Introduction to Game Theory for Law and Philosophy Students
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Problem Set 2 (Expected Utility)

1. A decision maker is an expected utility maximizer.

a) His preferences on the prizes a, b, c and d are $a \succ b \succ c \succ d$.

He is indifferent between the certain prize b and the lottery which yields the prize a with probability 0.8 and the prize d with probability 0.2.

He is indifferent between the certain prize c and the lottery which yields the prize a with probability 0.3 and the prize d with probability 0.7.

What is the probability α which make him indifferent between the certain prize c and the lottery which yields b with probability α and yields d with probability $1-\alpha$?

b) Assume that the decision maker is characterized by the vNM numbers:
$v(a) = 4,\ v(b) = 3,\ v(c) = 2$ and $v(d) = 1$
Would he be "different" if wer characterized by the vNM numbers:
$w(a) = 7,\ w(b) = 4,\ w(c) = 1$ and $w(d) = -2$?
(Try to generalize this observation.)

2. A decision maker is very optimistic. Whenever he faces a lottery he behaves as if he believes that chance will be in his favor and that he will win the best prize which can be realized (according to his preferences) with strictly positive probability. Is he rational in the economic sense? Do his preferences satisfy the Independence axiom?

3. A parent of two children has bought one gift. He is indifferent between giving the gift to one child or the other but strongly prefers tossing a fair coin to determine which child will receive the gift.

In what sense does this example violate expected utility theory?

4. A decision maker must choose an action from set A if the contingency a is realized and from set B if the contingency b is realized. The probabilities of the two contingencies are p_a and p_b respectively (where $p_a + p_b = 1$).

Assume that the decision maker makes a contingent decision before the realization of a or b is known (in other words, he makes a plan of the type "I will take the action x in A if a happens and the action y in B if b happens"). Show that if he is an expected utility maximizer he will not change his mind once he learns whether a or b has happened (this property is called time consistency.)

5. Do you recall your choice between

Roulette A

Chance	90%	6%	1%	3%
Prize	$0	$45	$30	$-15

and the Roulette B

Chance	90%	7%	1%	2%
Prize	$0	$45	-10	$-15

A majority of people choose A. What do you have to say about this choice?