A SEQUENTIAL CONCESSION GAME WITH ASYMMETRIC INFORMATION*

JANUSZ A. ORDOVER AND ARIEL RUBINSTEIN

I. INTRODUCTION

The essence of the situation to be studied here is this: two players are involved in a conflict that can be resolved in only two possible ways. Each player favors a different outcome. During the bargaining phase, which lasts until some finite time τ_0, each player has the option to concede. If none concedes, the game ends at time τ_0 in a way that is known to one of the players from the beginning of the game. The other player is uncertain about the outcome. Time is valuable, but each player prefers to receive his favored outcome at τ_0 than to concede immediately.

Thus, the game analyzed here is characterized by the asymmetric information about the outcome at the end point, i.e., at time τ_0. Whereas one player is perfectly informed about the possible outcome; the other is compelled to deduce the information from the actions of his informed opponent. The latter, in turn, can try to exploit his initial advantage by manipulating the flow of information. For example, the informed player may adopt a tough stance in order to create the impression that he is not afraid of forcing the resolution of the conflict at τ_0 and thereby builds his reputation.[1] At the same time, the uninformed player by not conceding can test the opponent's resolve.

This brief description suggests that the problem analyzed here is a variant of the game of attrition.[2] However, our problem differs from the standard game of attrition in two respects. First, we formulate the game in discrete time, which has important analytic consequences. These are fully developed in Hendricks and Wilson [1985]. Second, and more important perhaps, we study

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1. Reputation-building is extensively discussed in Wilson [1985].

2. Wars of attrition are studied, for example, in Riley [1980], Bliss and Nalebuff [1984], Osborne [1985], Weiss and Wilson [1984], and Hendricks and Wilson [1985]. In particular, Hendricks and Wilson provide an extensive discussion of equilibria in the war of attrition in discrete time. They do not, however, allow for private asymmetric information. Bliss and Nalebuff [1984] allow for asymmetric information about the time preferences in continuous time models.

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the game of attrition in a setting with asymmetric information. However, unlike Bliss and Nalebuff [1984], for example, we do not posit that private information pertains to the costs of *fighting* the game. These are assumed to be common knowledge. Here, the informational asymmetries pertain to the costs of failing to make the agreement before the prespecified time. This type of asymmetric information is, we think, of independent interest. Indeed, asymmetric information about the disagreement outcome is an essential feature of many realistic game situations, two of which are instanced below. Others can also be readily supplied.

The analytic approach adopted here is closely related to that of Kreps and Wilson [1982b], who also consider a game with one-sided imperfect information. The key common feature is the assumption that one of the informed players has a dominant strategy for the duration of the game. As a consequence, all nodes of a game are reached with a positive probability. Following these two authors, we also utilize sequential Bayesian equilibrium as the appropriate solution concept.

Besides asymmetric information, the key features of the situation are that full information will be revealed at the end of the bargaining horizon and that compromises are not possible. The absence of compromise permits a simple characterization of equilibria and facilitates comparative statics analysis of optimal strategies. It also bares clear the information-transmission and reputation-building features of the concession game.

The following are two possible interpretations of the model.

A. Two firms, an incumbent and an entrant, compete for control of a natural monopoly market. The entrant's product is of unknown quality. Its quality will be revealed at time τ_0. If it is of high quality, the incumbent will have to exit. Each firm would rather exit sooner than later, if it is to exit at all. On the other hand, each firm prefers to be the sole supplier. In this model exit is equivalent to concession, and imperfect information pertains to product quality.

B. In some legal conflicts the dispute can be resolved in only two ways (monetary transfers being difficult or impossible): for example, an accused can be found guilty or not; a parent can get child custody or not, etc.[3] In some of these cases the parties do not consider a compromise in the pre-trial stage. Alternatively

3. Static models of pre-trial bargaining are studied in Landes [1981], Gould [1973], Shavell [1982], and P'ng [1983].

they take a verdict as being exogenously given. It is also plausible that one party has better information about the likely verdict, in case a judge has to make a decision.

The rest of the paper is as follows. In the next section we introduce the model. Section III analyzes the sequential equilibrium of the model. There we show that equilibrium strategies entail the following: at the beginning of the game the informed player concedes with some "large" probability, and then the players concede with probabilities that make their opponents indifferent between conceding at their next decision point or proceeding for one more period. Section IV discusses the efficiency of the equilibrium and presents the comparative statics results. We show, for example, that as the bargaining horizon is lengthened, various efficiency measures decline.

II. The Model

Two players, 1 and 2, are involved in a dispute that can be resolved in only two possible ways, denoted by a and b. No compromise is possible. The interests of the players are such that 1's preferred resolution is a and 2's is b. Time is discrete: $t = 0,1,2,\ldots,$ $T > 2$. We assume that T is even. (As will become clear soon, the case when T is odd is almost identical to the one with $T - 1$ periods.) In the first T periods, each player in turn, starting with player 1, may concede. If either player concedes, the game stops. If 1 concedes in period t (t even), the *outcome* of the game is (b,t); if 2 concedes at t (t odd), the outcome is (a,t). In the event that neither concedes, the conflict is resolved in the last period in a way that depends on the state of nature ω. If $\omega = w$, player 1 is "weak," denoted by 1_w, and the resolution will be b. If $\omega = s$, the player is "strong," denoted by 1_s, and the resolution will be a. The information structure is such that 1 knows the value of ω. Player 2 does not know the state of nature, but it is common knowledge that 1 does. 2's prior estimate that $\omega = w$ is equal to p_0, where $0 < p_0 < 1$.

Players 1 and 2 have preferences $\gtrsim_1$ and $\gtrsim_2$, respectively, over the set of possible lotteries of outcomes of the game. (We use the notation $p0_1 + (1 - p)0_2$ for the lottery, "get 0_1 with the probability p and get 0_2 otherwise." We assume that players have von Neumann-Morgenstern utility functions of the form $u_i(c,t) = u_i(c)\delta_i^t, = 1,2$, and $0 < \delta_i < 1$. (This representation of preferences is quite general; see Fishburn and Rubinstein [1982].) Although it is not necessary, we assume that preferences are symmetric, namely,

$$u_1(a) = u_2(b) = M > 0, \quad u_1(b) = u_2(a) = m > 0,$$
$$M > m, \qquad \delta_1 = \delta_2 = e^{-r}.$$

We assume that preferences are such that each player would rather win at time T than concede immediately. Thus, 1_s never concedes; 1_w, who cannot prevail at time T, chooses his actions based on his belief that 2 will concede. This assumption also implies that player 2 will not concede if he is certain that 1 will; otherwise, 2's behavior depends, as will be shown below, on his belief that $\omega = w$ and on the actual behavior of his opponent.

A strategy for player 1_w is a sequence $\{x^t\}_{t=0,2,4,\ldots,T-2}$, where $0 \leqq x^t \leqq 1$ is the probability of conceding at time t. Player 1_s is a dummy player, since he has a dominant strategy never to concede. A strategy for player 2 is a sequence $\{y^t\}_{t=1,3,5,\ldots,T-1}$, where $0 \leqq y^t \leqq 1$ is the probability of conceding at time t.

Player 2 faces uncertainty as to the state of nature. A *belief system* is a sequence of numbers between 0 and 1, $\{p^t\}_{t=1}^{T-1}$, where p^t is interpreted as 2's belief at the beginning of the tth period, t odd, that $\omega = w$.

We use a version of the *Sequential Equilibrium* (see Kreps and Wilson [1982a]) as our solution concept. Consider a three-tuple (x,y,p), where x,y are strategies for 1_w and 2, respectively, and p is 2's belief system. Such a three-tuple is a sequential equilibrium if the belief system satisfies the Bayesian formula which is always applicable because $p^t \leqslant p_0 < 1$; and at any decision node the residual part of each of the strategies x and y is the best response against the residual parts of the other player's strategy.

III. The Main Proposition

For the statement of the proposition we need additional notation. Let α be the probability that makes player 1 indifferent between giving up at $t = 0,2,\ldots, T-2$, and accepting the following lottery: get a with probability α in period $t + 1$, and get b with probability $(1 - \alpha)$ in period $t + 2$. That is,

$$(b,t) \sim_1 \alpha(a,t + 1) + (1 - \alpha)\,(b,t + 2).$$

Stationarity of preferences implies that α is invariant with time. α also satisfies

$$(a,t) \sim_2 \alpha(b,t+1) + (1-\alpha)(a,t+2), \; t = 1,3,\ldots, T-3.$$

Since bargaining effectively ends in period T, and player 2 moves last, let us define $\bar{\alpha}$ to satisfy

$$(a,T-1) \sim_2 \bar{\alpha}(b,T) + (1-\bar{\alpha})(a,T).$$

Finally, let $\{p_*^t\}_{t=1}^{T-1}$, t odd, be a sequence such that

$$p_*^t = 1 - (1-\bar{\alpha})(1-\alpha)^{(T-t-1)/2}.$$

We now turn to the statement of the main result. Its proof is in the Appendix.

PROPOSITION 1. There exists a sequential equilibrium of the game. Any sequential equilibrium (x,y,p) satisfies the following conditions:
(i) if $p_0 < p_*^1$, then $x^0 = 0$, and $y^1 = 1$;
(ii) if $p_0 > p_*^1$, then $x^0 = (p_0 - p_*^1)/p_0(1 - p_*^1)$,
and for all $t > 1$ (t odd), $p^t = p_*^t$, $x^{t+1}p^t = \alpha$, and $y^t = \alpha$.

Discussion. Condition (i) states that if 2 is initially pessimistic, meaning that $p_0 < p_*^1 = 1 - (1-\bar{\alpha})(1-\alpha)^{(T-2)/2}$, then he gives up at the first opportunity, which is at $t = 1$. Anticipating that, 1_w sets $x^o = 0$ which does not allow 2 to revise his initial estimate.

In the event that 2 is optimistic, $p_0 > p_*^1$, then at his first opportunity 1_w concedes with probability x^0 which reduces 2's initial estimate as to who his opponent is from p_0 to some value p^1, which is precisely equal to p_*^1.

The sequence x^t is chosen to satisfy $p^t x^{t+1} + (1-p^t) \cdot 0 = \alpha$. Thus, player 2 is made indifferent between conceding or not at time t. Similarly, the choice of y^t makes player 1 indifferent between his two options at every decision node. The initial "jump" at $t = 0$ makes p_*^{T-1} equal to $\bar{\alpha}$ and thus makes 2 indifferent between conceding and moving into the stage in which asymmetric information is resolved.

In the model described in Section II, the selection of points at which concessions can actually occur is quite arbitrary. It is useful to analyze a case in which the elapsed (real) time between the moves becomes arbitrarily small.

Let τ be a continuous time variable—$0 \leqq \tau < \infty$. Let τ_0 be the length of the *game horizon.* That is, after τ_0 has lapsed, all the relevant information will be revealed, and the game ends. The players have von Neumann-Morgenstern utility functions of the form $u_i(c,\tau) = u(c)\delta^{\tau}$ defined on the set $(a,b) \times R_+$.

Let us divide the time interval $[0,\tau_0]$ into T equal periods with the length $\Delta = \tau_0/T$. At the beginning of each period, one of the players has the opportunity to concede. Each player's time preferences over the set $\{a,b\} \times \{0,1,\ldots, T\}$ are induced by $u(c,t) = u_i(c)\delta^{\Delta t}$, $t \varepsilon [0,T]$. Then for every T we have the game whose outcome has been characterized in Proposition 1.

Our task here is to characterize the outcomes of the game as $T \to \infty$ (i.e., as $\Delta \to 0$); that is, as the time between two concession points shrinks to zero.

Notice first that α, $\overline{\alpha}$, all depend on T:

$$\alpha(\Delta) = \frac{m(1 - \delta^{2\Delta})}{M\delta^{\Delta} - m\delta^{2\Delta}}$$

$$\overline{\alpha}(\Delta) = \frac{m(1 - \delta^{\Delta})}{M\delta^{\Delta} - m\delta^{\Delta}}.$$

The threshold probability p_*^1 is given by $p_*^1 = 1 - (1 - \overline{\alpha})(1 - \alpha)^{(T-2)/2}$. It satisfies

$$\lim_{T\to\infty} p_*^1 = 1 - \delta^{(m/M-m)\tau_0} = 1 - e^{-\tau_0/2\lambda}, \quad \lambda = (M - m)/2mr,$$

as can be demonstrated by applying l'Hopital rule to the expression for p_*^1.

Given that 1 does not concede immediately, the probability that one of the players will concede at *his* concession point is just $\alpha(\Delta)$. Consider now the quantity $\Delta/\alpha(\Delta)$ which is the expected time of the first concession. As $T \to \infty$, this ratio tends to

$$\lim_{T\to\infty} \frac{\Delta}{\alpha(\Delta)} = \frac{M - m}{2mr} = \lambda,$$

which does not depend on Δ.

For a small Δ, and for any τ, the probability that no concession occurs before time τ, provided that player 2 has not given up immediately can be approximated by $e^{-\Delta/\lambda}$. Therefore, provided that 1 does not concede immediately, in the limit as $T \to \infty$, the distribution of concession times tends to exponential distribution [Feller, 1966, vol. II]; namely, prob {a player concedes before time τ} $= 1 - e^{-\tau/\lambda}$, where λ^{-1} is the "hazard rate."

The limit distribution of the different outcomes of the game is summarized in

PROPOSITION 2. For τ_0 satisfying $Me^{-r\tau_0} > m$ and $p_0 > \lim_{T\to\infty} p_*^1(T)$, the limit outcomes of the sequential equilibrium of the game when $T\to\infty$ are (i) Player 1 concedes at time 0 with probability,

$$\frac{p_0 - 1 + e^{-\tau_0/2\lambda}}{e^{\tau_0/2\lambda}},$$

where $\lambda = (M - m)/2mr$. (ii) Conditional on player 1 not conceding immediately, the distribution of concessions is exponential with hazard rate $1/\lambda$; (iii)The probability that no player concedes before τ_0 is $(1 - p_0)e^{-\tau_0/2\lambda}$.

IV. COMPARATIVE STATICS RESULTS

The formula presented in Proposition 2 makes it easy to calculate several measures of efficiency properties of the equilibrium.

A. The probability that player 2 concedes to 1_w. This is given by $1/2\ p_0\ (1 - e^{-\tau_0/2\lambda})(1 - p_0)e^{\tau_0/2\lambda}$ if $p_0 > p_*^1(\tau_0)$ and is equal to p_0 if $p_0 < p_*^1$. Thus, lengthening the game horizon τ_0 increases the probability of the inefficient outcome of the game. For example, it increases the probability that an incumbent purveying a low quality good will prevail over the high quality incumbent. On the other hand, an increase in λ, which implies a fall in the hazard rate, decreases the probability of an inefficient outcome.

B. The expected utilities of the players. These are given in Table I.

From Table I we note that a lengthening of the bargaining horizon lowers the expected utilities of strong and uninformed players. A fall in the hazard rate (an increase in λ) benefits the uninformed player but has an ambiguous effect on the expected utility of the strong informed player.

TABLE I

Player	Utilities when	Utilities when
	$p_*^1 > p_0$	$p_*^1 < p_0$
1_s	M	$M \cdot E_k(e^{-rk}) + Me^{-r\tau_0}(1 - e^{-\tau_0/2\lambda})$
1_w	M	m
2	m	$p_0x^0M + (1 - p_0x^0)m$

E denotes the expectation operator, and k is distributed exponentially with the hazard rate $1/2\lambda$.

APPENDIX: PROOF OF PROPOSITION 1

We shall prove the Theorem in a series of steps. Notice the similarity of the proof with the line of reasoning in Kreps and Wilson [1982b].

Let (x,y,p) be a sequential equilibrium. Then,

Step 1. If $y^t = 1$, then $x^{t-1} = 0$, and $y^{t-2} = 1$.

Indeed, since $(a,t) \gtrsim_1 (b,t-1)$, 1_w will not concede at $t-1$ if he knows for sure that 2 will concede in the next period. Therefore, if $y^t = 1$, then $x^{t-1} = 0$. But if $x^{t-1} = 0$, then $p^t = p^{t-2}$. Since 2 is not more optimistic at $t-2$ than he is at t and since $(a,t-2) \gtrsim_2 (a,t)$, then it must be that $y^{t-2} = 1$. This means that if it is optimal for 2 to concede with probability 1 in some period $t > 1$, then it is optimal for him to concede immediately, i.e., at $t = 1$.

Step 2. If $y^t = 0$, then $x^{t+1} = 0$, and $y^{t+2} = 0$.

Assume that $x^{t+1} > 0$. This implies that along the equilibrium path continuing beyond period $t+1$ cannot yield 1_w a better outcome than $(b,t+1)$. This is because if it would, then 1_w should set $x^t + 1 = 0$. And, 1_w can assure himself $(b,t+1)$ by conceding at $t+1$. Now, since $(b,t-1) \gtrsim_2 (b,t+1)$ and since $y_t = 0$, 1_w should have given up in period $t-1$; i.e., $x^{t-1} = 1$. But if $x^{t-1} = 1$ and if 2 still has to make a move to time t, this means that he is playing against 1_s ($p^t = 0$). Hence 2's optimal strategy is to set $y^t = 1$, which contradicts our assumption that $y^t = 0$.

Similarly, if $y^{t+2} > 0$, then since $(a,t) \succ_2 (a,t+2)$ and $x^{t+1} = 0$, 2's optimal strategy is to set $y^t = 1$, again contradicting the assumption that $y^t = 0$.

Step 3. $y^t \neq 0$.

By Step 2, if $y^{t_0} = 0$, then for all $t > t_0$, $x^t = 0$, and $y^t = 0$. However, since $(b,t_0+1) \gtrsim_1 (b,T)$, it would be better for 1_w to concede at $t_0 + 1$ ($x^{t_0+1} = 1$), which is a contradiction. Consequently, we have shown that it is never optimal for 2 to be intransigent.

Step 4. If $y^{t_0} < 1$, then for all $t_0 \leqq t < T$ (t odd)

$$p^t x^{t+1} = \alpha, \text{ and } p^{T-1} = \overline{\alpha}.$$

If $y^{t_0} < 1$, then steps 1 and 3 imply that $0 < y^t < 1$ for all $t \geqq t_0$. Therefore, $p^{T-1} = \overline{\alpha}$. This is because at $T-1$ player 2 is indifferent between conceding and not conceding.

Since the continuation of the game after period t is preference-equivalent for 2 to getting (a,t), it must be that for $t_0 \leqq t < T-2$,

$$(a,t) \sim_2 p^t x^{t+1} (b,t+1) + (1 - p^t x^{t+1})(a,t+2).$$

Given that preferences are stationary, we can note that this implies $p^t x^{t+1} = \alpha$.

A similar argument establishes that $y^t = \alpha$ for all t, $t \geqq t_0 + 2$.

Step 5. If $y^t < 1$, then $p^t = p^t_*$, $t > t_0$.

By Step 4, if $y^{t_0} < 1$, $p^{T-1} = \overline{\alpha} = p_*^{T-1}$, and $p^t x^{t+1} = \alpha$ for all $t \geqq t_0$. Therefore, using the Bayesian formula in question (2), we can obtain

$$p^t = \frac{p^{t-2}(1 - x^{t-1})}{1 - p^{t-2}x^{t-1}} = \frac{p^{t-2} - \alpha}{1 - \alpha},$$

which implies that $p^t = p^t_*$.

Step 6. If $p_0 < p^1_*$, then $x^0 = 0$, and $y^1 = 1$.

From (1) it follows that $p^1 \leqq p_0$. Since $p^1_* > p_0$, then $p^1 < p^1_*$.From Step 5 this implies that $y^1 = 1$. Consequently, $x^0 = 0$.

Step 7. If $p_0 > p^1_*$, then $x^0 = (p_0 - p^1_*)/(p_0(1 - p^1_*))$, and $y^1 < 1$. First, we show that $p^{T-1} \neq p_0$; that is, that $x^t \neq 0$ for all t. Assume that $p^{T-1} = p_0$. Then we have that $p^1_* > \bar{\alpha}$. Hence 2's optimal strategy in period $T - 1$ is to set $y^{T-1} = 0$. This contradicts the result that $y^t \neq 0$.

Next we show that $y^1 \leqq 1$. Since $p^{T-1} \neq p_0$, there exists a minimal t_0 such that $x^{t_0} > 0$. That is, there exists some t_0 after which 1_w concedes with positive probability. From Steps 1 and 3 we have $0 < y^{t_0+1} < 1$. Therefore, by Step 5, $p^{t_0+1} = p^{t_0+1}_*$. Therefore,

$$x^{t_0} = (p^{t_0-1} - p^{t_0+1}_*)/(p^{t_0-1}(1 - p^{t_0+1}_*)).$$

Note that $p^{t_0-1} = p_0$, because t_0 is the first t such that $x^t > 0$. Hence, $p^{t_0-1}x^{t_0} = (p_0 - p^{t_0+1}_*)/(1 - p^{t_0+1}_*)$. Using the fact that $p_0 > p^1_*$, this implies that $p^{t_0-1}x^{t_0} > \alpha$. From which we conclude that unless $x^{t_0} = 0$, $y^{t_0-1} = 0$. But in Step 3 we have shown that $y^t \neq 0$. Therefore, $x^0 > 0$, and by Step 1, $y^1 < 1$.

Since $y^1 < 1$, then by Step 3, $0 < y^1 < 1$. By Step 5, $p^1 = p^1_*$. Hence

$$x^0 = (p_0 - p^1_*)/(p_0(1 - p^1_*)).$$

Step 7 also completes the proof.

NEW YORK UNIVERSITY
HEBREW UNIVERSITY

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