Introduction to Game Theory for Law and Philosophy Students
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Problem Set 6 (Zero Sum and Bayesian Nash Equilibrium)

0. Fun Reading

Read — http://slate.msn.com/id/2108640/ Does it ring bells?

1. A zero sum game

Here is a very simple game (it is too simple, but I made it as simple as possible in order to make your life easy):

Let us analyze the following board game. In the game two players, player 1 (the "X" player) and player 2 (the "+" player). Players alternate turns in putting an X or an + in an empty square in a 3x3 board.

Player 1 wins the game if he succeeds to put three X's in one row and player 2 wins the game if he succeeds to put three +'s in one column. If none of them makes it the play of the game is declared a draw.

a) Show that the game has a Nash equilibrium in which the outcome of the game is a draw.

b) Show that each player has a strategy which can guarantee a draw.

2. Information might be harmful!

Consider the following two-player game:

Player 1 must choose between U and D and Player 2 between L, M and R.

The payoffs depend on the state of the world which is either s_1 or s_2.

s_1	L	M	R
U	2,3	2,0	2,2
D	4,4	0,0	0,6

s_2	L	M	R
U	2,3	2,2	2,0
D	4,4	0,6	0,0

The state of the world is known only to player 1. Player 2 only knows that the probability of each state is $1/2$. In equilibrium we have to state the strategy of player 1 if the state is s_1, player 1's strategy if the state is s_2 and the strategy of player 2.

a) Show that the only equilibrium (with pure strategies) of the game involves both types of player 1 choosing D and player 2 choosing L.

b) Show that were player 2 to also know the state of the world, the expected equilibrium payoffs to both players would be lower!

(Compare with the discussion

in http://arielrubinstein.tau.ac.il/articles/machshavot.pdf)

3. Two-Envelope Exchange

The following problem is related to (but not identical to) the Two-Envelope Exchange Paradox (see for example, http://members.aol.com/kiekeben/envelope.html)

Someone draws a number 1,2,4,8,16 or 32. Denote the number by x. He then puts $\$x$ in one envelope and $\$2x$ in the other. Next, he randomly assigns one envelope to player 1 and the other to player 2. Once the envelopes have been handed out, each player opens his own envelope but does not reveal the contents to the other player. Next, each player must simultaneously decide whether he wishes to exchange envelopes. An exchange will be made only if both parties agree to the exchange. To persuade the players to make the exchange (and probably even to pay a few cents to the organizer...) it is claimed that if a player's envelope contains say 4$, then the other player holds an envelope containing either 2$ or $8 with equal probability and thus it is worthwhile to make the exchange.

Construct a game which describes the situation and show that in equilibrium, there will be no exchange.

4. Screening

At the outskirts of a remote village there is a bakery. The owner has just hired a baker and it is not known in the village whether he knows how to properly prepare sufganiot. The owner knows the quality of his sufganiot. Making the sufganiot entails expenses. A typical consumer has to decide whether to make the effort to go to the bakery. If he decides to go there, he will taste one before buying. If the sufganiut are good he will buy a large number; if not, he will only buy a few. The consumer would hate to go to the bakery if it does not sell sufganiot and would really hate the possibility of going there to buy bad sufganiot.

G	**Bake**	**Not**
go to shop	**10, 10**	**–5, 0**
does not	**0, –5**	**0, 0**

B	**Bake**	**Not**
go to shop	**–20, 5**	**–5, 0**
does not	**0, –5**	**0, 0**

Construct a non-trivial Nash equilibrium (namely not the one in which the consumer decides not to go to the bakery and the bakery owner decides not to make sufganiot).