LECTURE 3

Choice

Choice Functions

Until now we have avoided any reference to behavior. We have talked about preferences as a summary of the decision maker's mental attitude toward a set of alternatives. But economics is about action, and therefore we now move on to modeling "agent behavior". By a description of agent behavior we will refer not only to his actual choices, made when he confronts a certain problem, but to a full description of his behavior in all scenarios we imagine he might confront in a certain context.

Consider a *grand set* X of possible alternatives. We view a *choice problem* as a nonempty subset of X, and we refer to a choice from $A \subseteq X$ as specifying one of A's members.

Modeling a choice scenario as a set of alternatives implies assumptions of rationality according to which the agent's choice does not depend on the way the alternatives are presented. For example, if the alternatives appear in a list, he ignores the order in which they are presented and the number of times an alternative appears in the list. If there is an alternative with a default status, he ignores that as well. As a rational agent he considers only the set of alternatives available to him.

In some contexts, not all choice problems are relevant. Therefore we allow that the agent's behavior be defined only on a set D of subsets of X. We will refer to a pair (X, D) as a *context*.

Example:

1. Imagine that we are interested in a student's behavior regarding his selection from the set of universities to which he has been admitted. Let $X = \{x_1, \ldots, x_N\}$ be the set of all universities with which the student is familiar. A choice problem A is interpreted as the set of universities to which he has been admitted. If the fact that the student was admitted to some subset of universities does not imply his admission outcome for other universities, then D contains the $2^N - 1$ nonempty subsets of X. But if, for example, the universities are listed according to difficulty in

being admitted (x_1 being the most difficult) and if the fact that the student is admitted to x_k means that he is admitted to all less "prestigious" universities, that is, to all x_l with $l > k$, then D will consist of the N sets $A_1, \ldots, A_N$ where $A_k = \{x_k, \ldots, x_N\}$.

2. Imagine a scenario in which a decision maker is choosing whether to remain with the status quo s or choose an element in some set Y. We formalize such a scenario by defining $X = Y \cup \{s\}$ and identifying the domain of the choice function D as the set of all subsets of X that contain s.

We think about an agent's behavior as a hypothetical response to a questionnaire that contains questions of the following type, one for each $A \in D$:

Q(A): Assume you must choose from a set of alternatives A. Which alternative do you choose?

A permissible response to this questionnaire requires that the agent select a unique element in A for every question Q(A). We implicitly assume that the agent cannot give any other answer such as "I choose either a or b"; "the probability of my choosing $a \in A$ is $p(a)$"; or "I don't know".

Formally, given a context (X, D), a *choice function* C assigns to each set $A \in D$ a unique element of A with the interpretation that $C(A)$ is the chosen element from the set A.

Our understanding is that a decision maker behaving in accordance with the function C will choose $C(A)$ if he has to make a choice from a set A. This does not mean that we can actually observe the choice function. At most we might observe some particular choices made by the decision maker in some instances. Thus, a choice function is a description of hypothetical behavior.

Rational Choice Functions

It is typically assumed in economics that choice is an outcome of "rational deliberation". Namely, the decision maker has in mind a preference relation $\succsim$ on the set X and, given any choice problem A in D, he chooses an element in A that is $\succsim$ optimal. Assuming that it is well defined, we define the *induced choice function* $C_{\succsim}$ as the function that assigns to every nonempty set $A \in D$ the $\succsim$-best element of A. Note that the preference relation is fixed, that is, it is independent of the choice set being considered.

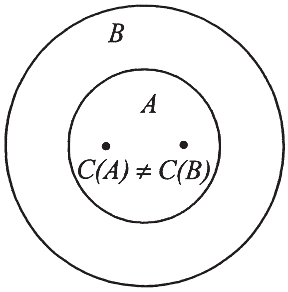

Figure 3.1
Violation of condition α.

Rationalizing

Economists were often criticized for making the assumption that decision makers maximize a preference relation. The most common response to this criticism is that we don't really need this assumption. All we need to assume is that the decision maker's behavior can be described *as if* he were maximizing some preference relation.

Let us state this "economic defense" more precisely. We will say that a choice function C *can be rationalized* if there is a preference relation $\succsim$ on X so that $C = C_{\succsim}$ (i.e., $C(A) = C_{\succsim}(A)$ for any A in the domain of C).

We will now identify a condition under which a choice function can indeed be presented as if derived from some preference relation (i.e., can be rationalized).

Condition α:

We say that C satisfies condition α if for any two problems $A,B \in D$, if $A \subset B$ and $C(B) \in A$, then $C(A) = C(B)$. (See fig. 3.1.)

Note that if $\succsim$ is a preference relation on X, then $C_{\succsim}$ (defined on a set of subsets of X that have a single most preferred element) satisfies condition α.

As an example of a choice procedure that does not satisfy condition α, consider the *second-best procedure*: the decision maker has in mind an ordering $\succsim$ of X (i.e., a complete, asymmetric and transitive binary relation) and for any given choice problem set A chooses the element from A, which is the $\succsim$-maximal from the nonoptimal alternatives. If A contains all the elements in B besides the $\succsim$-maximal, then $C(B) \in A \subset B$ but $C(A) \neq C(B)$.

We will show now that condition α is a sufficient condition for a choice function to be formulated *as if* the decision maker is maximizing some preference relation.

Proposition:

Assume that C is a choice function with a domain containing at least all subsets of X of size 2 or 3. If C satisfies condition α, then there is a preference $\succsim$ on X so that $C = C_{\succsim}$.

Proof:

Define $\succsim$ by $x \succsim y$ if $x = C(\{x, y\})$.

Let us first verify that the relation $\succsim$ is a preference relation.

Completeness: Follows from the fact that $C(\{x, y\})$ is always well defined.

Transitivity: If $x \succsim y$ and $y \succsim z$, then $C(\{x, y\}) = x$ and $C(\{y, z\}) = y$. If $C(\{x, z\}) \neq x$, then $C(\{x, z\}) = z$. By condition α and $C(\{x, z\}) = z$, $C(\{x, y, z\}) \neq x$. By condition α and $C(\{x, y\}) = x$, $C(\{x, y, z\}) \neq y$, and by condition α and $C(\{y, z\}) = y$, $C(\{x, y, z\}) \neq z$. A contradiction to $C(\{x, y, z\}) \in \{x, y, z\}$.

We still have to show that $C(B) = C_{\succsim}(B)$. Assume that $C(B) = x$ and $C_{\succsim}(B) \neq x$. That is, there is $y \in B$ so that $y \succ x$. By definition of $\succsim$, this means $C(\{x, y\}) = y$, contradicting condition α.

Following is a different version of the above proposition.

Proposition:

Let C be a choice function with a domain D satisfying that if $A, B \in D$, then $A \cup B \in D$. If C satisfies condition α, then there is a preference relation $\succsim$ on X such that $C = C_{\succsim}$.

Proof:

Define a binary relation as xRy if there is a set $A \in D$ such that $y \in A$ and $c(A) = x$. Note that R is not necessarily complete. We will see that the relation R does not have cycles.

The relation is antisymmetric. If xRy and yRx (for some $x \neq y$), then there is $A \in D$ containing y such that $C(A) = x$ and there is $B \in D$ containing x such that $C(B) = y$. The set $A \cup B$ is a member of D. By condition α both are true $C(A \cup B) = C(A) = x$ and $C(A \cup B) = C(B) = y$, a contradiction.

The relation is transitive. If xRy and yRz, then there is $A \in D$ containing y such that $C(A) = x$ and there is $B \in D$ containing z such that $C(B) = y$. The set $A \cup B$ is a member of D. The element $C(A \cup B)$ is in either A or B and thus by condition α it is either x or y. It is not y since if $C(A \cup B) = y \in A$ and by condition α, $C(A \cup B) = C(A) = y$. Thus, $C(A \cup B) = x$ and xRz.

A well-known proposition in Set Theory (see Problem 4 in Problem Set 1) guarantees that the acyclic relation R extends to a preference relation $\succsim$. By definition, $c(A) \succsim x$ for all $x \in A$ and thus it also follows that $c(A) \succsim x$ for all $x \in A$, which proves that $C_{\succsim} = C$.

Dutch Book Arguments

Some of the justifications for the assumption that choice is determined by "rational deliberation" are normative, that is, they reflect a perception that people should be rational in this sense and, if they are not, they should convert to reasoning of this type. One interesting class of arguments supporting this approach is referred to in the literature as "Dutch book arguments". The claim is that an economic agent who behaves according to a choice function that is not induced from maximization of a preference relation will not survive.

The following is a "sad" story about a monkey in a forest with three trees, a , b, and c. The monkey is about to pick a tree to sleep in. Assume that the monkey can assess only two alternatives at a time and that his choice function is $C(\{a, b\}) = b$, $C(\{b, c\}) = c$, $C(\{a, c\}) = a$. Obviously, his choice function cannot be derived from a preference relation over the set of trees. Assume that whenever he is on tree x it comes to his mind occasionally to jump to one of the other trees; namely, he makes a choice from a set $\{x, y\}$ where y is one of the two other trees. This induces the monkey to perpetually jump from one tree to another – not a particularly desirable mode of behavior in the jungle.

Another argument – which is more appropriate to human beings – is called the "money pump" argument. Assume that a decision maker behaves like the monkey with respect to three alternatives a, b, and c. Assume that, for all x and y, the choice $C(x, y) = y$ is strong enough so that whenever he is about to choose alternative x and somebody gives him the option to also choose y, he is ready to pay one cent for the opportunity to do so. Now, imagine a manipulator who presents the agent with the choice problem $\{a, b, c\}$. Whenever the decision maker is about to make the choice a, the manipulator allows him to revise his choice to b for one cent. Similarly, every time he is about to choose b or c, the manipulator sells him for one cent the opportunity to choose c or a accordingly. The decision maker will cycle through the intentions to choose a, b, and c until his pockets are emptied or until he learns his lesson and changes his behavior.

The above arguments are open to criticism. In particular, the elimination of patterns of behavior that are inconsistent with rationality require an environment in which the economic agent is indeed confronted with the above sequence of choice problems. The arguments are presented here as interesting ideas and not necessarily as convincing arguments for rationality.

What Is an Alternative

Some of the cases where rationality is violated can be attributed to the incorrect specification of the space of alternatives. Consider the following example taken from Luce and Raiffa (1957): a diner in a restaurant chooses *chicken* from the menu *steak tartare*, *chicken* but chooses *steak tartare* from the menu *steak tartare*, *chicken*, *frog legs*. At first glance it seems that he is not rational (since his choice conflicts with condition α). Assume that the motivation for the choice is that the existence of *frog legs* is an indication of the quality of the chef. If the dish *frog legs* is on the menu, the cook must then be a real expert, and the decision maker is happy ordering *steak tartare*, which requires expertise to make. If the menu lacks *frog legs*, the decision maker does not want to take the risk of choosing *steak tartare*.

Rationality is "restored" if we make the distinction between "*steak tartare* served in a restaurant where *frog legs* are also on the menu (and the cook must then be a real chef)" and "*steak tartare* in a restaurant where *frog legs* are not served (and the cook is likely a novice)". Such a distinction makes sense because the *steak tartare* is not the same in the two choice sets.

Note that if we define an alternative to be (a, A), where a is a physical description and A is the choice problem, any choice function C can be rationalized by a preference relation satisfying $(C(A), A) \succsim (a, A)$ for every $a \in A$.

The lesson to be learned from the above discussion is that care must be taken in specifying the term "alternative". An alternative a must have the same meaning for every choice problem A which contains a.

Choice Functions as Internal Equilibria

The choice function definition we have been using requires that a single element be assigned to each choice problem. If the decision maker follows the rational man procedure using a preference relation with indifferences, the previously defined induced choice function $C_{\succsim}(A)$ might be undefined because for some choice problems there would be more than one optimal element. This is one of the reasons that in some cases we use the alternative following concept to model behavior.

A *choice correspondence* C is required to assign to every nonempty $A \in D$ a nonempty *subset* of A, that is, $\emptyset \neq C(A) \subseteq A$. According to our interpretation of a choice problem, a decision maker has to select a unique element from every choice set. Thus, $C(A)$ cannot be interpreted as the choice made by the decision maker when he has to make a choice from A. The revised interpretation of $C(A)$ is the *set* of all elements in A that are satisfactory in the sense that if the decision maker is about to make a decision and choose $a \in C(A)$, he has no desire to move away from it. In other words, the induced choice correspondence reflects an "internal equilibrium": if the decision maker facing A considers an alternative outside $C(A)$, he will continue searching for another alternative. If he happens to consider an alternative inside $C(A)$, he will take it.

A related interpretation of $C(A)$ involves viewing it as the set of all elements in A that may be chosen under any of many possible particular circumstances not included in the description of the set A. Formally, let (A, f) be an extended choice set where f is the frame that accompanies the set A (like the default alternative or the order of the alternatives). Let $c(A, f)$ be the choice of the decision maker from the choice set A given the frame f. The (extended) choice function c induces a choice correspondence by $C(A) = \{x | x = c(A, f) \text{ for some } f\}$.

Given a preference relation $\succsim$ we define the induced choice correspondence (assuming it is never empty) as $C_{\succsim}(A) = \{x \in A \mid x \succsim y \ for\ all\ y \in A\}$.

When $x, y \in A$ and $x \in C(A)$, we say that x is revealed to be at least as good as y. If, in addition, $y \notin C(A)$, we say that x is revealed to be strictly better than y. Condition α is now replaced by condition WA, which requires that if x is revealed to be at least as good as y, then y is not revealed to be strictly better than x.

The Weak Axiom of Revealed Preference (WA):

We say that C satisfies WA if whenever $x, y \in A \cap B$, $x \in C(A)$, and $y \in C(B)$, it is also true that $x \in C(B)$ (fig. 3.2).

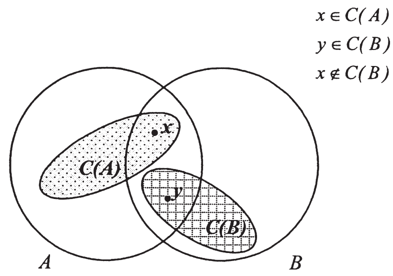

Figure 3.2
Violation of the weak axiom.

The Weak Axiom trivially implies two properties: *Condition* α: If $a \in A \subset B$ and $a \in C(B)$, then $a \in C(A)$. *Condition* β: If $a, b \in A \subset B$, $a \in C(A)$, and $b \in C(B)$, then $a \in C(B)$.

Notice that if $C(A)$ contains all elements that are maximal according to some preference relation, then C satisfies WA. Also, verify that conditions α and β are equivalent to WA for any choice correspondence with a domain satisfying that if A and B are included in the domain, then so is their intersection. Note also that for the next proposition, we could make do with a weaker version of WA, which makes the same requirement only for any two sets $A \subset B$ where A is a set of two elements.

Proposition:

Assume that C is a choice correspondence with a domain that includes at least all subsets of size 2 or 3. Assume that C satisfies WA. Then, there is a preference $\succsim$ so that $C = C_{\succsim}$.

Proof:

Define $x \succsim y$ if $x \in C(\{x, y\})$. We will now show that the relation is a preference:

Completeness: Follows from $C(\{x, y\}) \neq \emptyset$.

Transitivity: If $x \succsim y$ and $y \succsim z$, then $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$. Therefore, by condition β, if $y \in C(\{x, y, z\})$, then $x \in C(\{x, y, z\})$, and if $z \in C(\{x, y, z\})$, then $y \in C(\{x, y, z\})$. Thus, in any case, $x \in C(\{x, y, z\})$. By condition α, $x \in C(\{x, z\})$ and thus $x \succsim z$.

It remains to be shown that $C(B) = C_{\succsim}(B)$.

Assume that $x \in C(B)$. By condition α for every $y \in B$ we have $x \in C(\{x, y\})$ and thus $x \succsim y$. It follows that $x \in C_{\succsim}(B)$.

Assume that $x \in C_{\succsim}(B)$. Let $y \in C(B)$. If $y \neq x$ then $x \in C(\{x, y\})$ and by condition β we have $x \in C(B)$.

The Satisficing Procedure

The fact that we can present any choice function satisfying condition α (or WA) as an outcome of the optimization of some preference relation provides support for the view that the scope of microeconomic models is wider than simply models in which agents carry out explicit optimization. But have we indeed expanded the scope of economic models?

Consider the following "decision scheme", named *satisficing* by Herbert Simon. Let $v : X \to \mathbb{R}$ be a valuation of the elements in X, and let $v^* \in \mathbb{R}$ be a threshold of satisfaction. Let O be an ordering of the alternatives in X. Given a set A, the decision maker arranges the elements of this set in a list $L(A, O)$ according to the ordering O. He then chooses the first element in $L(A, O)$ that has a v-value at least as large as v^*. If there is no such element in A, the decision maker chooses the last element in $L(A, O)$.

Let us show that the choice function induced by this procedure satisfies condition α. Assume that a is chosen from B and is also a member of $A \subset B$. The list $L(A, O)$ is obtained from $L(B, O)$ by eliminating all elements in $B - A$. If $v(a) \geq v^*$, then a is the first satisfactory element in $L(B, O)$ and is also the first satisfactory element in $L(A, O)$. Thus, a is chosen from A. If all elements in B are unsatisfactory, then a must be the last element in $L(B, O)$. Since A is a subset of B, all elements in A are unsatisfactory and a is the last element in $L(A, O)$. Thus, a is chosen from A.

A direct proof that the procedure is rationalized can be obtained by explicitly constructing an ordering that rationalizes the satisficing procedure. Let $\succsim$ be the ordering that places on top the elements that satsifice, (namely, the members of $\{x|v(x) \geq v^*\}$) ordered according to O. The relation $\succsim$ puts the other alternatives at the bottom, ordered according to the reversed ordering O. For any set A, maximizing $\succsim$ will yield the first element (according to O) which is satisficing and if there isn't one then maximization will choose the last element in A (according to O).

Note, however, that even a "small" variation in this scheme can lead to a variation of the procedure such that it no longer satisfies condition α. For example:

Satisficing using two orderings: Let X be a population of university graduates who are potential candidates for a job. Given a set of actual candidates, count their number. If the number is smaller than 5, order them alphabetically. If the number of candidates is above 5, order them by their social security number. Whatever ordering is used, choose the first candidate whose undergraduate average is above 85. If there are none, choose the last student on the list.

Condition α is not satisfied. It may be that a is the first candidate with a satisfactory grade in a long list of students ordered by their social security numbers. Still, a might not be the first candidate with a satisfactory grade on a list of only three of the candidates appearing on the original list when they are ordered alphabetically.

To summarize, the satisficing procedure, though it is stated in a way that seems unrelated to the maximization of a preference relation or utility function, can be described as if the decision maker maximizes a preference relation. I know of no other examples of interesting general schemes for choice procedures that satisfy condition α other than the "rational man" and the satisficing procedures. However, later on, when we discuss consumer theory, we will come across several other appealing examples of demand functions that can be rationalized, though they appear to be unrelated to the maximization of a preference relation.

Psychological Motives Not Included within the Framework

The more modern attack on the standard approach to modeling economic agents comes from psychologists, notably from Amos Tversky and Daniel Kahneman. They have provided us with beautiful examples demonstrating not only that rationality is often violated but that there are systematic reasons for the violation resulting from certain elements within our decision procedures. Here are a few examples of this kind that I find particularly relevant.

Framing

The following experiment (conducted by Tversky and Kahneman (1986)) demonstrates that the way in which alternatives are framed may affect decision makers' choices. Subjects were asked to imagine being confronted by the following choice problem:
An outbreak of disease is expected to cause 600 deaths in the United States. Two mutually exclusive programs are expected to yield the following results:

a. 400 people will die.
b. With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die.

In the original experiment, a different group of subjects was given the same background information and asked to choose from the following alternatives:

c. 200 people will be saved.
d. With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved.

Whereas 78% of the first group chose b, only 28% of the second group chose d. These are "problematic" results since by any reasonable criterion a and c are identical alternatives, as are b and d. Thus, the choice from $\{a, b\}$ should be consistent with the choice from $\{c, d\}$.

Both questions were presented in the above order to 6,200 students taking game theory courses with the result that 73% chose b and 49% chose d. It seems plausible that many students kept in mind their answer to the first question while responding to the second one, and therefore the level of inconsistency was reduced. Nonetheless, a large proportion of students gave different answers to the two problems, which makes the findings even more problematic.

Overall, the results expose the sensitivity of choice to the framing of the alternatives. What is more basic to rational decision making than taking the same choice when only the manner in which the problems are stated is different?

Simplifying the Choice Problem and the Use of Similarities

The following experiment was also conducted by Tversky and Kahneman. One group of subjects was presented with the following choice problem:

Choose one of the two roulette games a or b. Your prize is the one corresponding to the outcome of the chosen roulette game as specified in the following tables:

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize \$	0	45	30	−15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize \$	0	45	−10	−15

A different group of subjects was presented the same background information and asked to choose between:

	Color	White	Red	Green	Blue	Yellow
(c)	Chance %	90	6	1	1	2
	Prize $	0	45	30	−15	−15

and

	Color	White	Red	Green	Blue	Yellow
(d)	Chance %	90	6	1	1	2
	Prize $	0	45	45	−10	−15

In the original experiment, 58% of the subjects in the first group chose a, whereas nobody in the second group chose c. When the two problems were presented, one after the other, to more than 3,000 students, 52% chose a and 7% chose c. Interestingly, the median response time among the students who answered a was 53 seconds, whereas the median response time of the students who answered b was 90 seconds.

The results demonstrate a common procedure people practice when confronted with a complicated choice problem. We often transfer the complicated problem into a simpler one by "canceling" similar elements. Although d clearly dominates c, the comparison between a and b is not as easy. Many subjects "cancel" the probabilities of White, Yellow, and Red and are left with comparing the prizes of Green, a process that leads them to choose a.

Incidentally, several times in the past when I presented these choice problems in class, I have had students (some of the best students, in fact) who chose c. They explained that they identified the second problem with the first and used the procedural rule: "I chose a from $\{a, b\}$. The alternatives c and d are identical to the alternatives a and b, respectively. It is only natural then, that I choose c from $\{c, d\}$". This observation brings to our attention the fact that the model of rational man does not allow dependence of choice on the previous choices made by the decision maker.

Reason-Based Choice

Making choices sometimes involves finding reasons to pick one alternative over the others. When the deliberation involves the use of reasons strongly associated with the problem at hand ("internal reasons"), we often find it difficult to reconcile the choice with the rational man paradigm.

Imagine, for example, a European student who would choose *Princeton* if allowed to choose from *Princeton*, *LSE* and would choose *LSE* if he had to choose from *Princeton*, *Chicago*, *LSE*. His explanation is that he prefers an American university so long as he does not have to choose between American schools – a choice he deems harder. Having to choose from $\{Princeton, Chicago, LSE\}$, he finds it difficult deciding between *Princeton* and *Chicago* and therefore chooses not to cross the Atlantic. His choice does not satisfy condition α, not because of a careless specification of the alternatives (as in the restaurant's menu example discussed previously), but because his reasoning involves an attempt to avoid the difficulty of making a decision.

A better example was suggested to me by a student Federico Filippini: "Imagine there's a handsome guy called Albert, who is looking for a date to take to a party. Albert knows two girls that are crazy about him, both of whom would love to go to the party. The two girls are called Mary and Laura. Of the two, Albert prefers Mary. Now imagine that Mary has a sister, and this sister is also crazy about Albert. Albert must now choose between the three girls, Mary, Mary's sister, and Laura. With this third option, I bet that if Albert is rational, he will be taking Laura to the party."

Another example follows Huber, Payne, and Puto (1982):

Let $a = (a_1, a_2)$ be "a holiday package of a_1 days in Paris and a_2 days in London". Choose one of the four vectors $a = (7, 4)$, $b = (4, 7)$, $c = (6, 3)$, and $d = (3, 6)$.

All subjects in the experiment agreed that a day in Paris and a day in London are desirable goods. Some of the subjects were requested to choose between the three alternatives a, b, and c; others had to choose between a, b, and d. The subjects exhibited a clear tendency toward choosing a out of the set $\{a, b, c\}$ and choosing b out of the set $\{a, b, d\}$.

A related experiment is reported in Shafir, Simonson, and Tversky (1993). A group of subjects was asked to imagine having to choose between a camera priced \$170 and a better camera, by the same producer, which costs \$240. Another group of subjects was asked to imagine having to choose between three cameras – the two described above and a third, much more sophisticated camera, priced at \$470. The addition of the third alternative significantly increased the proportion of subjects who chose the \$240 camera. The commonsense explanation for this choice is that subjects faced a conflict between two desires, to buy a better camera and to pay less. They resolved the conflict by choosing the "compromise alternative".

To conclude, decision makers look for reasons to prefer one alternative over the other. Typically, making decisions by using "external reasons" (which do not refer to the properties of the choice set) will not cause violations of rationality. However, applying "internal reasons" such as "I prefer the alternative a over the alternative b since a clearly dominates the other alternative c while b does not" might cause conflicts with condition α.

Mental Accounting

The following intuitive example is taken from Kahneman and Tversky (1984). Members of one group of subjects were presented with the following question:

1. Imagine that you have decided to see a play and paid the admission price of $10 per ticket. As you enter the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?

Members of another group were asked to answer the following question:

2. Imagine that you have decided to see a play where the admission is $10 per ticket. As you arrive at the theater, you discover that you have lost a $10 bill. Would you still pay $10 for a ticket for the play?

If the rational man cares only about seeing the play and his wealth, he should realize that there is no difference between the consequence of replying Yes to question 1 and replying Yes to question 2 (in both cases he will own a ticket and will be poorer by $20). Similarly, there is no difference between the consequence of replying No to question 1 and replying No to question 2. Thus, the rational man should give the same answer to both questions. Nonetheless, only 46% said they would buy another ticket after they had lost the first one, whereas 88% said they would buy a ticket after losing the banknote. In the data I collected (about 2,000 participants) the gap is much smaller: 64% and 79%, accordingly. It is likely that in this case subjects have conducted a calculation where they compared the "mental price" of a ticket to its subjective value. Many of those who decided not to buy another ticket after losing the first one attributed a price of $20 to the ticket rather than $10. This example demonstrates that decision makers may conduct "mental calculations" that are inconsistent with rationality.

Modeling Choice Procedures

There is a large and growing body of evidence that decision makers systematically use procedures of choice which violate the classical assumptions and that the rational man paradigm is lacking. As a result we have seen in recent years the introduction of economic models in which economic agents are assumed to use alternative procedures of choice. In this section, we focus on one particular line of research that attempts to incorporate such decision makers into economic models.

Classical models have characterized economic agents using a choice function. The statement $c(A) = a$ means that the decision maker selects a when choosing from the set of alternatives A. We wish to enrich the concept of a choice problem such that it will include not only the set of alternatives but also additional information that is irrelevant to the interests of the decision maker though it may nevertheless affect his choice. In what follows the additional information consists of a *default option*. The statement $c(A, a) = b$ means that when facing the choice problem A with a default alternative a the decision maker chooses the alternative b. Experimental evidence and introspection tell us that a default option is often viewed positively by a decision maker, a phenomenon known as the *status quo bias*.

Let X be a finite set of alternatives. Define an *extended choice function* to be a function that assigns a unique element in A to every pair (A, a) where $A \subseteq X$ and $a \in A$.

Following are some examples of extended choice functions which demonstrate the richness of the concept:

1. The decision maker has in mind a partial ordering D where aDb is interpreted as "a clearly dominates b" and an additional ordering $\succsim$ interpreted to be the real preference relation of the decision maker. The alternative $C(A, a)$ is the $\succsim$-best element in the set of alternatives that dominate a (i.e.,$\{x|\ xDa\}$).
2. Let d be a distance function on X. The decision maker has in mind a preference relation $\succsim$. The element $C(A, a)$ is the $\succsim$-best alternative that is not too far from a (i.e., it lies within $\{x \mid d(x, a) \leq d^*\}$ for some d^*).
3. The decision maker has in mind a preference relation $\succsim$ on X. The element $C(A, a)$ is an alternative in A that is the alphabetically first alternative after a which is $\succsim$-better than the default alternative a (and in the absence of such an alternative he sticks with the default).
4. Buridan's donkey: The decision maker has a preference relation in mind. If there is a unique alternative which is better than the default, then it is chosen. If not, then the decision maker stays

with the default option (since he cannot make up his mind) (see http://en.wikipedia.org/wiki/Buridan's_ass).

5. A default bias: The decision maker is characterized by a utility function u and a "bias function" β, which assigns a non-negative number to each alternative. The function u is interpreted as representing the "true" preferences. The number $\beta(x)$ is interpreted as the bonus attached to x when it is a default alternative. Given an extended choice problem (A, a), the procedure denoted by $DBP_{u,\beta}$, selects:

$$DBP_{u,\beta}(A,a) = \begin{cases} x \in A - \{a\} & if \quad u(x) > u(a) + \beta(a) \; and \; u(x) > u(y) \\ & \qquad for \; any \; y \in A - \{a, x\} \\ a & if \quad u(a) + \beta(a) > u(x), \forall x \in A - \{a\} \end{cases} .$$

Our aim is to characterize the set of extended choice functions that can be described as $DBP_{u,\beta}$ for some u and β. We will adopt two assumptions:

The Weak Axiom (WA)

We say that an extended choice function c satisfies the Weak Axiom if there are no sets A and B, $a, b \in A \cap B$, $a \neq b$ and $x, y \notin \{a, b\}$ (x and y are not necessarily distinct) such that:

1. $c(A, a) = a$ and $c(B, a) = b$ or
2. $c(A, x) = a$ and $c(B, y) = b$.

The Weak Axiom states that:

1. If a is revealed to be better than b in a choice problem where a is the default, then there cannot be any choice problem in which b is revealed to be better than a when a is the default.
2. If a is revealed to be better than b in a choice problem where neither a nor b is a default, then there cannot be any choice problem in which b is revealed to be better than a when again neither a nor b is the default.

Comment:

WA implies that for every a there is a preference relation $\succ_a$ such that $c(A, a)$ is the $\succ_a$-maximal element in A. To see this let

$Y_a = \{x | \; x \neq a$ and there exists a set B such that $c(B, a) = x\}$.

Now, consider the choice function on the grand set Y_a defined by $D(Y) = c(Y \cup \{a\}, a)$ for any $Y \subseteq Y_a$. By applying WA regarding the extended choice function c, the choice function D is well defined and satisfies condition α. Thus, there is an ordering $\succ_a$ on Y_a such that $D(Y)$ is the $\succ_a$-maximum in Y. Finally, extend $\succ_a$ so that a will be just below all the elements in Y_a and above all elements outside Y_a, which can be ordered in any way.

Default Tendency (DT)

We say that an extended choice function c satisfies Default Tendency if for every set A, if $c(A, x) = a$, then $c(A, a) = a$.

The second assumption states that if the decision maker chooses a from a set A when $x \neq a$ is the default, he does not change his mind if x is replaced by a as the default alternative.

Proposition:

An extended choice function c satisfies WA and DT if and only if it is a default-bias procedure.

Proof:

Consider a default-bias procedure c characterized by the functions u and β. It satisfies:

- DT: if $c(A, x) = a$ and $x \neq a$, then $u(a) > u(y)$ for any $y \neq a$ in A. Thus, also $u(a) + \beta(a) > u(y)$ for any $y \neq a$ in A and $c(A, a) = a$.
- WA: for any two sets A, B, $a, b \in A \cap B$, $a \neq b$:
    1. if $c(A, a) = a$ and $c(B, a) = b$, then both $u(a) + \beta(a) > u(b)$ and $u(b) > u(a) + \beta(a)$.
    2. if $c(A, x) = a$ and $c(B, y) = b$ ($x, y \notin \{a, b\}$), then both $u(a) > u(b)$ and $u(b) > u(a)$.

In the other direction, let c be an extended choice function satisfying WA and DT. Define a relation $\succ$ on $X \times \{0, 1\}$ as follows:

- For any pair (A, x) for which $c(A, x) = x$ and for any $y \in A - \{x\}$, define $(x, 1) \succ (y, 0)$.
- For any pair (A, x) for which $c(A, x) = y \neq x$ and for any $z \in A - \{x, y\}$, define $(y, 0) \succ (x, 1)$ and $(y, 0) \succ (z, 0)$.
- For all $x \in X$, $(x, 1) \succ (x, 0)$.

The relation is not necessarily complete or transitive, but by WA it is asymmetric. We will see that $\succ$ can be extended to a full ordering over $X \times \{0, 1\}$ denoted by $\succ^*$. Using problem 4 in Problem Set 1, we only need to show that the relation does not have cycles.

First note that:

a. For no x and y, $(x,0) \succ (y,0) \succ (x,1)$ since otherwise there is a set A containing x and y and another alternative $z \in A$ such that $c(A,z) = x$. By DT, also $c(A,x) = x$ and thus $(x,1) \succ (y,0)$ contradicting WA.

Assume that $\succ$ has a cycle and consider a shortest cycle. By WA, there is no cycle of length two, and thus the shortest cycle has to be at least of length three. Steps (b) and (c) establish that it is impossible for the shortest cycle to contain a consecutive pair $(x,0) \succ (y,0)$.

b. Assume that the cycle contains a consecutive segment $(x,0) \succ (y,0) \succ (z,1)$. By (a), $z \neq x$ and then there is a set A such that $c(A,z) = y$. Since $(x,0) \succ (y,0)$, $c(A \cup (x\}, z) = x$ and $(x,0) \succ (z,1)$. Thus, we can shorten the cycle.
c. Assume that the cycle contains a consecutive segment of the type $(x,0) \succ (y,0) \succ (z,0)$. By WA, the three elements are distinct. Since $(y,0) \succ (z,0)$, there exists a set A containing y and z and a different $a \in A$ such that $c(A,a) = y$. By (a), $a \neq x$ and then $c(A \cup \{x\}, a) = x$ and $(x,0) \succ (z,0)$, allowing us to shorten the cycle.

The next two steps establish that it is impossible for the shortest cycle to contain a consecutive pair $(x,0) \succ (y,1)$.

d. $(x,0) \succ (y,1) \succ (z,0)$ and $y \neq z$. If this were the case, then $c(\{x,y,z\}, y) = x$ and $(x,0) \succ (z,0)$, thus allowing us to shorten the cycle.
e. $(x,0) \succ (y,1) \succ (y,0) \succ (z,1)$. By DT, $z \neq x$ and by definition $z \neq y$. Consider $c\{\{x,y,z\}, z\}$. By WA and $(y,0) \succ (z,1)$, it cannot be z. If it is x, then $(x,0) \succ (y,0)$ and we can shorten the cycle. If it is y, then $(y,0) \succ (x,0)$ and we can shorten the cycle.

We can conclude that $\succ$ does not have a cycle. Now, let v be a utility function representing $\succ^*$. Define $u(x) = v(x,0)$ and $\beta(x) = v(x,1) - v(x,0)$ to obtain the result.

1. If $c(A,a) = a$, then $(a,1) \succ (x,0)$ for all $x \in A - \{a)$ and thus $u(a) + \beta(a) > u(x)$ for all x, that is, $c(A,a) = DBP_{u,\beta}(A,a)$.
2. If $c(A,a) = x$, then $(x,0) \succ (a,1)$ and $(x,0) \succ (y,0)$ for all $y \in A - \{a,x\}$ and therefore $u(x) > u(a) + \beta(a)$ and $u(x) > u(y)$ for all $y \in A - \{a,x\}$. Thus, $c(A,a) = DBP_{u,\beta}(A,a)$.

Comments on the Significance of Axiomatization

1. There is something aesthetically attractive about the axiomatization. However, I doubt that such an axiomatization is necessary in order to develop a model in which the procedure appears. As with other conventions in the profession, this practice appears to be a barrier to entry that places an unnecessary burden on researchers.
2. A necessary condition for an axiomatization of this type to be of importance is (in my opinion) the possibility of coming up with examples of sensible procedures of choice that satisfy the axioms and are not specified explicitly in the language of the procedure we are axiomatizing. Can one find such a procedure for the above axiomatization? I myself am unable to. Indeed, many of the axiomatizations in this field lack such examples, and therefore, in spite of their aesthetic value (and although I have done some axiomatizations myself), I find them to be futile exercises.

Bibliographic Notes

An excellent book on the lecture's subject is Kreps (1988). For the sources of consistency in choice and revealed preference assumptions, see Samuelson (1948), Houthakker (1950), and Richter (1966). Simon (1955) is the source of the discussion of satisficing. For a discussion of the bounded rationality approach to choice, see Rubinstein (1998). Sen (1993) provides a more philosophical discussion of the subject of this chapter. An excellent introduction to the Dutch Books arguments is Yaari (1985). Kahneman and Tversky (2000) is a definitive textbook on the psychological criticism of the economic approach to rationality. Rabin (1998) surveys the modern economics and psychology approach. The DBP procedure was studied by Massatliglu and Ok (2005). Here we have followed a simpler axiomatization due to Rubinstein and Salant (2006b).

Problem Set 3

Problem 1. (*Easy*)
The following are descriptions of decision-making procedures. Discuss whether the procedures can be described in the framework of the choice model discussed in this lecture and whether they are compatible with the "rational man" paradigm.

a. The decision maker chooses an alternative in order to maximize another person's suffering.
b. The decision maker asks his two children to rank the alternatives and then chooses the alternative that is the best on average.
c. The decision maker has an ideal point in mind and chooses the alternative that is closest to it.
d. The decision maker looks for the alternative that appears most often in the choice set.
e. The decision maker has an ordering in mind and always chooses the median element.

Problem 2. (*Moderately difficult*)
A choice correspondence C satisfies the *path independence* property if for every set A and a partition of A into A_1 and A_2 ($A_1, A_2 \neq \emptyset$, $A = A_1 \cup A_2$ and $A_1 \cap A_2 = \emptyset$) we have $C(A) = C(C(A_1) \cup C(A_2))$. (Of course this definition applies also for choice functions).

a. Show that the rational decision maker satisfies path independence.
b. Find examples of choice procedures that do not satisfy this property.
c. Show that if a choice function satisfies path independence, then it satisfies condition alpha.
d. Find an example of a choice correspondence satisfying path independence that cannot be rationalized.

Problem 3. (*Easy*)
Let X be a finite set. Check whether the following three choice correspondences satisfy WA:

$C(A) = \{x \in A|$ *the number of* $y \in X$ *for which* $V(x) \geq V(y)$ *is at least* $|X|/2\}$, and if the set is empty, then $C(A) = A$.

$D(A) = \{x \in A|$ *the number of* $y \in A$ *for which* $V(x) \geq V(y)$ *is at least* $|A|/2\}$.

$E(A) = \{x \in A | x \succ_1 y \text{ for every } y \in A \text{ or } x \succ_2 y \text{ for every } y \in A\}$ where $\succ_1$ and $\succ_2$ are two orderings over X.

Problem 4. (*Moderately difficult*)
Consider the following choice procedure: A decision maker has a strict ordering $\succsim$ over the set X and assigns to each $x \in X$ a natural number $class(x)$ to be interpreted as the "class" of x. Given a choice problem A, he chooses the best element in A from those belonging to the most common class in A (i.e., the class that appears in A most often). If there is more than one most common class, he picks the best element from the members of A that belong to a most common class with the highest class number.

a. Is the procedure consistent with the "rational man" paradigm?
b. Define the relation: xPy if x is chosen from $\{x, y\}$. Show that the relation P is a strict ordering (complete, asymmetric, and transitive).

Problem 5. (*Moderately difficult. Based on Kalai, Rubinstein, and Spiegler (2002).*)
Consider the following two choice procedures. Explain the procedures and try to persuade a skeptic that they "make sense". Determine for each of them whether they are consistent with the rational man model.

a. The primitives of the procedure are two numerical (one-to-one) functions u and v defined on X and a number v^*. For any given choice problem A, let $a^* \in A$ be the maximizer of u over A and let b^* be the maximizer of v over A. The decision maker chooses a^* if $v(a^*) \geq v^*$ and b^* if $v(a^*) < v^*$.
b. The primitives of the procedure are two numerical (one-to-one) functions u and v defined on X and a number u^*. For any given choice problem A, the decision maker chooses the element $a^* \in A$ that maximizes u if $u(a^*) \geq u^*$, and the element $b^* \in A$ that maximizes v if $u(a^*) < u^*$.

Problem 6. (*Moderately difficult. Based on Rubinstein and Salant (2006a).*)
The standard economic choice model assumes that choice is made from a *set*. Let us construct a model where the choice is assumed to be made from a *list*. (Note that the list $< a, b >$ is distinct from $< a, a, b >$ and $< b, a >$.)

Let X be a finite grand set. A *list* is a nonempty finite vector of elements in X. In this problem, consider a *choice function* C to be a function that assigns a single element from $\{a_1, \ldots, a_K\}$ to each vector $L =< a_1, \ldots, a_K >$. Let $< L_1, \ldots, L_m >$ be the concatenation of the m lists $L_1, \ldots, L_m$ (note that if the length of L_i is k_i, the length of the concatenation is $\Sigma_{i=1,\ldots,m} k_i$). We say that L' *extends* the list L if there is a list M such that $L' =< L, M >$.

We say that a choice function C satisfies Property I if for all $L_1, \ldots, L_m$, $C(< L_1, \ldots, L_m >) = C(< C(L_1), \ldots, C(L_m) >)$.

a. Interpret Property I. Give two examples of choice functions that satisfy I and two examples that do not.
b. Define formally the following two properties of a choice function:
Order Invariance: A change in the order of the elements in the list does not alter the choice.
Duplication Invariance: Deleting an element that appears elsewhere in the list does not change the choice.
Show that Duplication Invariance implies Order Invariance.
c. Characterize the choice functions that satisfy Duplication Invariance, and property I.

Assume now that at the back of the decision maker's mind there is a value function u defined on the set X (such that $u(x) \neq u(y)$ for all $x \neq y$). For any choice function C, define $v_C(L) = u(C(L))$.

We say that C *accommodates a longer list* if, whenever L' extends L, $v_C(L') \geq v_C(L)$ and there is a pair of lists L' and L such that L' extends L and $v_C(L') > v_C(L)$.

d. Give two interesting examples of choice functions that accommodate a longer list.
e. Give two interesting examples of choice functions that satisfy property I but do not accommodate a longer list.

Problem 7. (*Difficult. Based on Rubinstein and Salant (2006a).*)
Let X be a finite set. We say that a choice function c is *lexicographically rational* if there exists a profile of preference relations $\{\succ_a\}_{a \in X}$ (not necessarily distinct) and an ordering O over X such that for every set $A \subset X$, $c(A)$ is the $\succ_a$-maximal element in A, where a is the O-maximal element in A.

A decision maker who follows this procedure is attracted by the most notable element in the set (as described by O). If a is that element, he applies the ordering $\succ_a$ and chooses the $\succ_a$-best element in the set.

We say that c satisfies the *reference point property* if, for every set A, there exists $a \in A$ such that if $a \in A'' \subset A' \subset A$ and $c(A') \in A''$, then $c(A'') = c(A')$.

a. Show that a choice function c is lexicographically rational if and only if it satisfies the *reference point property*.
b. Try to come up with a procedure satisfying the reference point axiom that is not stated explicitly in the language of the lexicographically rational choice function (no idea about the answer).

Problem 8. (*Difficult. Based on Cherepanov, Fedderson, and Sandroni (2008).*)
Consider a decision maker who has in mind a set of rationales and an asymmetric complete relation over a finite set X. Given $A \subset X$, he chooses the best alternative in that he can rationalize.

Formally, we say that a choice function c is *rationalized* if there is an asymmetric complete relation $\succ$ (*not* necessarily transitive!) and a set of partial orderings (asymmetric and transitive) $\{\succ_k\}_{k=1...K}$ (called rationales) such that $c(A)$ is the $\succ$ -maximal alternative from among those alternatives found to be maximal in A by at least one rationale (given a binary relation $\succ$ we say that x is $\succ$ -maximal in A if $x \succ y$ for all $y \in A$). Assume that the relations are such that the procedure always leads to a solution.

We say that a choice function c satisfies *The Weak Weak Axiom of Revealed Preference (WWARP)* if for all $\{x, y\} \subset B_1 \subset B_2$ $(x \neq y)$ and $c\{x, y\} = c(B_2) = x$, then $c(B_1) \neq y$.

a. Show that a choice function satisfies WWARP if and only if it is rationalized. For the proof, construct rationales, one for each choice problem.
b. What do you think about the axiomatization?

Consider the "warm-glow" procedure: The decision maker has two orderings in mind: one moral $\succsim_M$ and one selfish $\succsim_S$. He chooses the most moral alternative m as long as he doesn't "lose" too much by not choosing the most selfish alternative. Formally, for every alternative s there is some alternative $l(s)$ such that if the most selfish alternative is s, then he is willing to choose m as long as $m \succsim_S l(s)$. If $l(s) \succ_S m$, he chooses s.

The function l satisfies $(i) s \succsim_S l(s)$ and $(ii) s \succsim_S s'$ implies $l(s) \succsim_S l(s')$.

c. Show that WWARP is satisfied by this procedure.
d. Show directly that the "warm-glow" procedure is rationalized (in the sense of the definition in this problem).