Section 3.X: Modeling Choice Procedures

There is a large body of evidence pointing to the failure of the rational man paradigm by showing that decision makers systematically use procedures of choice that violate the classical assumptions. The accumulated evidence has an affect on the development of economic theory. In the last decade we have seen the introduction of economic models in which economic agents are assumed to follow alternative procedures of choice. In this section, we focus on one line of example research that aims attempts to incorporate such decision makers in to economic models.

Classical models have chacterized economic agents using a choice function. The statement $c(A) = a$ means that the decision maker selects a when choosing from the set of alternatives A. The problem set includes several examples of choice functions that are not consistent with the rational man paradigm.

In this section, we adopt some of the new models of choice in order to enrich the concept of a choice problem. Thus, a choice problem will include not only the set of alternatives but also additional information, which though irrelevant to the interests of the decision maker may nevertheless affect his choice. An extra would be the ordering in which the alternatives are presented.

We will be dealing with a special case in which the additional information is the identity of a *default option*. The statement $c(A,a) = b$ will mean that when facing the choice problem A with a default alternative a the decision maker chooses b. There is some evidence, supported by introspection, that a default option is often evaluated positively by decision makers, a phenomenon known as the *status quo bias*.

Let X be a finite set of alternatives. Define an extended choice *function* to be a function that assigns a unique element in A to every pair (A,a) where $A \subseteq X$ and $a \in A$. A *default bias procedure* is an extended choice function characterized by a utility function u (which represents the true preferences) and a "bias function" b which assigns a non-negative number to each alternative ($b(x)$ is interpreted as the bonus attached to x when it is a default point) such that for any given an extended choice problem (A,a) he selects:

$x \in A - \{a\}$ if $u(x) > u(a) + b(a)$ and $u(x) > u(y)$ for any $y \in A - \{a,x\}$ and a otherwise.

Denote the procedure by $SBP_{u,b}$. We will characterize the set of extended choice functions that can be described as $SBP_{u,b}$ for some u and b using two assumptions:

The Weak Axiom (WA). We say that an extended choice function c satisfies the weak axiom if there are no sets A and B, $a,b \in A \cap B$, $a \neq b$ and $x,y \notin \{a,b\}$ (x and y are not necessarily distinct) such that:

(i) $c(A,a) = a$ and $c(B,a) = b$, or

(ii) $c(A,x) = a$ and $c(B,y) = b$.

The Weak Axiom states that:

(i) If alternative a is revealed to be better than b in a choice problem where a is the default, thaen there cannot be have any choice problem in which b is revealed to be better than a when a is the default.

(ii) If alternative a is revealed to be better than b in a choice problem where neither a nor b is a default, there cannot be any choice problem in which b is revealed to be better than a when again neither a nor b is the default.

Comment: WA implies that for every a there is a preference relation $\succ_a$ such that $c^*(A,a)$ is the $\succ_a$ -maximal element in A. To see this let $Y_a = \{x \in X \mid (x,0) \succ (a,1)\}$. Y_a contains all elements in X that are revealed to be better than a when a is the default. Consider the choice function on Y_a defined by $D(Y) = c(Y \cup \{a\}, a)$. By WA, it satisfies condition α and thus there is an ordering $\succ_a$ on Y_a such that $D(A)$ is the $\succ_a$ -maximum in Y. Extend $\succ_a$ so that a is better than all elements outside Y_a. Finally, WA implies that $c(A,a) = c(Y \cup \{a\}, a) = D(A)$.

The second assumption states that if the decision maker chooses a from a set A when is the default and $x \neq a$, he does not change his mind if x is replaced by a as the default point.

Default Tendency (DT). If $c(A,x) = a$, then $c(A,a) = a$.

Proposition. An extended choice function c satisfies WA and DT if and only if it is a default-bias procedure.

Proof. Consider a default-bias procedure c characterized by the functions u and b that It satisfies the two axioms.

DT: if $c(A,x) = a$ and $x \neq a$, then $u(a) > u(y)$ for any $y \neq a$ in A. Thus, also

$u(a) + b(a) > u(y)$ for any $y \neq a$ in A and $c(A,a) = a$.

WA: if (i) $c(A,a) = a$ and $c(B,a) = b$, then $u(a) + b(a) > u(b)$ and $u(b) > u(a) + b(a)$ and if (ii) $c(A,x) = a$ and $c(B,y) = b$ ($x,y \notin \{a,b\}$), then $u(a) > u(b)$ and $u(b) > u(a)$.

In the other direction, let c^* be an extended choice function satisfying WA and DT. Define a relation $\succ$ on $X \times \{0,1\}$ as follows:

-For any pair (A,x) for which $c^*(A,x) = x$, define $(x,1) \succ (y,0)$ for all $y \in A - \{x\}$.

-For any pair (A,x) for which $c^*(A,x) = y \neq x$, define $(y,0) \succ (x,1)$ and $(y,0) \succ (z,0)$ for all $z \in A - \{x,y\}$.

-Extend the relation so that $(x,1) \succ (x,0)$ for all $x \in X$.

The relation is not necessarily complete and transitive but by WA it is asymmetric. We will see that $\succ$ can be extended to $\succ^*$ i.e.. a full ordering over $X \times \{0,1\}$. By Problem 1 below, we only need to show that the relation does not have cycles. Then, let v be a utility function representing $\succ^*$. Define $u(x) = v(x,0)$ and $b(x) = v(x,1) - v(x,0)$ to obtain the result. If $c^*(A,a) = a$, then $(a,1) \succ (x,0)$ for all $x \in A - \{a\}$ and thus $u(a) + b(a) > u(x)$ for all x and thus $c^*(A,a) = SBP_{u,b}(A,a)$. Similarly, if $c^*(A,a) = x$, then $(x,0) \succ (a,1)$ and $(x,0) \succ (y,0)$ for all $y \in A - \{a,x\}$ and thus $u(x) > u(a) + b(a)$ and $u(x) > u(y)$ for all $y \in A - \{a,x\}$. Thus, $c^*(A,a) = SBP_{u,b}$.

Assume that $\succ$ has a cycle. Consider a shortest cycle. By WA there is no cycle of length two and thus the shortest cycle has at least three elements in it.

(a) Assume that there is a segment $(x,0) \succ (y,0) \succ (z,1)$ in the cycle.

If $z = x$ we get a contradiction to DT ($(x,0) \succ (y,0)$ means that there is a set A containing x and y and a third alternative a such that $c^*(A,a) = x$. Then, also $c^*(A,x) = x$ and $(z,1) \succ (y,0)$. If $z \neq x$, then there is a set A such that $c^*(A,z) = y$. Since $(x,0) \succ (y,0)$, $c^*(A \cup (x\},z) = x$ and $(x,0) \succ (z,1)$ and therefore we can shorten the cycle.

(b) Assume that the cycle contains a segment of the type $(x,0) \succ (y,0) \succ (z,0)$. By WA, the three elements are distinct. Since $(y,0) \succ (z,0)$, there exists a set A containing y and z and $a \in A$ such that $c(A,a) = y$. If $a \neq x$, then $c^*(A \cup \{x\},a) = x$ and $(x,0) \succ (z,0)$ allowing us to shorten the cycle. If $a = x$, that is if $c(A,x) = y$, then $(x,0) \succ (y,0) \succ (x,1)$ thus, contradicting DT.

(c) If $(x,0) \succ (y,1) \succ (z,0)$ and $y \neq z$, then $c^*(\{x,y,z\},y) = x$ and $(x,0) \succ (z,0)$, thus shortening the cycle.

(d) If $(x,0) \succ (y,1) \succ (y,0) \succ (z,1)$, then let A contain y and z such that

$c^*(A, z) = y$. If $c^*(A \cup \{x\}, y) = x$, then $(x, 0) \succ (z, 1)$ and if $c^*(A \cup \{x\}, y) = y$, then $(y, 1) \succ (x, 0)$, thus violating WA.

Since we have proved that all cycles of size three or more can be reduced, the shortest cycle must be of length 2; however, by WA, cycles of length 2 are not possible. Thus, there cannot be any cycles to begin with.

Coffee!!:)

Comments on the significance of axiomatization

(1) There is something aesthetic in the axiomatization, but... I doubt that such an axiomatization is necessary for an economist to develop a model in which the procedure will appear. As with other conventions in the profession, this practice appears to be a barrier to entry which places an unnecessary burden on the shoulders of researchers. (to be expanded).

(2) A necessary condition for an axiomatization of this type to be of importance is (in my opinion) that we come up with examples of sensible procedures of choice which satisfy the axioms and are not specified explicitly in the language of the procedure we are axiomatizing. Can you find such a procedure for the above axiomatization? Many of the axiomatizations in this field lack such examples and thus, in spite of their aesthetic value, I find them to be futile exercises.

Bibliographic Comments

The above procedure was studied by Massatliglu and Ok (2005). Here we have followed a simpler axiomatization due to Rubinstein and Salant (unpublished). Problem 2 in is taken from Rubinstein and Salant (2006) and Problem 3 from Cherepanov, Fedderson and Sandroni (2008).

Problem Set 3X

Problem 1: Let $\succ$ be an asymmetric binary relation on finite X that does not have cycles. Show (by induction on the size of X) that $\succ$ can be extended to a complete ordering.

Problem 2: Think about a decision maker who chooses among vacation packages a is maximizing either the entertainment value or its historic value. The presence of Eilat among the vacation packages will lead him to decide according to entertainment value. The presence of Jerusalem will lead him to decide according to historic value.

We say that a choice function c is *lexicographically rational* if there exists a profile of preference relations $\{\succ_a\}_{a\in X}$ (not necessary distinct) and an ordering O over X such that for every set $A \subset X$:

$C(A)$ is the $\succ_a$ -maximal element in A, where a is the O-maximal element in A.

A decision maker who follows this procedure is attracted by the most notable element in the set (as described by O). If a is the most attractive element in the choice set, he applies the ordering $\succ_a$ and chooses the $\succ_a$ -best element in the set.

We say that c satisfies the *reference point property* if for every set A, there exists $a \in A$ such that:

If $a \in A'' \subset A' \subset A$ and $c(A') \in A''$, then $c(A'') = c(A')$.

(a) Show that a choice function c is lexicographically rational if and only if it satisfies the *reference point* property.

(b) Try to come up with a procedure satisfying the reference point axiom which is not stated explicitly in the language of the lexicographical rational choice function (No idea about the answer).

Problem 3:

We say that a choice function c is a *rationalized choice function* if there is an asymmetric relation $\succ$ (not necessarily transitive) and a set of partial orderings $\{\succ_k\}_{k=a...K}$ (called rationales) such that $c(A)$ is the $\succ$ maximal from among those alternatives found to be maximal in A by at least one rationale. (The decision maker has in mind a set of rationales and a preference relation. He chooses the best alternative that he can rationalize.)

Weakening Weak Axiom of Revealed Preference (WWARP): If $\{x,y\} \subset B_1 \subset B_2$ ($x \neq y$) and $C\{x,y\} = C(B_2) = x$, then $C(B_1) \neq y$.

(a) Show that a choice function satisfies WWARP if and only if it is a rationalized choice function.

(b) What do you think about the axiomatization? Note that the rationales were modeled here as asymmetric binary relations (without structure) and that there is one rationale for each choice set.

(c) Consider the "warm-glow" procedure: The decision maker has two complete orderings in mind: one moral and one selfish. He chooses the more moral alternative m as long as he doesn't "lose" too much by not choosing the more selfish alternative. Formally, for every s there is some $s \succsim l(s)$ such that if the most selfish alternative is s then he is willing to choose the most moral alternative as long as it is not worse than $l(s)$. The function l satisfies that $s \succsim s'$ iff $l(s) \succsim l(s')$.

(i) Show that WWARP is satisfied by this procedure.

(ii) Show directly that the warm-glow procedure is a rationalized choice function.