CHAPTER 6

Choice of conjectures in a bargaining game with incomplete information

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6.1 Introduction

The axiomatic approach to bargaining may be viewed as an attempt to predict the outcome of a bargaining situation solely on the basis of the set of pairs of utilities that corresponds to the set of possible agreements and to the nonagreement point.

The strategic approach extends the description of a bargaining situation. The rules of bargaining are assumed to be exogenous, and the solution is a function not only of the possible agreements but also of the procedural rules and the parties' time preferences.

The aim of this chapter is to show that in the case of incomplete information about the time preferences of the parties, the bargaining solution depends on additional elements, namely, the players' methods of making inferences when they reach a node in the extensive form of the game that is off the equilibrium path.

The solution concept commonly used in the literature on sequential bargaining models with incomplete information is one of sequential equilibrium (see Kreps and Wilson (1982)). Essentially, this concept requires that the players' strategies remain best responses at every node of decision in the extensive form of the game, including nodes that are not expected to be reached. The test of whether a player's strategy is a best response depends on his updated estimation of the likelihood of the uncertain elements in the model. For nodes of the game tree that are reachable, it is plausible to assume that the players use the Bayesian formula. Off the equilibrium path, the Bayesian formula is not applicable. The formulation of a game with incomplete information does not provide the description of how the players modify their beliefs when a "zero-probability"

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event has occurred. (A zero-probability event occurs when the players reach a node in the extensive form of the game that is off the equilibrium path.) The concept of sequential equilibrium requires that the *solution* specifies the players' new beliefs after a zero-probability event occurs. The new beliefs that a player adopts after a zero-probability event is called a *conjecture.* The sequential-equilibrium concept also requires that a conjecture be the basis for continuing the updating of the player's beliefs unless another zero-probability event occurs, in which case the player must choose another conjecture.

Although we have great freedom to select conjectures to support strategies to be best responses, ideally the sequential-equilibrium concept should enable selection of a unique outcome out of the set of sequential-equilibrium outcomes. Indeed, several sequential bargaining models reach uniqueness of the sequential equilibrium (see Sobel and Takahashi (1983), Ordover and Rubinstein (1983), and Perry (1985)). The uniqueness of sequential-equilibrium outcomes in sequential bargaining models is not robust to changes in the procedural bargaining rules or the informational structure. Even in simple models such as Fudenberg and Tirole's (1983) two-period seller-buyer bargaining game, where only the seller makes offers, the incomplete information about the seller's reservation price makes a multiplicity of equilibria possible.

In the current paper, I argue that the multiplicity of equilibria is not a drawback either of the model or of the solution concept, but rather an outcome of the arbitrariness of the choice of conjectures. Specification of rules that the players use to choose conjectures enables us to restrict the set of outcomes of the sequential equilibria. A comparison between the set of sequential-equilibrium outcomes under various assumptions about the properties of the choice of conjectures, clarifies the connection between the choice of conjectures and the outcome of the game.

To present a more concrete discussion of the conjectures problem, I analyze a special case of the model for bargaining over a partition of a dollar that I presented earlier (Rubinstein (1982)). In the present version of the model, each bargainer bears a constant cost per period of negotiation. One of the players has incomplete information about the bargaining cost of his opponent, which may be higher or lower than his own. The inferences that the player with the incomplete information makes about his opponent's bargaining cost lie at the center of the following discussion.

6.2 The model

The basic model used here is a subcase of the model analyzed in Rubinstein (1982). Two players, 1 and 2, are bargaining on the partition of one dollar. Each player in turn makes an offer; his opponent may agree to the offer (Y) or reject it (N). Acceptance of an offer terminates the game. Rejection ends a period, and the rejecting player makes a counteroffer, and so on without any given time limit.

Let $S = [0,1]$. A partition of the dollar is identified with a number s in S by interpreting s as the proportion of the dollar that player 1 receives.

A strategy specifies the offer that a player makes whenever it is his turn to do so, and his reaction to any offer made by his opponent. Let F be the set of all strategies available to a player who starts the bargaining. Formally, F is the set of all sequences of functions $f = \{f^t\}_{t=1}^{\infty}$, where

$$\text{For } t \text{ odd}, f^t\colon S^{t-1} \to S,$$

$$\text{For } t \text{ even}, f^t\colon S^t \to \{Y,N\},$$

where S^t is the set of all sequences of length t of elements of S. (In what follows, G is the set of all strategies for a player whose first move is a response to the other player's offer.)

A typical outcome of the game is a pair (s,t), which is interpreted as agreement on partition s in period t. Perpetual disagreement is denoted by $(0,\infty)$.

The outcome function of the game $P(f,g)$ takes the value (s,t) if two players who adopt strategies f and g reach an agreement s at period t, and the value $(0,\infty)$ if they do not reach an agreement. The players are assumed to bear a fixed cost per period. Player 1's utility of the outcome (s,t) is $s - c_1 t$, and player 2's utility of the outcome (s,t) is $1 - s - c_2 t$. The number c_i is player i's bargaining cost per period. The outcome $(0,\infty)$ is assumed to be the worst outcome (utility $-\infty$). It is assumed that the players maximize their expected utility.

Assume one-sided incomplete information. Player 1's cost, $c_1 = c$, is common knowledge. Player 2's cost might be either c_w or c_s, where $c_w > c > c_s > 0$. Assume that ω_0 is player 1's subjective probability that player 2's cost is c_w, and that $1 - \omega_0$ is his probability that player 2's cost is c_s. If player 2's cost is c_w, it is said that he is of type 2_w, or the "weaker" type; if player 2's cost is c_s, it is said that he is of type 2_s, or the "stronger" type. Consider these numbers to be small; specifically, assume that $1 > c_w + c + c_s$.

It was shown in Rubinstein (1982) that if it is common knowledge that player 2 is of type 2_w, then the only perfect equilibrium is for player 1 to demand and receive the entire one dollar in the first period. If player 1 knows he is playing against 2_s, the only perfect equilibrium is for him to demand and receive c_s in the first period.

The game just described is one with incomplete information. Let

$(f,g,h) \in F \times G \times G$ be a triple of strategies for player 1, player 2_w, and player 2_s, respectively. The outcome of the play of (f,g,h) is

$$P(f,g,h) = \langle P(f,g), P(f,h) \rangle,$$

that is, a pair of outcomes for the cases of player 2 actually being type 2_w or 2_s.

The set of Nash equilibria in this model is very large. In particular, for every partition s, the pair $\langle (s,1),(s,1) \rangle$ is an outcome of a Nash equilibrium (see Rubinstein (1985)).

We turn now to the definition of sequential equilibrium. Define a belief system to be a sequence $\omega = (\omega^t)_{t=0,2,4,\ldots}$, such that $\omega^0 = \omega_0$ and ω^t: $S^t \rightarrow [0,1]$. The term $\omega^t(s^1, \ldots, s^t)$ is player 1's subjective probability that player 2 is 2_w after the sequence of offers and rejections $s^1, \ldots, s^{t-1}$, after player 2 has made the offer s^t and just before player 1 has to react to the offer s^t.

A sequential equilibrium is a four-tuple $\langle f,g,h,\omega \rangle$ satisfying the requirement that after any history, a player's residual strategy is a best response against his opponent's residual strategy. The belief system is required to satisfy several conditions: It has to be consistent with the Bayesian formula; a deviation by player 1 does not change his own belief; after an unexpected move by player 2, player 1 chooses a new conjecture regarding player 2's type, which he holds and updates at least until player 2 makes another unexpected move.

So far, the choice of new conjectures is arbitrary. In Section 6.4, several possible restrictions on the choice of new conjectures are presented. The study of these restrictions is the central issue of the present paper.

6.3 Review of the complete-information model

In this section, the characterization of the perfect-equilibrium outcomes in the complete-information model (where the bargaining costs are common knowledge) is reviewed.

Proposition 1 (Conclusion 1 in Rubinstein (1982)). Assume that c_1 and c_2 are common knowledge. If $c_1 < c_2$, the outcome $(1,1)$ (i.e., player 1 gets the whole dollar in the first period) is the only perfect-equilibrium outcome, and if $c_1 > c_2$, the outcome $(c_2,1)$ (i.e., player 2 gets $1 - c_2$ in the first period) is the only perfect-equilibrium outcome.

Remark. The asymmetry is due to the procedure of the bargaining. If the size of the costs is "small," the dependence of the bargaining outcome on the bargaining order is negligible.

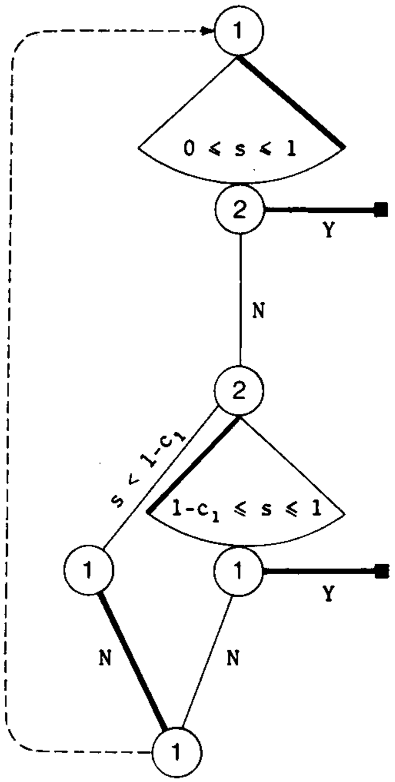

Figure 6.1

Proof.

ASSERTION 1A. If $c_1 < c_2$, the outcome $(1,1)$ is a perfect-equilibrium outcome.

PROOF. Define a pair of strategies $(\hat{f},\hat{g})$, such that

For t odd, $\hat{f}^t \equiv 1$ and $\hat{g}^t \equiv Y$,

For t even, $\hat{g}^t \equiv 1 - c_1$,

and

$$\hat{f}^t \equiv \begin{cases} Y & \text{if } s^t \geq 1 - c_1, \\ N & \text{otherwise.} \end{cases}$$

The procedure for checking that $(\hat{f},\hat{g})$ is a perfect equilibrium is straightforward and is illustrated diagrammatically in Figure 6.1. The circled numbers in the diagram are the "names" of the players who must move at the corresponding node of the game. The edges correspond to moves in the game. Heavy edges correspond to moves planned by the pair of strategies $(\hat{f},\hat{g})$. Light edges are deviations. Whenever the continuation of the strategies is the same for a range of moves, one of two types of notation is used. A light edge with a formula like $s < 1 - c_1$ means that the continuation is the same for every offer s satisfying the formula $s < 1 - c_1$. An arch with a formula like $0 \leq s \leq 1$ means that the continuation of the strategies is the same for every offer s satisfying the formula $0 \leq s \leq 1$. The heavy edge of the segment corresponds to the only offer in the range that is the planned offer. The small solid squares designate terminal points of the game.

ASSERTION 1B. If $c_1 < c_2$, the outcome $(1,1)$ is the only perfect-equilibrium outcome.

PROOF. Let U^1 be the set of all $u = s - c_1(t-1)$, where (s,t) is a perfect-equilibrium outcome in a subgame starting with player 1's offer. Let U^2 be the set of all $u = s - c_1(t-1)$, where (s,t) is a perfect-equilibrium outcome of a subgame starting with player 2's offer. By assertion 1A, $1 \in U^1$ and $1 - c_1 \in U^2$. Since player 1 always accepts an offer $s \geq 1 - c_1$, then $1 - c_1 = \max U^2$.

Next, it is proved that $\inf U^2 \geq \inf U^1 - c_1$. Assume that $\inf U^1 - c_1 > \inf U^2$. Pick $u \in U^2$, such that $u < \inf U^1 - c_1$, and select a perfect equilibrium that corresponds to this u. It must be that player 2's first offer in this perfect equilibrium is u and that player 1 accepts it; otherwise, $u - c_1 \in U^1$. However, player 1 gains if he deviates and rejects this offer, since then he receives at least $\inf U^1$, and $\inf U^1 - c_1 > u$.

Assume that $\inf U^1 < 1$. Let $u \in U^1$, $u < 1$, and $\epsilon > 0$. Pick a perfect equilibrium that corresponds to this u. Player 2 must reject a demand by player 1 of $u + \epsilon$. Thus, for every $\epsilon > 0$, $\inf U^2 \leq u + \epsilon - c_2$, and therefore $\inf U^2 \leq \inf U^1 - c_2$, which contradicts $\inf U^2 \geq \inf U^1 - c_1$. Consequently, in $U^1 = 1$ and $U^1 = \{1\}$.

The rest of proposition 1 is proved in similar fashion.

The players' dilemma is now clearer. If it is common knowledge that player 2 is type 2_w, then player 1 gets the entire dollar. If it is common knowledge that player 2 is type 2_s, then player 1 gets only c_s. These are the two extreme possible outcomes of the bargaining. Here, player 1 does not know player 2's identity, and the solution is likely to depend on ω_0. In the rest of the chapter, we study possible ways in which the bargaining outcome depends on player 1's initial beliefs.

6.4 Conjectures

The sequential-equilibrium concept allows the free choice of an arbitrary new conjecture when a zero-probability event occurs. It seems reasonable that adopting new conjectures is not an arbitrary process. In this section, several possible consistency requirements for the choice of conjectures are described.

(C-1) Optimistic conjectures

The conjectures of $\langle f,g,h,\omega \rangle$ are said to be *optimistic conjectures* if, whenever a zero-probability event occurs, player 1 concludes that he is playing against type 2_w (i.e., the weaker type). Thus, a player whose conjectures are optimistic has the prejudgment that a deviator is type 2_w. Such conjectures serve as a threat to player 2. Any deviation by player 2 will make player 1 "play tough." It is shown in Section 6.6 that optimistic conjectures support many sequential-equilibrium outcomes. In the complete-information game, the (subgame) perfectness notion eliminates many unreasonable threats. In the incomplete-information game, many of these threats are possible, being supported by the optimistic conjectures. Optimistic conjectures have often been used in bargaining literature (see Cramton (1982), Fudenberg and Tirole (1983), and Perry (1985)). They are very useful in supporting equilibrium outcomes because they serve as the best deterring conjectures.

(C-2) Pessimistic conjectures

The conjectures of $\langle f,g,h,\omega \rangle$ are said to be *pessimistic conjectures* if, whenever a zero-probability event occurs, player 1 concludes that he is playing against type 2_s (i.e., the stronger type).

In what follows, denote by $\gtrsim_1$, $\gtrsim_w$, and $\gtrsim_s$ the preferences of players 1, 2_w, and 2_s on the set of all lotteries of outcomes.

(C-3) Rationalizing conjectures

The conjectures of $\langle f,g,h,\omega \rangle$ are said to be *rationalizing conjectures* if

1. Whenever $\omega^{t-2}(s^{t-2}) \neq 1$, $(s^t,1) \gtrsim_s (s^{t-1},0)$, and $(s^{t-1},0) >_w (s^t,1)$, then $\omega^t(s^t) = 0$, and,
2. In any other zero-probability event, $\omega^t(s^t) = w^{t-2}(s^{t-2})$.

In order to understand condition (1), imagine that player 1 makes the offer s^{t-1}, and player 2 rejects it and offers s^t, which satisfies that

$(s^t,1) \gtrsim_s (s^{t-1},0)$ and $(s^{t-1},0) >_w (s^t,1)$. That is, player 2 presents a counteroffer that is better for type 2_s and worse for type 2_w than the original offer, s^{t-1}. Then, player 1 concludes that he is playing against type 2_s.

The rationalizing conjectures enable player 2_s to sort himself out by rejecting s^{t-1} and demanding an additional sum of money that is greater than c_s but less than c_w.

The underlying assumption here is that a player makes an offer hoping that his opponent will accept it. Thus, making an offer s^t, where $(s^{t-1},0) >_w (s^t,1)$ and $(s^t,1) \gtrsim_s (s^{t-1},0)$, is not rational for type 2_w, and is rational for type 2_s. Therefore, player 1 adopts a new conjecture that rationalizes player 2's behavior.

By condition (2), in the case of any other unexpected move made by player 2, player 1 does not change his prior.

The analysis of a weaker version of the rationalizing requirement for a more general framework of the bargaining game with incomplete information is the issue of a previous paper (Rubinstein (1985)).

There are many reasonable requirements on conjectures that are not discussed here. Let me briefly mention three other requirements found in the literature.

(C-4) Passive conjectures

The conjectures of $\langle f,g,h,\omega \rangle$ are *passive* if $\omega^t(\underset{\sim}{s}^t) = \omega^{t-2}(\underset{\sim}{s}^{t-2})$ whenever neither type 2_w nor type 2_s plans to reject s^{t-1} and to offer s^t after the history $\underset{\sim}{s}^{t-2}$ and after player 1 offered the partition s^{t-1}. In other words, unless the Bayesian formula is applicable, player 1 does not change his beliefs.

It should be noted that in complete-information game-theoretic models, it is usually assumed that players react passively about the basic conjecture, that is, that all of the players behave rationally. Even when a player makes a move that is strongly dominated by another move, all of the other players continue to believe that he is a rational player.

(C-5) Monotonic conjectures

The conjectures of $\langle f,g,h,\omega \rangle$ are said to be *monotonic* if, for every $s^1, \ldots, s^{t-1}$ and $x < y$ (t even), $\omega^t(s^1, \ldots, s^{t-1},y) \geq \omega^t(s^1, \ldots, s^{t-1},x)$. In other words, the lower player 2's offer, the greater player 1's probability that he is playing against type 2_s.

(C-6) Continuous conjectures

The belief system ω is said to be *continuous* if, for every t, $\omega^t(s^1, \ldots, s^t)$ is a continuous function.

Note that although the preceding consistency requirements are defined in terms of the present bargaining game, the definitions may be naturally extended to a wider class of games. In particular, it is easy to define the analogs of these properties for seller–buyer games in which the buyer's or the seller's reservation price is unknown.

6.5 Several properties of sequential equilibrium in this model

The following several properties of sequential equilibrium in this model are valid without further assumptions about the choice of conjectures.

Proposition 2. In any sequential equilibrium,

1. Whenever it is player 2's turn to make an offer, players 2_w and 2_s make the same offer (although they might respond differently to player 1's previous offer);
2. If player 1 makes an offer and player 2_s accepts it, then player 2_w also accepts the offer;
3. If player 1 makes an offer, x, that player 2_w accepts and player 2_s rejects, then player 2_s makes a counteroffer, y, which is accepted by player 1 where $x - c_s \geq y \geq x - c_w$.

Outline of the proof (For a full proof, see Rubinstein (1985)).

1. Assume that there is a history after which players 2_w and 2_s make two different offers, y and z, respectively. After player 2 makes the offer, player 1 identifies player 2's type. Player 1 accepts z because otherwise he gets only c_s in the next period. If in the sequential equilibrium player 1 rejects the offer y, then he would get the whole dollar in the next period and player 2_w does better by offering z. If player 1 accepts both offers, y and z, the type that is supposed to make the higher offer (the worst for player 2) deviates to the lower offer and gains.
2. Note that if player 2_s accepts player 1's offer and player 2_w rejects it, then player 2_w reveals his identity and player 1 receives the whole dollar in the next period. Player 2_w gains by accepting player 1's offer.
3. If player 2_w accepts x and player 2_s offers y, player 1 identifies player 2_s and accepts y. Thus, if $y < x - c_w$, player 2_w does better by rejecting x; and if $y > x - c_s$, player 2_s does better by accepting x.

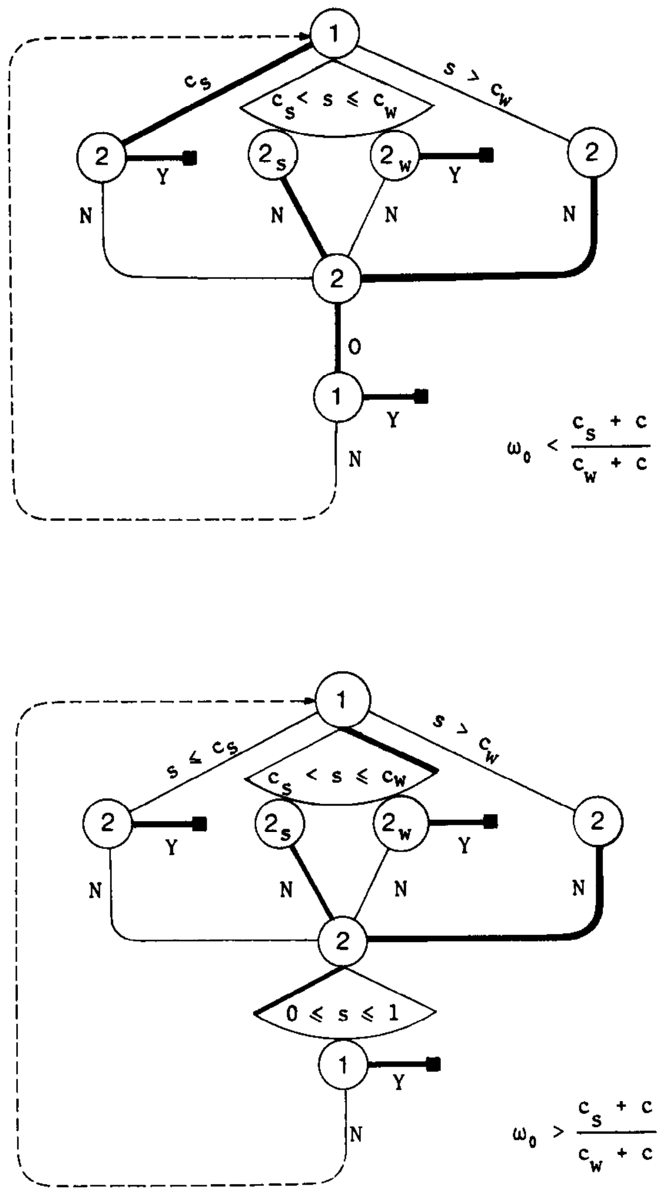

Figure 6.2

6.6 Pessimistic and optimistic conjectures

The following two propositions show how dramatically the equilibrium outcomes vary under different methods of choosing conjectures.

Proposition 3. In any sequential equilibrium with pessimistic conjectures, the outcome is

$$\langle (c_s,1),(c_s,1)\rangle \text{ if } \omega_0 < \frac{c_s + c}{c_w + c},$$

$$\langle (c_w,1),(0,2)\rangle \text{ if } \frac{2C}{C_w + C} > \omega_0 > \frac{c_s + c}{c_w + c}.$$

Proof. Figure 6.2 illustrates sequential equilibrium with pessimistic conjectures in both cases. By proposition 2, both types of player 2, 2_w and 2_s, always make the same offer. In sequential equilibrium with pessimistic conjectures, the offer must be 0 and has to be accepted by player 1; otherwise, player 2 would deviate, offering some small positive ϵ. This persuades player 1 that player 2 is type 2_s, and player 1 accepts the offer. Since player 1 accepts the offer of 0 in the second round, the only two possible outcomes of a sequential equilibrium are $\langle (c_w,1),(0,2)\rangle$ and $\langle (c_s,1),(c_s,1)\rangle$. The exact outcome is determined by the relationship between ω_0 and $(c_s + c)/(c_w + c)$.

Proposition 4.

1. If $\omega_0 \leq 2c/(c + c_w)$, then, for every $1 - c + c_s \geq x^* \geq c$, $\langle (x^*,1),(x^*,1)\rangle$ is a sequential-equilibrium outcome with optimistic conjectures.
2. If $\omega_0 > (c_s + c)/(c_w + c)$, then for every $1 - c + c_s \geq x^* \geq c_w$, $\langle (x^*,1),(x^* - c_w,2)\rangle$ is a sequential-equilibrium outcome with optimistic conjectures.

Proof.

1. Figure 6.3 is a diagrammatic description of a sequential equilibrium with optimistic conjectures whose outcome is $\langle (x^*,1),(x^*,1)\rangle$. The symbol $1 \Leftrightarrow 2_w$ stands for the continuation of the equilibrium as in the complete-information game with players 1 and 2_w. Note that a deviation by player 1, by demanding more than x^*, is not profitable since the most that he can hope for from a deviation is

$$\omega_0(x^* - c + c_w) + (1 - \omega_0)(x^* - 2c) = x^* + \omega_0 c_w - c(2 - \omega_0) \leq x^*.$$

The restriction $x^* \leq 1 - c + c_s$ is needed for assuming that player 2_s will not prefer to reject the offer x^*.

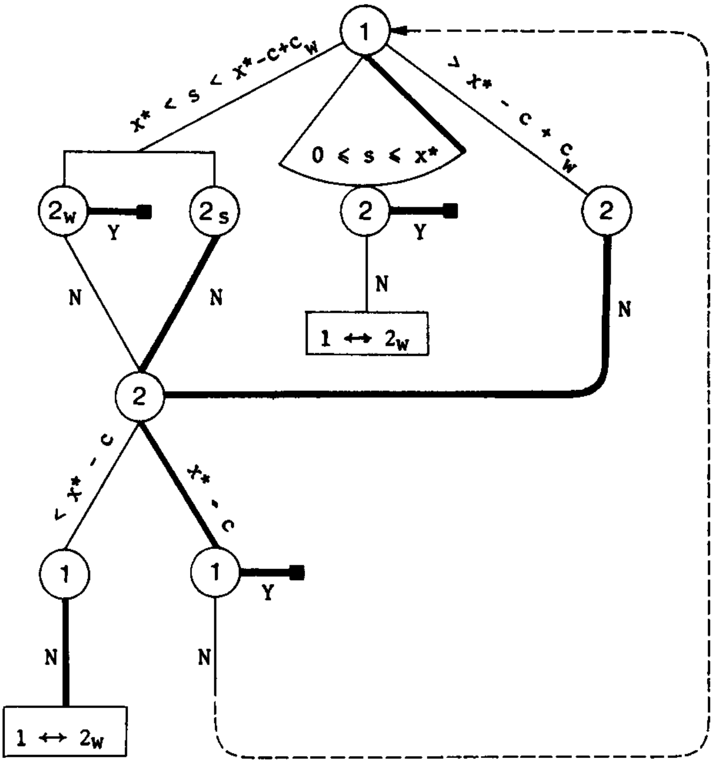

Figure 6.3

2. Figure 6.4 is an illustration of a sequential equilibrium with optimistic conjectures whose outcome is $\langle(x^*,1),(x^* - c_w,2)\rangle$. Note that if player 1 demands only $x^* - c_w + c_s$, he does not gain, since if $(c + c_s)/(c + c_w) < \omega_0$,

$$x^* - c_w + c_s < \omega_0 x^* + (1 - \omega_0)(x^* - c_w - c).$$

We have shown that optimistic conjectures turn almost every outcome into a sequential-equilibrium outcome. A very small ω_0 is sufficient to support a sequential equilibrium in which player 1 receives almost as much of the dollar as he would receive had he known with certainty that he was playing against player 2_w. On the other hand, pessimistic conjectures shrink the set of sequential equilibrium outcomes such that player 1 receives virtually nothing, since player 2 is always able to persuade him that he is type 2_s.

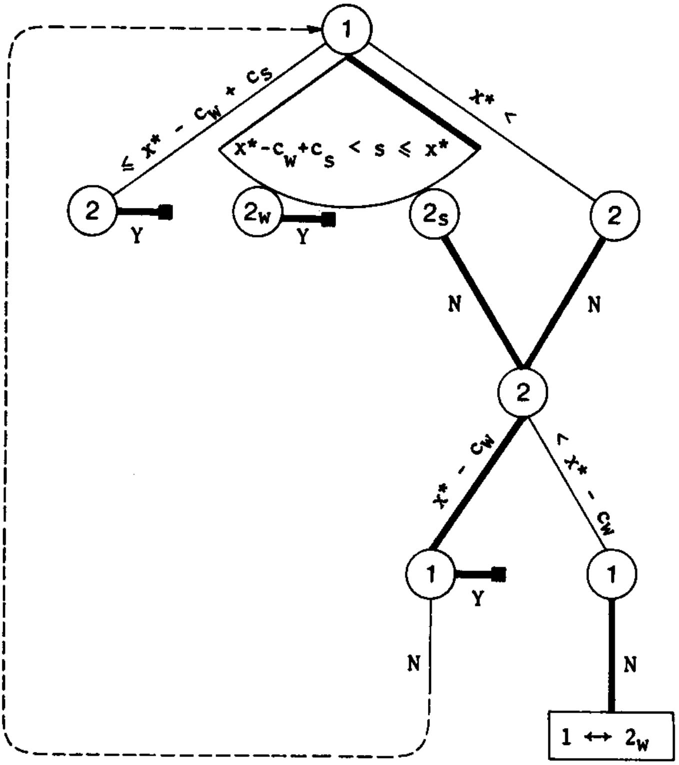

Figure 6.4

The sharp differences between the set of sequential-equilibrium outcomes under pessimistic and optimistic conjectures is no coincidence. It points to a sensible connection between conjectures and the bargaining outcome: Optimism strengthens player 1's position by limiting player 2's prospects of deviation.

6.7 Rationalizing conjectures

The next proposition states that for almost all ω_0, there is a unique (C-3) sequential-equilibrium outcome. If ω_0 is small enough (under a certain cutting point, ω^*), player 1 receives almost nothing. If ω_0 is high enough (above ω^*), player 1 receives almost the entire dollar.

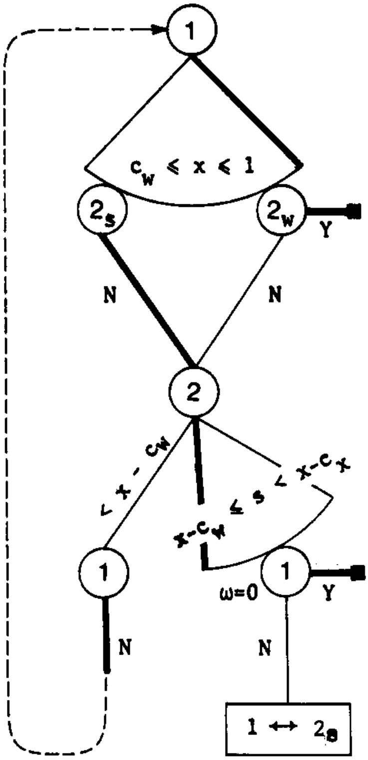

Figure 6.5

Proposition 5. For any sequential equilibrium with rationalizing conjectures,

1. If $\omega_0 > 2c/(c + c_w)$, its outcome is $\langle(1,1),(1 - c_w,2)\rangle$;
2. If $2c/(c + c_w) > \omega_0 > (c + c_s)/(c + c_w)$, its outcome is $\langle(c_w,1),(0,2)\rangle$;
3. If $(c + c_s)/(c + c_w) > \omega_0$, its outcome is $\langle(c_s,1),(c_s,1)\rangle$.

The proof of this proposition follows the basic logic of the main theorem in Rubinstein (1985). Here, many simplifications are possible because the time preferences are very simple. In this review, I will settle for presenting a sequential equilibrium with the outcome $\langle(1,1),(1 - c_w,2)\rangle$ for the case where $\omega_0 > 2c/(c + c_w)$. Figure 6.5 describes this sequential equilibrium.

Player 1 offers the partition 1, player 2_w accepts the offer, and player 2_s rejects it and offers $1 - c_w$. The offer $1 - c_w$ persuades player 1 that he is playing against type 2_s, and he accepts the offer. Even if player 1 demands $x < 1$, player 2_s rejects the offer x (unless $x \le c_s$) and makes the offer $\max\{0, x - c_s\}$, which persuades player 1 that player 2 is type 2_s. If player 2 offers player 1 less than $x - c_w$, then player 1 rejects it without changing his subjective probability, ω_0. The rejection is optimal for player 1 because $\omega_0(1 - c) + (1 - \omega_0)(1 - 2c - c_w) > 1 - c_w$, since $\omega_0 > 2c/(c + c_w)$.

Remark. The characterization of sequential equilibrium remains valid when we replace (C-3,b) with a weaker condition, (C-3,b*).

(C-3,b) Monotonicity with respect to insistence*

The conjectures of $\langle f,g,h,\omega\rangle$ are said to be *monotonic with respect to insistence* if, whenever $\omega^{t-2}(s^{t-2}) \neq 1$, and player 2 rejects an offer s^{t-1} and offers the partition s^t satisfying that for both types, $(s^t,1)$ is better than $(s^{t-1},0)$ (i.e., $s^t \le s^{t-1} - c_w$), then $\omega^t(s^t) \le \omega^{t-2}(s^{t-2})$.

The role of condition (C-3,b*) is to prevent player 1 from "threatening" player 2 that insistence will increase player 1's probability that he is playing against player 2_w.

Remark: I have little to say about sequential equilibrium with passive conjectures. However, the following partial observations indicate a strengthening in player 1's position relative to sequential equilibrium with rationalizing conjectures. This occurs because, with rationalizing conjectures, player 2_s could identify himself only by rejecting an offer x and making a new offer $x - c_w$. With passive conjectures, it might also be that in equilibrium, player 2_s identifies himself by rejecting x and offering a certain y_0 satisfying $x - c_s > y_0 > x - c_w$.

Proposition 6. The following are possible outcomes of sequential equilibrium with passive conjectures:

1. If $\omega_0 \ge c/c_w$, $\langle(1,1),(1-\epsilon,2)\rangle$ is a sequential-equilibrium outcome for $c/\omega_0 \ge \epsilon \ge c_w$.
2. If $\omega_0 \le 2c/(c + c_w)$, either $\langle(c_w,1),(0,2)\rangle$ or $\langle(c_s,1),(c_s,1)\rangle$ is a sequential-equilibrium outcome.

The proof is omitted since it repeats ideas that appear in the construction of equilibria in previous proofs.

6.8 Final remarks

The results in Sections 6.6 and 6.7 reveal systematic differences between bargaining outcomes due to systematic differences in the choice of conjectures. What has been done here is partial in many ways:

1. The informational structure is very special: one-sided uncertainty and only two possible types.
2. A special class of time preferences (fixed bargaining costs) is used.
3. A special bargaining problem is studied: partition of a dollar.
4. Only three sets of conjectures are analyzed.

However, I believe that the results indicate the spirit of more general results pertaining to the influence of the choice of conjectures on the bargaining outcome.

It seems that the next important task in extending the analysis is a systematic study of the choice of conjectures. Interesting partial orderings on conjectures-choice methods are likely to derive interesting comparative static results.

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