Introduction to Game Theory for Law and Philosophy Students
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Problem Set 7 (Extensive Games)

0. Last moment extra problem

Show that in all Nash equilibibira of the centepede game (discssued in class) player 1 stops the game right away

1. Match Game

In front of two players there is a pile of n matches. Each player in turn takes 1 or 2 matches from the pile. The last player to take a match loses the game.

Draw the game for the case $n = 5$.

Who is the winner of this game (as a function of n)?

2. **The chain store paradox**

Initially, a chain of stores operates in T cities numbered $1, 2, \ldots, T$ and makes $\$4$ in each city. In each of the cities there is one competitor who has to decide whether to enter the market or not. If the competitor does enter the market, then the chain store must decide whether:

(F) to fight the competitor (by conducting an aggressive price war) which will lead to a loss of $\$3$ for both the chain store and the competitior;

or

(A) to accommodate the competitor in which case both will earn $2.

Each of the competitors must decide whether to enter the market or not on a specific date. Without loss of generality, assume that the dates for decision making are ordered according to the players' numbers $1, 2, \ldots, T$.

When making a decision a player knows exactly what has happened previously. The chain is interested in maximizing the sum of its profits in the T cities while each competitor is interested only in its own profits.

Draw the game for $T = 2$.

Show that the game has numerous Nash equilibria but only one subgame perfect equilibrium (regardless of T).

3. **Auction**

Two potential buyers are competing for one indivisible item worth $\$10$.

Buyer 1 has $\$9$ and buyer 2 has only $\$6$. The seller declares that he will not accept any offer lower than $\$3$.

The buyers take turns in bidding. All bids are in integers and cannot exceed $\$10$. Player can bid above their cash holding but if they win they are punished severely (worst event for the bidder). When one of them does not raise the bid, the auction is over and the other player gets the item for the last bid he made.

Show that in all subgame perfect equilibria of the game player 1 gets the item.

Show that there is a subgame perfect equilibrium in which the item is sold for less than $\$6$.

Show that there is a subgame perfect equilibrium in which the item is sold for $\$8$.