Review Problems

The following is a collection of questions I gave in exams during the last few years. There are some mistakes in the questions and I will not be surprised if some of the questions were used already in problems sets. Please let me know about any major mistakes.

Problem 1 (Princeton 02)

Consider a consumer with a preference relation in a world of two goods: X (an aggregated consumption good) and M ("membership in a club", for example), which can be consumed or not. In other words, the consumption of X can be any non-negative real number while the consumption of M must be either 0 or 1.

Assume that consumer preferences are strictly monotonic, continuous and satisfy property

E: For every x there is y such that $(y,0) \succ (x,1)$ (that is, there is always some amount of money which can compensate for the loss of membership).

■A) Show that any consumer's preference relation can be represented by a utility function of the type

$$u(x,m)= \begin{cases} x & if \;\; m=0 \\ x+g(x) & if \;\; m=1 \end{cases}$$

■B) (Less easy) Show that the consumer's preference relation can also be represented by a utility function of the type $u(x,m)= \begin{cases} f(x) & if \;\; m=0 \\ f(x)+v & if \;\; m=1 \end{cases}$

■C) Explain why continuity and strong monotonicity (without E) are not sufficient for A.

■D) Compute the consumer's demand function.

■E) Taking the utility function to be of the form described in part (A), compute the consumer's indirect utility function. For the case that the function g is differentiable verify the Roy equality in respect to commodity M.

Problem 2 (Princeton 02)

The standard economic choice model assumes that choice is made from a *set*. Let us construct a model where the choice is assumed to be from a *list*.

Let X be a finite "grand set". A *list* is a non-empty finite vector of elements in X. In this problem, consider a *choice function* C to be a function which assigns to each vector $L =< a_1,\ldots,a_K >$ a single element from $\{a_1,\ldots,a_K\}$. (Thus, for example, the list $< a,b >$ is distinct from $< a,a,b >$ and $< b,a >$). For all $L_1,\ldots,L_m$ define $< L_1,\ldots,L_m >$ to be the list which is the concatenation of the m lists. (Note that if the length of L_i is k_i the length of the concatenation is $\Sigma_{i=1,..,m}k_i$). We say that L' *extends* the list L if there is a list M such that $L' =< L,M >$.

We say that a choice function C satisfies property I if for all $L_1,\ldots,L_m$
$C(< L_1,\ldots,L_m >) = C(< C(L_1),\ldots,C(L_m) >)$.

■A) Interpret property I. Give two (distinct) examples of choice functions which satisfy I and two examples of choice functions which do not.

■B) Define formally the following two properties of a choice function:

Order Invariance: A change in the order of the elements of the list does not alter the choice and

Duplication Invariance: Deleting an element which appears in the list elsewhere does not change the choice.

Characterize the choice functions which satisfy Order Invariance, Duplication Invariance and condition I.

Assume now that in the back of the decision maker's mind is a value function u defined on the set X (such that $u(x) \neq u(y)$ for all $x \neq y$). For any choice function C define $v_C(L) = u(C(L))$.

We say that *C accommodates a longer list* if whenever L' extends L, $v_C(L') \geq v_C(L)$ and there is a list L' which extends a list L for which $v_C(L') > v_C(L)$.

■C) Give two interesting examples of a choice function which accommodates a longer list.

■D) Give two interesting examples of choice functions which satisfy property I but which do not accommodate a longer list.

Problem 3 (Princeton 01)

A consumer has to make his decision *before* he is informed about whether a certain event, which is expected with probability α, happened or not. He assigns the vNM utility $v(x)$ to the consumption of the bundle x in case the event occurs and he assigns the number

$w(x)$ to the consumption of x when the event does not occur. The consumer maximizes his expected utility. Both v and w satisfy the standard assumptions about the consumer. Assume also that v and w are concave.

■A) Show that the consumer's preference relation is convex.

■B) Find a connection between the consumer's indirect utility function and the indirect utility functions derived from v and w.

■C) A new commodity appears in the market: "a discrete piece of information which tells the consumer whether the event occurred or not". The commodity can be purchased prior to the consumption decision. Use the indirect utility functions to characterize the demand function for the new commodity.

Problem 4 (Princeton 01)

■A. Define a formal concept for "$\succsim_1$ is close to $\succsim_0$ more than $\succsim_2$ is close to $\succsim_0$".

■B. Apply your definition to the class of preference relations represented by $U_1 = tU_2 + (1-t)U_0$ where the function U_i represents $\succsim_i$ $(i = 0,1,2)$.

■C. Consider the above definition in the consumer context. Denote by $x_k^i(p,w)$ the demand function of $\succsim_i$ for good k. Is it true that if $\succsim_1$ is close $\succsim_0$ more than $\succsim_2$ is close to $\succsim_0$ than $|x_k^1(p,w) - x_k^0(p,w)| \le |x_k^2(p,w) - x_k^0(p,w)|$ for any commodity k and for every price vector p and wealth level w?

Problem 5 (Princeton 01)

Consider the following procedure of choice. A decision maker has a strict ordering $\succsim$ over the set X and he assigns to each $x \in X$ a natural number $class(x)$ interpreted as the "class" of x. Given a choice problem A he chooses the element in A which is the best among those elements in A which belong to the "most popular" class in A (that is, the class which appears in A most often). If there is more than one most popular class, he picks the best element from the members of A which belong to a most popular class with the highest class number.

■A) Is the procedure consistent with the "rational man" paradigm?

■B) Can every choice function be "explained" as an outcome of such a procedure?

(Try to formalize a "property" which is satisfied by such procedures of choice and clearly is not satisfied by some other choice functions)

Problem 6 (Princeton 97**)**

A decision maker forms preferences over the set X of all possible distribution of a population over two categories (like living in two locations). An element in X is a vector (x_1, x_2) where $x_i \geq 0$ and $x_1 + x_2 = 1$. The decision maker has two considerations in mind.

■He thinks that if $x \succsim y$ then for any z the mixture of α of x with $(1-\alpha)$ of z should be at least as good as the mixture of α of y with $(1-\alpha)$ of z.

■He is indifferent between a distribution which is fully concentrated in location 1 and one which is fully concentrated in location 2.

▸A. Show that the only preference relation which is consistent with the two principles is the degenerate indifference relation ($x \sim y$ for any $x, y \in X$).

▸B. The decision maker claims that you are wrong as his preference relation is represented by a utility function $|x_1 - 1/2|$. Why is he wrong?

Problem 7 (Princeton 00) (Based on Fishburn and Rubinstein (1982))

Let $X = \Re^+ \times \{0, 1, 2, \ldots\}$ where (x, t) is interpreted as getting \$$x$ at time t. A preference relation on X having the following properties:

■There is indifference between getting \$0 at time 0 or at any other time.

■For any positive amount of money it is better to get it as soon as possible.

■Money is desirable.

■The preferences between (x, t) and $(y, t+1)$ is independent of t.

■continuity.

▸A. Define formally the continuity assumption for this context.

▸B. Show that the preference relation has utility representation.

▸C. Verify that preferences induced from a utility function $u(x)\delta^t$ (with $\delta < 1$ and u continuous and increasing) satisfies the above properties.

D. Formulate a concept "one preference relation is more impatient than another"

E. Discuss the claim that preferences represented by $u_1(x)\delta_1^t$ is more impatient than preferences represented by $u_2(x)\delta_2^t$ if and only if $\delta_1 < \delta_2$.

Problem 8 (Tel Aviv 03**)**

Consider the following consumer problem: There are two goods, 1 and 2. The consumer has a certain endowment. Before the consumer are two "exchange functions": he can exchange x units of good 1 for $f(x)$ units of good 2, or he can exchange y units of good 2 for $g(y)$ units of good 1. Assume the consumer can only make one exchange.

A) Show that if the exchange functions are continuous and the consumer's preference relation satisfies monotonicity and continuity, then a solution to the consumer problem exists.

B) Explain why strong convexity of the preference relation is not sufficient to guarantee a unique solution if the functions are increasing and convex.

C) What does the statement "the function f is increasing and convex" mean (answer in only one line)?

D) Suppose both functions f and g are differentiable and concave and the product of their derivatives at the point 0 is 1. Suppose also that the preference relation is strongly convex. Show that, under these conditions, the agent will not find it optimal to make two different exchanges, one exchanging good 1 for good 2, and one exchanging good 2 for good 1.

E) Now assume $f(x) = ax$ and $g(y) = by$. Explain this assumption. Find a condition which will ensure that it is not profitable for the consumer to make more than one exchange.

Problem 9 (Tel Aviv 99)

Consider a consumer in a world with K goods and preferences satisfying the standard assumptions we assumed regarding the consumer. At the start of trade, the consumer is endowed with a basket of goods e and he chooses the best basket from the budget set $B(p,e) = \{x|px = pe\}$. The consumer's preference over baskets of goods can be represented by a utility function, u. Define $V(p,e) = \max\{u(x)|px = pe\}$.

A) Explain the meaning of the function V and show that $V(tp,e) = V(p,e)$ where t is any positive number.

B) Show that for every basket e, the set of vectors p such that $V(p,e)$ is less than or equal to $V(p^*,e)$ is convex.

C) Consider the graph of the function V on the plane p_i, e_i. Assume (p,e) is a vector satisfying $x_i(p,e) = e_i$, that is, the demand for good i is equal to the amount the agent was originally endowed with. What is the slope of the curve of V at the point (p_i, e_i)?

Problem 10 (Tel Aviv 98)

A consumer with wealth $w = 10$ "must" obtain a book from one of three stores. Denote the prices at each store p_1,p_2,p_3. All prices are below w in the relevant range. The consumer takes the following strategic decision: First he compares the prices at the first two stores and obtains the book from the first store if it's price is not greater than the price at the second store. If $p_1 > p_2$, he compares the prices of the second and third stores and obtains the book from the second store if it's price is not greater than the price at the third store. He uses the remainder of his wealth to purchase other goods.

A) What is this consumer's "demand function"?

B) Does this consumer satisfy the "rational man" assumptions?

C) Consider the function $v(p_1,p_2,p_3) = w - p_{i^*}$, where i^* is the store the consumer purchases the book from if the prices are (p_1,p_2,p_3). What does this function represent?

D) Explain why $v(\bullet)$ is not monotonically decreasing in prices. Compare with the indirect utility function of the classical consumer model.

Problem 11 **(**Princeton 98**)**

Consider a consumer in a world with K commodities (with preferences over bundles that satisfy the standard assumptions we make on a consumer). The consumer gets his income in form of a bundle of commodities z and he chooses the best bundle from among the set $B(p,z) = \{x|px = pz\}$. Given that the consumer's preferences are represented by a utility function u, define $V(p,z) = max\{u(x)|px = pz\}$.

Interpret the function V.

What can you say on $V(\lambda p, w)$?

Show that V is quasi convex in p.

Fix all prices but commodity p_i, and all quantities of in the initial bundle but w_i. Show that the slope of the indifference curve of V in the two dimensional space where the parameters on the axes are p_i and w_i is $(x_i(p,w) - w_i)/p_i$ where $x(p,w)$ is the solution to the consumer's problem $B(p,w)$.

Problem 12 (Tel Aviv 98) (Based on Miyamato, Wakker, Bleichordt, and Peters (19??)).

"In life" we often face decisions where we must choose between longevity and quality of life. Consider the preference relation of a decision maker deciding on matters of "longevity and quality of life." This preference relation ranks lotteries on the set of all certain outcomes of the form (q, t), defined as "a life of quality q and length t" (where q and t are nonnegative numbers). Assume the preference relation satisfies the von Neumann-Morgenstern assumptions. It also satisfies:

a) Indifference between "high" and "low" quality of life when longevity is 0.

b) For a given quality of life level, the decision maker maximizes expected longevity.

c) A higher quality of life is better.

A) Formalize the three aforementioned assumptions.

B) Show that the preference relation derived from maximizing the expectation of the function $v(q)t$, where $v(\cdot)$ is a strictly increasing function and $v(q) > 0$ for all q, satisfies all three assumptions.

C) Show that all preference relations satisfying the above assumptions can be represented by an expected utility function of the form $v(q)t$, where $v(\cdot)$ is a positive and increasing function.

Problem 13 (Tel Aviv 99)

Kahneman and Tversky (1986) report on the following experiment: Each participant receives a questionnaire asking him to make two choices, one from $\{A, B\}$ and the second from $\{C, D\}$.

(A) A sure profit of $240.

(B) A lottery between a profit of $1000 with probability 25% and 0 with probability 75%.

(C) A sure loss of $750.

(D) A lottery between a loss of $1000 with probability 75% and 0 with probability 25%.

The participant will receive the sum of the outcomes of the two lotteries he chooses. Seventy-three percent of participants chose the combination A and D. What do you make of this result?