Explain why the relation may be incomplete.
4. Define a choice correspondence $C(A) = \{a | \text{thereexists} x \in A \text{suchthat} c^*(A, x) = a\}$, that is, $C(A)$ is the set of all elements in A which are chosen given some default alternative. Show that $C(A)$ is the set of all $\succ$ maximal elements and interpret this result.

Problem 17 (NYU 2006)

Consider a world with balls of K different colors. An object is called a bag and is specified by a vector $x = (x_1, .., x_K)$ (where x_k is a non-negative integer indicating the number of balls of color k.) For convenience denote by $n(x) = \sum x_k$ the number of balls in bag x.

An individual has a preference relation over bags of balls.

1. Suggest a context where it will make sense to assume that:
   i. For any integer λ, $x \sim \lambda x$.
   ii. If $n(x) = n(y)$ then $x \succsim y$ iff $x + z \succsim y + z$.
2. Show that any preference relation which is represented by $U(x) = \sum x_k v_k / n(x)$ for some vector of numbers $(v_1, .., v_k)$ satisfies the two axioms and interpret it.
3. Find a preference relation which satisfies the two properties which cannot be represented in the form suggested in (2).

Problem 18 (Tel Aviv 2006)

Imagine a consumer who lives in a world with $K + 1$ commodities, and behaves in the following manner: The consumer is characterized by a vector D, consisting of the commodities $1, .., K$. If he can purchase D, he will consume it and spend the rest of his income on commodity $K + 1$. If he is unable to purchased D, he will not consume commodity $K + 1$, and purchase the bundle tD $(t \leq 1)$ where t is the largest that he can afford.

1. Show that there exists a monotonic and convex preference relation which explains this pattern of behavior.
2. Show that there is no monotonic, convex and continuous preference relation that explains this pattern of behavior.

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