CHAPTER 5

A sequential strategic theory of bargaining

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1 Introduction

The purpose of this survey is to review the development of the sequential strategic approach to the bargaining problem, and to explain why I believe that this theory may provide a foundation for further developments in other central areas of economic theory.

John Nash started his 1950 paper by defining the bargaining situation:

> A two person bargaining situation involves two individuals who have the opportunity to collaborate for mutual benefit in more than one way. . . . The two individuals are highly rational, . . . each can accurately compare his desire for various things . . . they are equal in bargaining skill.

Given a bargaining situation, we look for a theoretical prediction of what agreement, if any, will be reached by the two parties.

I began with this clarification because of the existing confusion in some of the literature among the above problem and the following (nonexclusive) ethical questions: "What is a just agreement?" "What is a reasonable outcome for an arbitrator's decision?" and "What agreement is optimal for society as a whole?" These questions differ from the current one mainly in that they allow derivation of an answer from a social welfare optimization. An a priori assumption that a solution to the bargaining problem satisfies collective rationality properties seems inappropriate.

The survey was prepared while I visited the Department of Economics at the University of Western Ontario, and the IMSSS, Stanford University. I would like to thank Ken Binmore and Asher Wolinsky for their encouragement and for the insights I got from them during the past five years. Asher Wolinsky, Maria Herrero, Christopher Harris, Motty Perry, Stephen Turnbull, and in particular Ken Binmore provided many useful comments on an earlier draft of the paper.

The list of references in this paper does not purport to be comprehensive. It includes mainly papers in which the infinite-alternating-offers, sequential-bargaining model is used.

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The bargaining problem was presented by Edgeworth (1881) who considered it the most fundamental economic situation. Edgeworth did not go beyond identifying the entire "contract curve" (the set of individually rational and Pareto optimal agreements) as the set of possible agreements. For years economists tended to agree that further specification of a bargaining solution should depend on the vague notion of the "bargaining ability" of the players. Among the exceptions are Zeuthen (1930) and Hicks (1932), but their models assumed patterns of concession behavior that were not derived from rational behavior assumptions.

The theory of von Neumann and Morgenstern inspired Nash to suggest two approaches to solving the bargaining problem: first, the axiomatic approach [see Nash (1950); for a survey of the axiomatic approach literature see Roth (1979)]. The axiomatic method is explained in Nash (1953):

> One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely.

The drawback of the axiomatic approach is that it is too general. The general, abstract terms it uses and the minimal information it assumes make it hard to check the reasonableness of the axioms. In particular, Nash's axioms of Pareto optimality and Independence of Irrelevant Alternatives (IIA) have the flavor of "collective rationality" and are therefore controversial.

Nash (1953) describes the second approach to the bargaining problem, namely the strategic (noncooperative) approach:

> . . . one makes the players' steps of negotiation . . . become moves in the non-cooperative model. Of course, one cannot represent all possible bargaining devices as moves in the non-cooperative game. The negotiation process must be formalized and restricted, but in such a way that each participant is still able to utilize all the essential strength of his position.

The main difficulty with the strategic approach lies in the need to specify the moves in the game. Bargaining situations do not have a unique procedure. Therefore any bargaining game can be accused of being too special.

Nash himself felt the need to complement his axiomatic solution by a strategic model. He suggested a static strategic model in which the players make simultaneous, once-and-for-all demands. If the demands are compatible, they form the terms of the agreement; incompatible demands cause disagreement. Every agreement on the contract curve is an equilibrium outcome for this game; however, by requiring some sort of stability related to uncertainty about the compatibility of demands near the Pareto frontier, Nash established that his solution is the only necessary limit of equilibrium outcomes when the uncertainty becomes negligible. [For a clearer modification of Nash's demand game, see Binmore (1986a).] Some other pioneering strategic models of bargaining were studied by Harsanyi and Selten and are not in the scope of this survey.

Nash (1950) created the standard informational framework for the axiomatic bargaining approach. The approach is based solely on information about the bargaining preferences over lotteries in which the outcomes are taken from among the set X of possible agreements and the disagreement outcome D. These preferences are assumed to satisfy the von Neumann–Morgenstern (VM) assumptions. Nash's symmetry axiom excludes any information, other than the attitude toward risk, from being relevant to the solution.

The sequential strategic approach to bargaining is motivated by the desire to construct a bargaining theory built on information about the time preferences of the players. However, as will be explained later, it leads to unexpected support for the Nash bargaining solution. This linkage supports Nash's (1953) assertion that "the two approaches to the problem, via the negotiation model or via the axioms, are complementary; each helps to justify and clarify the other."

Let us start by reviewing the sequential bargaining model of Rubinstein (1982), which is the basis for the current survey.

2 A sequential bargaining model

2.1 *The bargaining situation*

The cornerstone of the model is the following bargaining situation: Two players, named 1 and 2, are bargaining over the set of feasible agreements $X = [0, 1]$. The players have opposing preferences over X. If $x > y$, player 1 prefers x to y and player 2 prefers y to x.

Classical economic situations that fit this bargaining situation include:

1. Two people would like to divide 1 dollar that they own jointly. An element $x \in X$ is the portion of the dollar that player 1 receives.
2. A seller of one unit of a good with reservation price 0 wishes to sell the good to a buyer with reservation price 1. A number $x \in X$ stands for the price the buyer pays the seller.
3. An employer faces a stream of profits. A member $x \in X$ is the proportion of the profits that is given to the employees.
4. Two agents in a bartering economy own initial bundles $(1, 0)$ and $(0, 1)$. The contract curve of the proper Edgeworth box can be made equivalent to X by identifying $x \in X$ with the point on the curve where 1 is left with x units of his initial commodity.

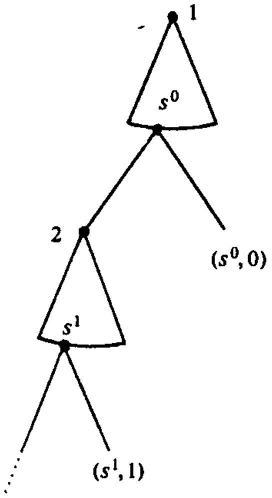

Figure 1

Remark: Note that x and $1-x$ are not necessarily to be identified with VM utilities. In fact, by varying the associated VM utility functions, the situation can be mapped onto any Nash bargaining problem.

2.2 *The bargaining procedure*

Events in the bargaining procedure are confined to times in the set $N=\{0,1,2,3,\dots\}$. Each bargainer, in turn, offers a possible agreement and his opponent may agree to the offer, "Y," or reject it, "N." Acceptance ends the bargaining. Rejection leads to the next period when the rejecting player makes a counteroffer, and so on, without any predetermined limit on the number of repetitions of the process. There are no rules that bind the players to previous offers they made. It is assumed that the players are indifferent to the path of rejected offers made during the negotiation. An outcome of the process might be either an agreement $x \in X$ at stage n, denoted by (x, n), or perpetual disagreement, denoted by D. A diagram of the extensive form of the game is shown in Figure 1 (a radius of an arc is a choice from among the set X).

The alternating offers model was first studied (with finite horizon and fixed bargaining costs) by Ingolf Stahl (1972). An advantage of this procedure is that although it is a game form with perfect information (no simultaneous moves), the game form is "almost" symmetric. The only asymmetry arises because of the need to specify who is the first player to make an offer. Stipulating a "small" period of negotiation will eliminate this asymmetry.

2.3 *Time preferences*

The new informational element that does not appear in the Nash bargaining theory is the parties' attitudes toward time. Let $\succsim_i$ be player i's preference on the set of possible outcomes $X \times N \cup \{D\}$. The preferences are assumed to satisfy the following assumptions:

(A-1) The preferences extend the preferences in the basic bargaining situation: For all n_0,

$$(x, n_0) \succsim_1 (y, n_0) \text{ iff } (x, n_0) \precsim_2 (y, n_0) \text{ iff } x \geq y.$$

(A-2) Time is valuable: For all $1 > x > 0$ and $n_1 > n_2$,

$$(x, n_2) \succ_i (x, n_1).$$

(A-3) Continuity: $\succsim_1$ has a closed graph in the product topology.

(A-4) Stationarity: For all $x, y \in X$ and $n \in N$,

$$(x, n) \gtreqless_i (y, n+1) \text{ iff } (x, 0) \gtreqless_i (y, 1).$$

By the above assumptions, $\succsim_i$ is determined uniquely by i's preference on the outcomes in the two first periods. These assumptions guarantee the existence of a utility representation $u_i(x)\delta^n$ for all arbitrary δ [see Fishburn and Rubinstein (1982)]. They are sufficient for the existence part of Theorem 1 below. However, for the uniqueness result it will be assumed further that:

(A-5) Existence of present value: For all $x \in X$ there exists a $v_i(x) \in X$ such that $(v_i(x), 0) \sim_i (x, 1)$.

(A-6) Increasing compensation for delay: The difference $x - v_i(x)$ is a strictly increasing function of x.

Remark: One should be careful not to apply Theorem 1 automatically to all the examples listed in Section 2.1. Assumption (A-6) is quite strong in some contexts, particularly in the bartering economy example.

A leading family of time preferences that satisfies the above assumptions includes the preferences represented by the utility functions $x\delta_i^n$ and $(1-x)\delta_2^n$ (δ_i is referred to as a fixed discounting factor).

The time preferences represented by the utility functions $x - c_1 n$ and $1 - x - c_2 n$ do not satisfy (A-5) and (A-6), but will be used to illustrate the theorems because they are covered by the original paper's conditions. For a discussion of the model without (A-4), see Binmore (1986c).

2.4 *The solution*

The Nash equilibrium concept is a very weak notion of a solution to the sequential bargaining model. Every outcome (x, n) is a Nash equilibrium outcome. As usually happens in sequential games, a Nash equilibrium analysis admits the use of incredible threats; for example, a player may insist forever on a particular large demand. Such possibilities allow the support of a large class of Nash equilibria. Using a stronger concept – such as Selten's (subgame) Perfect Equilibrium (P.E.) – would not be sufficient without the assumptions in Section 2.3 about the time preferences. Both notions are necessary for Theorem 1.

> *Theorem 1* [Rubinstein (1982)]. *Let* (x^*, y^*) *be the (unique!) solution for the pair of equations*
>
> $$(y,0) \sim_1 (x,1), \qquad (x,0) \sim_2 (y,1).$$
>
> *Then the* unique *perfect equilibrium of the game is the pair of strategies in which player 1 (player 2) always makes the offer* x^* (y^*), *accepts any offer which leaves him better off than* y^* (x^*), *and rejects any offer which is strictly worse for him than* y^* (x^*).

We refer to the two equations of Theorem 1 as the *fundamental equations.*

Remark: Under somewhat weaker assumptions [dropping (A-5) and weakening (A-6)] it was originally shown that the P.E. agreements are those $x^* \in X$ and $y^* \in X$ satisfying

$$y^* = \min\{y \mid (x^*, 1) \lesssim_1 (y, 0)\}$$

and

$$x^* = \max\{x \mid (y^*, 1) \lesssim_2 (x, 0)\}.$$

2.5 *Proofs for Theorem 1*

It is easy to verify that the pair of strategies described in the theorem is a P.E. [Just note that player 2 behaves optimally when he rejects an offer x made by player 1 if $x > x^*$, because $(x, 0) <_2 (x^*, 0) \sim_2 (y^*, 1)$.]

It is also easy to show that, given the assumptions about the preferences, the fundamental equations have a unique solution. The diagram (Figure 2) of the indifference curves $(y, 0) \sim_1 (x, 1)$ and $(x, 0) \sim_2 (y, 1)$ is useful in illustrating this fact.

The more interesting part of the proof is to show the uniqueness of the P.E. In what follows, two proofs are sketched. They reveal different in-

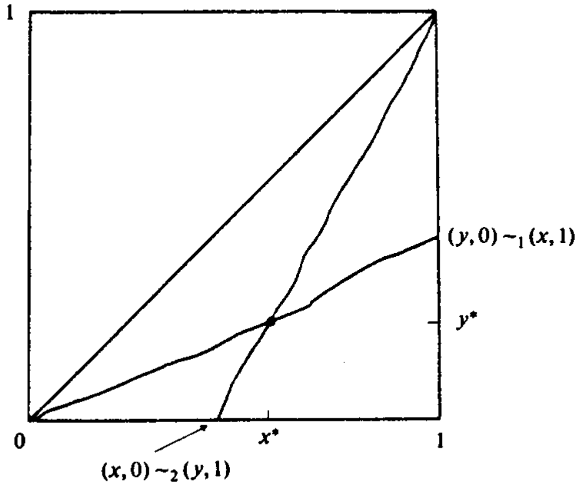

Figure 2

sights into the theorem and provide useful techniques for proving other theorems as well.

Outline of Proof 1 [this is essentially Shaked and Sutton's (1984a) simplification of the original proof]: Given an outcome (x, n), denote by $v_i^n(x)$ player i's present value of (x, n); that is, $(v_i^n(x), 0) \sim_i (x, n)$. Let M_i be the supremum of $v_i^n(x)$ over all the outcomes (x, n) of P.E. in the game where i makes the first move. Similarly, define m_i as the infimum of the same set. Note that all the subgames after a rejection of an offer are equivalent to one of the two game that start with one of the bargainers making an offer.

Step 1 $(M_2, 0) \lesssim_1 (M_1, 1)$. By perfection, whenever it is player 1's turn to react, he accepts any $y \in X$ that satisfies $(y, 0) >_1 (M_1, 1)$. If $(u_2, 0) >_1 (M_1, 1)$ and u_2 is 2's present value of a P.E. starting by 2's offer, then 2 will do better by offering some y satisfying $u_2 > y$ and $(y, 0) >_1 (M_1, 1)$.

Step 2 $(M_2, 0) \gtrsim_1 (M_1, 1)$. For any $u_1 \leq M_1$ that is 1's present value of a P.E. starting with 1's offer, it is easy to construct a P.E. that starts with player 2 making an offer y satisfying $(y, 0) \sim_1 (u_1, 1)$. Therefore, $(M_2, 0) \gtrsim_1 (M_1, 1)$.

Step 3 $(M_1, 0) \gtrsim_2 (M_2, 1)$. If player 1 demands more than $v_2(M_2)$, player 2 rejects the offer. Player 2 will never offer more than $v_1(M_1) =$

M_2, which is less than $v_2(M_2)$. Therefore, $M_1 \le v_2(M2)$ and $(M_1, 0) \gtrsim_2 (M_2, 1)$.

By the construction of the P.E. with the outcome $(x^*, 0)$, $x^* \le M_1$. By steps 1 and 2, $(M_1, 1) \sim_1 (M_2, 0)$. It follows from Figure 2 that $x^* \le M_1$, $(M_1, 1) \sim_1 (M_2, 0)$, and $(M_1, 0) \gtrsim_2 (M_2, 1)$ imply that $M_1 = x^*$ and $M_2 = y^*$. Similarly, $m_1 = x^*$ and $m_2 = y^*$. ■

Outline of Proof 2 [this proof is essentially Binmore's (1986c)]:

Step 1 Define the sequences:

	$x_0 = 1$	$y_0 = 0$
for n odd	$x_n = v_1(x_{n-1})$,	$y_n = v_2(y_{n-1})$,
for n even	$x_n = v_2(x_{n-1})$,	$y_n = v_1(y_{n-1})$.

On the even numbers, $x_n \to x^*$. On the odd numbers, $y_n \to x^*$. (In Figure 2, the sequences converge to a solution of the fundamental equations.)

Step 2 There is no P.E. in which player 1's present value exceeds any of the x_N (for N even) or falls below any of the y_N (for N odd).

Step 3 By steps 1 and 2, the only present value of a P.E. of the game for player 1 is x^* and the rest follows easily. ■

Remark: Let G_N be the bargaining game with finite horizon which ends at the end of period N. Then, for N odd (even) the only P.E. of G_N is one in which player 1 offers y_N (x_N) in the first period and player 2 accepts.

2.6 *Examples and remarks*

A *The bargaining ends immediately*

There is no inefficiency in the P.E. characterized by Theorem 1 because the bargaining ends immediately. [This is not a mere consequence of the perfectness. Unless the fundamental equations have a unique solution, this is incorrect. See Binmore (1986c) for a discussion of the circumstances under which the bargaining does not end immediately.] Thus the model cannot explain the fact that sometimes we observe long negotiations between parties. I do not find this fact disturbing because the existence of prolonged interchanges during negotiation is more naturally attributed to problems of incomplete information.

B *Pareto optimality*

If we extend the set of possible agreements to $X=\{(x_1,x_2)\mid 0\le x_i$ and $x_1+x_2\le 1\}$ and assume that player i cares only about x_i, then the proofs of Theorem 1 validate the conclusion that in the unique P.E., player 1 offers Pareto optimal outcome $(x^*,1-x^*)$. Thus, even without assuming Pareto optimality, we are led to an efficient outcome.

C *It pays to be more patient*

Define $\succsim_1$ to be *more impatient* than $\succsim_1'$ if whenever $(y,0)\succsim_1'(x,1)$ then $(y,0)\succsim_1(x,1)$. A glance at Figure 2 reveals that $x^*(\succsim_1', \succsim_2)\ge x^*(\succsim_1, \succsim_2)$. Thus, as expected, being more patient pays in this model.

D $x^*>y^*$

The asymmetry between the players in the bargaining procedure gives player 1 an advantage over player 2 in the sense that, given the players' time preferences, player 1 is better off if he starts the bargaining $(x^*>y^*)$. In particular, if the players' attitudes toward time is identical - that is, if

$$(y_1,n_1)\sim_1(y_2,n_2) \text{ iff } (1-y_1,n_1)\sim_2(1-y_2,n_2)$$

- then $x^*>1/2$ and $y^*<1/2$.

E *The P.E. of the game as a limit of the P.E. of G_N*

Binmore's proof of Theorem 1 reveals an interesting property of the game: The unique P.E. outcome is a limit of the sequence of P.E. outcomes of the finite-horizon games G_N. For more general conditions under which an infinite extensive form game is the limit of finite games, see Fudenberg and Levine (1983) and Harris (1985a).

F *Example: fixed discount rates*

Assume that the players have time preferences induced from the utility functions $x\delta_1^t$ and $(1-x)\delta_2^t$. Then the P.E. partition is

$$x^*=(1-\delta_2)/(1-\delta_1\delta_2).$$

Notice the limit cases like $\delta_1=1$ and $1>\delta_2$. Although the case $\delta_1=1$ does not satisfy all the assumptions, it is easy to see that the limit of the above formula $(x^*=1)$ is indeed the unique P.E. outcome for this case.

G *Accelerating the bargaining process*

Assume that players have the continuous-time, present-value formula xe^{-r_1t} and $(1-x)e^{-r_2t}$, where t denotes "real time." Given that the length of one period of negotiation is Δ, then $\delta_i = e^{-r_i\Delta}$. Fixing the rates r_1 and r_2, the formula $(1-\delta_2)/(1-\delta_1\delta_2)$ becomes a function of Δ, $x^*(\Delta)$. If we accelerate the bargaining, we obtain

$$\lim_{\Delta\to 0} x^*(\Delta) = \lim_{\Delta\to 0} y^*(\Delta) = r_2/(r_1+r_2).$$

In the limit we see no asymmetry: The solution depends only on time preferences, not on who moves first.

H Example: fixed bargaining costs

Assume that player i bears a fixed bargaining cost per bargaining round, c_i. If $c_1 < c_2$ (player 1 is the stronger) then in the P.E., player 1 achieves his best outcome, "1." If $c_1 > c_2$ then the outcome is c_2. When $c_1 = c_2 = c$, for every x in the interval $c \leq x \leq 1$, $(x, 0)$ is a P.E. outcome. Furthermore, in this case the bargaining may continue beyond the first period. In Rubinstein (1982), an example is constructed where the P.E. agreement is reached in the second round. In that example, the first move by player 1 could be viewed as a signal to player 2 about the P.E. that they are playing in the subgame starting with 2's offer.

3 The axiomatic and strategic approaches to the bargaining problem

3.1 *The Nash program*

The Nash bargaining solution has dominated bargaining theory since 1950. The solution is attractive; it is simple, it requires little information, and it has a beautiful axiomatization. However, the drawbacks of the Nash solution are clear as well: (1) Some of the axioms are not easily defended in the abstract. (2) Additional information, such as the negotiation time preferences, seems to be relevant to the solution. This information is excluded by the axioms of Symmetry and Invariance under Affine Transformations of Utility Scales. Underlying these axioms is the assumption that the only relevant information is the players' VM utilities. (3) As economists we often find the Nash set-up too abstract to guide us in the selection of the disagreement point from among several available options.

This is the point where the strategic approach provides insight into the bargaining process.

The idea of supporting cooperative solutions by noncooperative models and solutions is now called the "Nash program." Nash presented this task and executed it by the demand game mentioned in the introduction. Binmore (1986b) was the first to observe that the sequential model of Section 2 has a strong relationship to the Nash solution [see also McLennan

(1982)]. This discovery seemed paradoxical because the bases for the two models were very different. This paradox was resolved in Binmore, Rubinstein, and Wolinsky (1986), which is the source for most of the rest of this section.

3.2 *Nash-(VM)-bargaining solution and Nash-(time preference)-bargaining solution*

The primitives of Nash bargaining theory are $(X, D, \succsim_1, \succsim_2)$, where X is the set of possible agreements, D is the "disagreement point," and $\succsim_i$ are the preferences of player i over the set of lotteries where the certain outcomes are elements of X. Assume that $\succsim_i$ satisfies the VM assumptions (i.e., is represented by the expectation of a utility function $u_i: X \to R$). Assume that the players are risk-averse in the sense that they prefer the average $px+(1-p)y$ to a lottery that awards prizes x and y with probabilities p and $(1-p)$, respectively. Now we fit to $(X, D, \succsim_1, \succsim_2)$ a Nash bargaining problem (S, s^0):

$$S = \{(u_1, u_2) = (u_1(x), u_2(x)) \mid \text{for some } x \in X\}$$

and

$$s^0 = (u_1(D), u_2(D)).$$

Define the Nash-VM-bargaining solution,

$$x^{\mathrm{VM}}(\succsim_1, \succsim_2) = \arg\max_{x \in X} (u_1(x) - u_1(D))(u_2(x) - u_2(D)).$$

Clearly $x^{\mathrm{VM}}(\succsim_1, \succsim_2)$ is well-defined, because Nash solution is invariant to the choice of the utility representations and the risk aversion assures that S is a convex set.

For the Nash-time preference (TP)-bargaining solution, define the primitives of the model as above with the only change that $\succsim_i$ is i's preference over the set $X \times T$, where $T = [0, \infty]$ is the time space. Assume that $\succsim_i$ satisfies the assumptions made in Section 2 about time preferences (adjusted to the change in the time space). Assume that both players are indifferent to the time dimension with respect to D; that is, $(D, t) \sim_i (D, 0)$ for all t. It was shown in Fishburn and Rubinstein (1982) that for δ large enough there exist concave functions $u_1(x)$ and $u_2(x)$ such that $u_i(x)\delta^t$ represent the time preferences. Notice that $u_1(D) = u_2(D) = 0$. Now let us fit to $(X, D, \succsim_1, \succsim_2)$ a Nash bargaining problem (S, s^0) and define the Nash-TP-bargaining solution

$$x^{\mathrm{TP}}(\succsim_1, \succsim_2) = \arg\max_{x \in X} u_1(x) u_2(x).$$

It is easy to check that $x^{\mathrm{TP}}(\succsim_1, \succsim_2)$ is well-defined. [Notice that if $v_i(x)\epsilon^t$ also represents $\succsim_i$ then there exists $k > 0$ such that $v_i(x) = k u_i(x)^{\log \epsilon / \log \delta}$.]

3.3 *Nash-TP solution and the strategic models*

We are ready to describe the exact sense in which the sequential strategic model (Section 2) is related to the Nash solution. The strategic model uses the same information as the Nash-TP solution, and indeed the strategic model approaches the Nash-TP solution and not the regular Nash-VM solution.

The exact relationship is as follows: Assume that the real time length of one period of bargaining is Δ, where $\Delta > 0$. The time preferences on the set $X \times T$ induce preferences on $X \times N$ by $(x_1, n_1) \gtrsim_i (x_2, n_2)$ if $(x_i, n_1\Delta) \gtrsim_i (x_2, n_2\Delta)$. For a given Δ, the sequential bargaining game has a unique P.E. outcome $(x^*(\Delta), 0)$ or $(y^*(\Delta), 0)$ depending on the identity of the player who starts the bargaining.

The following theorem is essentially due to Binmore (1986b) and was modified in Binmore, Rubinstein, and Wolinsky (1986).

Theorem 2

$$\lim_{\Delta \to 0} x^*(\Delta) = \lim_{\Delta \to 0} y^*(\Delta) = x^{\mathrm{TP}}(\gtrsim_1, \gtrsim_2).$$

Proof: Choose concave functions $u_i(x)$ such that $u_i(x)\delta^t$ represents $\gtrsim_i$ $(i = 1, 2)$. Let $u_2 = \Psi(u_1)$ be the function describing the frontier of the set S. By Theorem 1, the pair $x^*(\Delta), y^*(\Delta)$ is the solution to the equations:

$$u_1(y^*(\Delta)) = \delta^\Delta u_1(x^*(\Delta)), \qquad u_2(x^*(\Delta)) = \delta^\Delta u_2(y^*(\Delta)).$$

Denote $u_i^\Delta = u_i(x^*(\Delta))$. Then

$$\Psi(u_1^\Delta) = \delta^\Delta u_2(y^*(\Delta)) = \delta^\Delta \Psi(u_1(y^*(\Delta)) = \delta^\Delta \Psi(\delta^\Delta u_1^\Delta).$$

Let $u_1^{\Delta(n)}$ be a sequence that converges to $\bar{u}_1$. Then

$$\lim_{n \to \infty} \frac{\Psi(u_1^\Delta) - \Psi(\delta^\Delta u_1^\Delta)}{u_1^\Delta - \delta^\Delta u_1^\Delta} = \lim_{n \to \infty} \left(-\frac{(\delta^\Delta - 1)\Psi(\delta^\Delta u_1^\Delta)}{(\delta^\Delta - 1) u_1^\Delta} \right) = -\frac{\Psi(\bar{u}_1)}{u_1}$$

and thus $\bar{u}_1$ is the arg max of $u_1 \Psi(u_1)$. ■

Remark: Shortening the period of negotiation eliminates the asymmetry in the bargaining procedure. In the limit, there is no difference between the game outcomes when players 1 or 2 are the first to move.

Figure 3 summarizes Section 3.3.

3.4 *Nash-VM solution and the strategic models*

Carrying out the Nash program for the Nash-VM solution requires modification of the model's basic strategic structure. A change suggested in

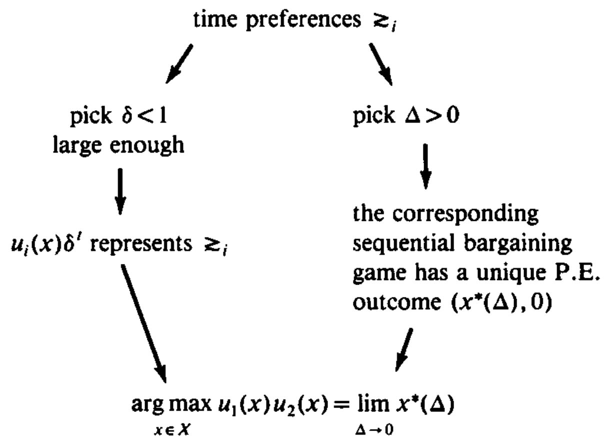

Figure 3

Binmore, Rubinstein, and Wolinsky (1986) is to introduce an exogenous risk of breakdown in the negotiations. At the beginning of every period, before an offer is made, the bargaining will end without an agreement with an exogenous positive probability p. In case of a breakdown, the outcome is D. Time is not valuable directly $[(x_1, n_1) \sim_i (x_1, n_2)$ for all n_1 and $n_2]$ but the longer the players plan to negotiate the larger the chances of a breakdown. To complete the description of the modified game, we must specify the players' preferences on the set of lotteries in which the pure outcomes are elements in X. Let $\succsim_1$ and $\succsim_2$ be extensions of the preference ordering on X to preferences on the set of lotteries over elements in X. We assume that the preferences satisfy the VM assumptions and that they display risk aversion. Formally, the strategic model with risk of breakdown closely resembles the model presented in Section 2. A pair of strategies in the bargaining determine a lottery of the type in which an outcome x is agreed with probability $(1-p)^n$ and disagreement, D, is achieved by probability $1-(1-p)^n$. Such a lottery is denoted $\langle x, n \rangle$. The game has a unique P.E. that is determined as before by the unique solution to the fundamental equations of Theorem 1, $\langle x^*, 1 \rangle \sim_1 \langle y^*, 0 \rangle$ and $\langle y^*, 1 \rangle \sim_2 \langle x^*, 0 \rangle$. Denote the solution by $x^*(p), y^*(p)$. The following theorem then provides a noncooperative foundation for the Nash-VM bargaining solution.

Theorem 3. *(a) The sequential game with a risk of breakdown has a unique P.E. The outcome is $x^*(p)$ if 1 starts the game and $y^*(p)$ if 2 starts the bargaining.*

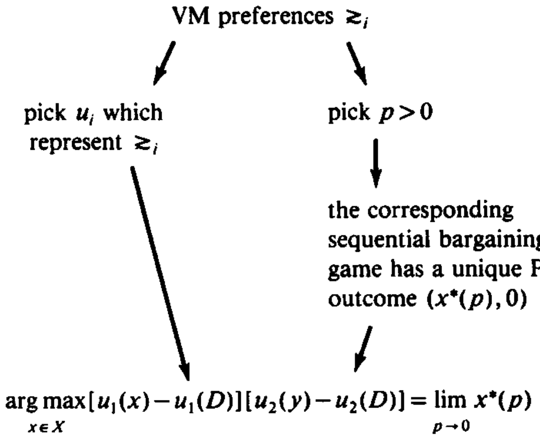

Figure 4

(b) $\lim_{p \to 0} x^*(p) = \lim_{p \to 0} y^*(p) = x^{\mathrm{VM}}(\succsim_1, \succsim_2)$.

Figure 4 summarizes Section 3.4.

Remark: The analysis of the strategic games with time preferences and with risks of breakdown can be unified and extended in several directions. For example, in Wolinsky (in press) the probability of breakdown depends on the choice of search intensities.

3.5 *The choice of disagreement point and the outside option principle*

Theorems 2 and 3 help us to identify the proper disagreement point in Nash bargaining theory for those cases where it is reasonable to regard the Nash solution as an approximation to the equilibrium outcome of an appropriate strategic model. If time impatience is the significant friction in the model, we ought to take the status quo agreement (the time-indifferent partition) as the disagreement point. Thus, if a strike is in progress during which the workers and the firm negotiate about the value of the post-strike wage stream, then the disagreement point is taken to be the consequence of prolonging the negotiations forever – that is, at the disagreement point, each negotiator is assigned the income stream he receives as long as the dispute continues. When the friction in the model is due to the risk of an exogenous breakdown of the negotiation, we ought to take as D the event that a breakdown occurs. Thus if two parties bargain over the gain from a business opportunity that they are able to exploit together, then D is the event that a third party will snatch the opportunity away.

Another natural candidate for indentification with the element D in Nash's framework is the outside option outcome, denoted by e. By the outside option outcome, we refer to an outcome that results if a party withdraws from the negotiation (a more general definition will allow the outcome to depend on the identity of the player who withdraws). In many applications of the Nash solution, e has been identified with D. The strategic approach suggests that this is wrong.

Outside options may be incorporated into the strategic model by modifying the strategic games, that of Section 2 (time impatience) and of Section 3.4 (a risk of a breakdown). At each node in which a player must respond to an offer, we add the alternative of withdrawing and forcing e. Binmore (1986c) and Shaked and Sutton (1984b) discovered that including an outside option does not alter the solution to the game without outside option, if both x^* and y^* are preferable to e by both parties. Thus, adopting the sequential strategic model leads to the conclusion that it is not proper to identify D with e. The presence of e merely restricts the domain under which the Nash-TP or VM bargaining solutions - with the choice of D as the status quo or the breakdown event - approximate the strategic models when friction elements become negligible.

4 Bargaining with incomplete information

4.1 *The role of incomplete information in the model*

A critical assumption of the sequential bargaining model as developed so far is that each player has complete information about the other player's preferences. Hence it is not surprising that in P.E. the bargaining ends immediately, and it is natural to try to explain the prolongation of the bargaining process by a game with incomplete information.

When incomplete information exists, the series of offers and reactions become a communication system between the players. Each player tries to conclude from the other's moves who his opponent is, and each may try to mislead the other to believe that he has a better bargaining position than he actually has. A player's readiness to delay an agreement may be interpreted as a signal about the unknown information. Impatience is an incentive to compromise, but introducing delays is the only means a player has under his control to test the credibility of what the opponent says.

Following Harsanyi (1967), the situation is modeled as a Bayesian game. The sequential structure of the strategic bargaining game calls for an extension of the subgame perfectness notion to games with incomplete information. Kreps and Wilson's (1982) Sequential Equilibrium (S.E.) is a natural convenient solution concept for extending the analysis of Section 2 to incomplete information situations. However, in contrast to the usefulness of perfectness in bargaining games with complete information, the set of S.E. is enormously large. Let us demonstrate this point by an example taken from Rubinstein (1985b). [A more general discussion appears in Rubinstein (1985a).]

4.2 *An example of a bargaining game with incomplete information*

Assume that the players' utility functions are $x-c_1t$ and $1-x-c_2t$ (the case of fixed bargaining costs) and that these functions are the players' VM utility functions as well. Player 1's cost $c_1=c$ is common knowledge. Player 2's cost c_2 is known only to player 2. Both parties are aware of the random process which selects c_2. With probability ω_0, $c_2=c_w>c$. In such a case, it is said that player 2 is the weak type, 2_w, because if player 1 were in possession of exact information about 2's cost then the outcome of the bargaining would be the worse one for player 2. With probability $1-\omega_0$, $c_2=c_s<c$ and it is said that 2 is 2_s, the stronger type.

Candidates for equilibrium are triples (f, g, h), where f is a strategy for player 1 and g and h are strategies for types 2_w and 2_s respectively. A candidate for S.E. is a four-tuple (f, g, h, ω), where the added element ω is a function that assigns a number to every possible history after which 1 has to move. This number is interpreted as player 1's belief that 2 is of the weak type. To the assumptions on S.E. we add the requirement that if 1 concludes with certainty that he is playing against a certain type, he continues to hold this belief forever and therefore uses his unique P.E. strategy in the complete information game against this type.

The concept of S.E. requires that the solution specify the players' new beliefs after a zero-probability event occurs. The belief that a player adopts after a zero-probability event is called a conjecture. The S.E. concept allows great freedom in selecting conjectures to support strategies as best responses and thereby to rationalize threats. Player 1's most severe credible threat is that a deviation by player 2 will lead him to play the game as if he were playing against type 2_w. To support this threat, player 1 may use the rule that a deviation by player 2 is an indication that he belongs to type 2_w. These "optimistic conjectures" support many S.E. In Rubinstein (1985b) it is shown that for all ω_0 and all $1-c+c_s\geq x^*\geq c_w$, there is a S.E.

in which the bargaining ends immediately with the agreement x^* or lasts for two periods, where in the first period 2_w agrees to x^* and in the second period player 1 accepts player 2_s's offer $x^* - c_w$. In the S.E., any move of player 2 which was not expected from either of the two types makes player 1 believe that he plays against 2_w. A deviation of 2_s is prevented by player 1's threat to believe that he plays against 2_w.

Thus the set of S.E. is very large. The perfectness notion eliminates incredible threats in the model with complete information, but the severe threats return through the back door in the game with incomplete information, in that the freedom to select conjectures is left open.

4.3 *Selection of S.E.*

When presenting and analyzing the concept of S.E., Kreps and Wilson (1982) wrote: ". . . the formulation [of S.E.] in terms of players' beliefs gives the analyst a tool for choosing among S.E." The idea is to impose additional restrictions on the beliefs, restrictions that are reasonable at least in the special context of bargaining games. As things stand, I do not feel that we have a firm enough theory to justify the selection of one particular S.E. from among the many. Let me mention just one approach, suggested in Rubinstein (1985a, b). No claims are made that this is the only viable approach.

Three additional requirements are imposed on S.E. The first is quite straightforward. If player 1 makes an offer s^{t-1} that is rejected and followed by 2's offer s^t satisfying $s^t > s^{t-1} - c_w$, then player 1 does not strictly strengthen his belief that he is playing against 2_w. The second assumption is a tie-breaking assumption. If an offer has been made and the receiver is indifferent between accepting it or continuing the bargaining, then he accepts the offer. The third requirement is the crucial one: Assume that 1 offers s^{t-1}, and that player 2 rejects it and makes a counteroffer s^t. Assume that 2's reaction was not an expected one from any type. If $s^t \leq s^{t-1} - c_s$ and $s^t > s^{t-1} - c_w$, it is required that player 1 conclude that 2 is 2_s. Thus, it is assumed that 2_s can sort himself out by rejecting s^{t-1} and making an offer that, if accepted in the next period, is better for 2_s and worse for 2_w than accepting s^{t-1}. The idea is that player 2_s is implicitly saying "you ought to believe that I am 2_s because if you do then your consequent outcome will make me better off, whereas if 2_w sent the same message he would be worse off." Under the above circumstances, if 1 believes the message "I am 2_s" then he accepts s^t. Indeed, it is rational for 2_s to reject s^{t-1} and offer $s^t \leq s^{t-1} - c_s$ in the following period; it is not rational for 2_w to do so because, for 2_w, $(s^{t-1}, 0)$ is preferred to $(s^t, 1)$.

A sequential equilibrium satisfying the above three assumptions is referred to as a *bargaining sequential equilibrium* (B.S.E.). The next theorem [Rubinstein (1985a, b)] is a characterization of the B.S.E. outcomes.

Theorem 4. In any B.S.E.,

if $w_0 > 2c/(c+c_w)$, *the outcome is* $(1,0)$ *if 2 is* 2_w *and* $(1-c_w, 1)$ *if 2 is* 2_s;

if $2c/(c+c_w) > w_0 > (c+c_s)/(c+c_w)$ *the outcome is* $(c_w, 0)$ *if 2 is* 2_w *and* $(0,1)$ *if 2 is* 2_s;

if $(c+c_s)/(c+c_w) > w_0$ *the outcome is* $(c_s, 0)$ *whatever 2's type.*

The B.S.E. of Theorem 4 has some attractive features: The negotiation does not end immediately and it satisfies the expected comparative statics properties. However, the foundations for the approach remain shaky. One can bring arguments against the strong inference assumptions made in Theorem 4; for instance, the belief functions are not continuous. There are other intuitive arguments that can restrict the beliefs of an uninformed player. For example, Bikhchandani (1985) assumes that if only one of the two types of player 2 is supposed to reject a given offer, then the rejection reveals the informed player's type independently of whether his counteroffer is consistent with this type or not. This change enables Bikhchandani to build an example of S.E. (with mixed strategies) that is qualitatively different than Rubinstein's (1985a). In particular, in Bikhchandani's equilibrium the bargaining process may last for an arbitrarily large finite number of periods if player 1 assesses probability close to 1 that player 2 is of the weak type.

Extending the notion of B.S.E. to the case where the number of player-2 types is larger than 2 requires new ideas. Grossman and Perry (1986) suggest (for a buyer–seller model) such an extension leading to a unique play of the game in which the negotiation continues for many periods. However, for some parameters of the model, the requirements are too strong and lead to non-existence problems.

4.4 *The state of the art*

The topic of bargaining with incomplete information has received a lot of attention in the last five years. It was inspired by development of the "economics of information" and attempts to refine the S.E. concept. Recent surveys of the vast literature are Fudenberg, Levine, and Tirole (1985) and Cramton (1984). Among the works within the infinite alternating-offers model, let me mention especially Grossman and Perry (1986) as well as Chatterjee and Samuelson (1985), Cramton (1984), Harris (1985b), and Perry (1986). Most of the other literature simplifies the bargaining procedure to avoid too many zero-probability events. Often it has been done by giving the ability to make offers only to one party.

Recall the feature of the model with complete information that there is no delay in reaching an agreement. A central target of the literature on bargaining with incomplete information is to check the hypothesis that empirically observed delays are due to the lack of complete information. Achieving this target seems now less plausible; very recently Gul and Sonnenschein (1985) showed that, for a large class of S.E., if the length of a period of negotiation is small then it is almost certain that the bargaining ends almost instantaneously.

In my opinion, we are far from having a definitive theory of bargaining with incomplete information for use in economic theory. The problems go deeper than bargaining theory and appear in the literature on refinements of S.E., an issue explored thoroughly in the last few years. My intuition is that something is basically wrong in our approach to games with incomplete information and that the "state of the art" of bargaining reflects our more general confusion.

5 Markets and bargaining

5.1 *Bargaining as the central activity in a market*

Bargaining theory provides the building blocks for models of markets in which the transactions are made within small groups (like pairs), in the absence of trading institutions like auctioneers. The process of determining the terms of any particular contract is a bargaining situation influenced by the market environment (outside options) and by the process that matches the agents. The study of those markets where the basic activity is bargaining has the aim of providing a mini-micro foundation for market analysis and for investigation of the economic phenomena that underly price formation. In particular, we look for clarification of the sense, if any, in which the market solution approximates the competitive outcome in the case of many small agents.

5.2 *The static approach*

Diamond (1981, 1982), Mortensen (1982a, 1982b), and Diamond and Maskin (1979) have studied static models of the economy in a steady state. The next model follows their approach: There are two types of agents in the market, 1 and 2. A pair of agents of opposite types can agree on one element from among the set X. A pair of agents that is matched and reaches an agreement leaves the market. The probabilities of agents in the market being matched are kept constant. Each agent of type 1 or 2 has a probability, α or β, of meeting an agent of type 2 or 1. The players have a common discount rate δ, that is, the functions $x\delta^t$ and $(1-x)\delta^t$ are the players' VM utility functions of an agreement x at the tth period of life.

Whenever a pair of agents is matched, it is assumed that they reach an agreement $\bar{x}$. This agreement is the Nash bargaining solution with respect to the threat point $(\delta V_1, \delta V_2)$, where V_i is type i's expected value of existing in the market at the point of his arrival to the market and before it is known whether the agent is matched. Thus,

$$V_1 = (1-\alpha)\delta V_1 + \alpha\bar{x}, \qquad V_2 = (1-\beta)\delta V_2 + \beta(1-\bar{x}),$$

and

$$\bar{x} = \delta V_1 + \frac{1-\delta V_1 - \delta V_2}{2}.$$

Therefore,

$$\bar{x} = \frac{1-\delta+\delta\alpha}{2(1-\delta)+\delta\alpha+\delta\beta}.$$

This formula has two interesting limit cases. When the impatience element is small, we obtain

$$\lim_{\delta\to 1} \bar{x} = \frac{\alpha}{\alpha+\beta}.$$

When we take $\alpha = a\Delta$, $\beta = b\Delta$, and $\delta = e^{-r\Delta}$, and shrink the length of one period to zero, we obtain

$$\lim_{\Delta\to 0} \bar{x} = \frac{r+a}{r+a+b}.$$

Essentially, the constant elements in the market are the probabilities α and β. Diamond and Mortensen took α and β to be functions of a constant number of agents in the market, N_1 and N_2. The matching technology, which is the specification of the functional relationship between α, β and N_1, N_2, is needed for a comparison between the above results and a competitive market outcome. Specifically, for the matching technology, when $\alpha = M/N_1$ and $\beta = M/N_2$ we have

$$\lim_{\delta\to 1} \bar{x} = \frac{N_2}{N_1+N_2},$$

and if $\alpha(\Delta) = (M/N_1)\Delta$ and $\beta(\Delta) = (M/N_2)\Delta$ then

$$\lim_{\Delta \to 0} \bar{x} = \frac{(rN_1N_2/M)+N_2}{(2rN_1N_2/M)+N_1+N_2}.$$

Diamond and Mortensen assumed that whenever a pair of agents is matched, they immediately agree on the Nash bargaining solution with the disagreement point $(\delta V_1, \delta V_2)$. The foundations for this assumption were not clear. If it is assumed that the bargaining is separated from the matching and occurs sequentially in the interval between any two matching periods, then Section 3 casts doubts on the plausibility of this use of the Nash bargaining solution. A better assumption is that the bargaining and matching processes are simultaneous. Then, whenever a pair of agents is matched, they take into account both the time impatience and the possibilities that one of them will pass to a new match and abandon the bargaining process. An agent's evaluations of the event that he or an opponent cease bargaining are affected by the equilibrium in the market. A priori there is no reason for the bargaining outcome in the environment to be identified with the Nash solution with a disagreement point $(\delta V_1, \delta V_2)$.

5.3 *The sequential approach*

In Rubinstein and Wolinsky (1985), we tried to look into the bargaining "black box" in this market. We specified in detail the order of events in the market and derived the market equilibrium from noncooperative behavioral assumptions. In the construction of the model we adhered as closely as possible to the spirit of competitive analysis.

We keep the assumptions of Section 5.2 except with respect to the bargaining process. The time periods of the bargaining are the same as the periods in which the random element matches the agents. A particular bargaining situation between a pair of agents is a modified version [see Binmore (1986c)] of the model of Section 2. At each stage, one of the bargainers is selected (with equal probabilities for the two agents) to make a proposal. Before the random draw selects the player who will make the proposal, the players may meet new partners and abandon the old partners. Any agent of type 1 (independently of any other element in the model) abandons his opponent with probability α. With probability $(1-\alpha)\beta$ he is abandoned and is left without a partner until he finds a new member of the opposite type. With probability $(1-\alpha)(1-\beta)$ both partners continue the bargaining. An agent cannot bargain simultaneously with more than one opponent. The decision to replace an opponent is not strategic in this model, but will apparently be a rational choice in equilibrium.

We carefully avoid calling this model a game, because the set of players is not specified. An agent in the model is born and participates in the matching and in the bargaining process until he reaches an agreement with an agent of the opposite type. Then he leaves the market. He has a perfect recall of his personal history. A strategy for an agent is a rule for how to behave after any possible personal history.

Consider a pair of strategies, one for each type, that prescribes the same bargaining tactics for every player of a particular type against any opponent. [The last restriction seems unnecessary; see Binmore and Herrero (in press a).] The pair is a Market Equilibrium (M.E.) if no agent can gain by deviating from his strategy after some personal history (assuming that all agents of the opposite type follow their original strategy). Notice that the probabilities α and β are fixed in the model independently of the outcomes of the agents.

> ***Theorem 5*** [Rubinstein and Wolinsky (1985)]. *There is a unique Market Equilibrium. In the equilibrium, an agent of type 1 (or 2) always makes the offer x^* (or y^*) which is accepted.*

The numbers x^* and y^* were calculated and it was shown that

$$\lim_{\Delta \to 0} x^* = \lim_{\Delta \to 0} \bar{x}, \qquad \lim_{\delta \to 1} x^* = \lim_{\delta \to 1} \bar{x},$$

and furthermore that $\bar{x} = \frac{1}{2}x^* + \frac{1}{2}y^*$.

Thus, the sequential model leads to the same outcome as in Section 5.2: The M.E. outcome is the Nash bargaining solution relative to the expected values of being in the market unmatched. This seems a "razor's edge" result. Wolinsky (in press) reveals that the coincidence is due to the assumption that a bargainer's matching options are unaffected by him being matched or not. If these matching options are not the same, the limit M.E. outcome is the Nash bargaining solution relative to a convex combination of the values of being unmatched and being matched with a partner who is not ready to make an agreement.

Before comparing the M.E. with the competitive equilibrium, I would like to mention several economic models in which the sequential bargaining model is a cornerstone and which are not mentioned later. Shaked and Sutton (1984a) assume a market where one employer can hire one worker from a pool of n workers. Once the employer starts to bargain with a worker, he must continue the negotiation for T periods; only then can he move on to a new worker. In the limit, diminishing the impatience factor of the players results in the employer's share of the surplus being $(T+1)/2T$.

Binmore (1985) is an attempt to extend the two-bargainer sequential model to an n-player situation in which different coalitions of players are to divide different surpluses. Binmore explored several bargaining procedures where an agreement requires agreement of a group containing more than two players. Wilson (1985) studied one of those procedures more extensively. In an economy with many buyers and sellers, each period has all the sellers or all the buyers making public offers to the opposite kind of agents, who must respond by accepting one of the offers or rejecting all of them. Wilson finds that in the subgame P.E. all the accepted prices are Walrasian.

Other game-theoretic analyses of trading processes are surveyed intensively by Wilson in this volume. [A pioneering work in this direction is an unpublished paper by Butters (1980).]

5.4 *The strategic approach and the competitive equilibrium*

Unless there is an auctioneer in the market, the competitive equilibrium analysis does not specify a mechanism that forms prices. The competitive solution is often justified by the so-called competitive conditions. These include smallness of the agents, negligible transactions costs, full rationality, symmetric information, and so on. It is usually claimed that under almost frictionless conditions, any reasonable mechanism of price formation will approximately implement the competitive equilibrium prices. The construction of our model in Section 5.3 maintained these characteristics. Therefore, it seems meaningful to compare the competitive equilibrium with the M.E. when the frictions in the model (the time impatience) become negligible.

The comparison may be made in the following example: In the market for an indivisible good, there are two types of agents, sellers (type 1) and buyers (type 2). Each seller holds one unit of the good and his reservation value is 0. Each buyer is interested in buying one unit and his reservation value is 1. The market is in a steady state, in the sense that there is a constant number N_i of agents of type i. There is a linear matching technology; that is, $\alpha(\Delta)=(M/N_1)\Delta$ and $\beta(\Delta)=(M/N_2)\Delta$, where Δ is the length of one period and M is the fixed number of matches that occur per unit of time. When the time discount factor approaches one, the M.E. price is $N_2/(N_1+N_2)$. In contrast, it looks as if the competitive equilibrium price is 0 or 1 according to the relative size of N_1 and N_2. This observation leads us [Rubinstein and Wolinsky (1985)] to conclude that there is a difference between the competitive and the sequential strategic solutions even when the market's frictions are negligible.

This statement seems puzzling to some; others claim that this is a misuse of the concept of competitive equilibrium. Recent works of Binmore, Herrero, and Gale shed light on this result and further identify conditions under which the competitive equilibrium is approached by the strategic approach. In all these models, the set of players is specified, and is identified with the continuum set.

In their main result, Binmore and Herrero (in press b) examine a market with $N_1(0)$ sellers and $N_2(0)$ buyers at time 0 who remain in the market until all the agents of one of the types make a transaction. A trade outcome at time t is shown [in Binmore and Herrero (in press a)] to be a Nash bargaining solution as the values of being in the market matched and unmatched at time t. Most important is the assumption of a linear technology; the probabilities of being matched are endogenous and vary over time. This leads to the competitive price result: The short side collects all the surplus. If $N_1(0)=N_2(0)$ then each pair splits the surplus equally.

In one of his results, Gale (in press) extends Binmore and Herrero (in press b) to the case where sellers' and buyers' reservation values are distributed in the interval $[0,1]$. Gale obtains the competitive result - the price is at the intersection of demand and supply curves. This result depends on a simplification of the bargaining process; one of the bargainers is randomly selected to make a take-it-or-leave-it offer, and if the offer is rejected the pair return to the pool of unmatched agents. An important assumption in Gale's model is that the agents who are matched have full information on their reservation values.

In Gale (1986) each agent is coming to the market with an initial bundle. Gale allows each agent to make a series of transactions until he decides to consume his bundle. He assumes that the bargaining procedure has the take-it-or-leave-it form. Time impatience is eliminated; the players are indifferent about the timing of their consumption. Finally, Gale assumes that the support of the set of agents is very diverse both in terms of utility functions and the initial bundles, and that this diversity remains forever.

In the second part of Gale (in press), Rubinstein and Wolinsky (1985) is extended in the following sense: Assume that each instant each seller has probability α_i of matching a buyer of type i with reservation value x_i, and that each buyer has probability β_j of being matched with a seller who has reservation price y_j. Let p be a number such that

$$\sum \alpha_i \max\{(x_i-p),0\} = \sum \beta_j \max\{(p-y_j),0\}.$$

Then p is the limit of the M.E. when $\delta \to 1$. If α_i and β_j are proportional to N_{1i} and N_{2j} (the steady-state numbers of buyers and sellers of types i and j, accordingly), then the above formula is transformed to

$$\sum N_{1i} \max\{(x_i-p),0\} = \sum N_{2j} \max\{(p-y_j),0\},$$

which has the interpretation that buyer surplus is equal to seller surplus. Thus Gale (in press) shows that a price exists even where the supply and demand curves have the regular monotonicity properties; this is what Gale calls the "stock equilibrium" price.

The main insight of Gale (in press) is the following observation: Assume that N_{1i} and N_{2j} reflect the flow rates of agents who consider entering the market. There is a small positive entrance fee. Let p be the price at the intersection of the demand and supply curves induced by these numbers. Gale calls this a "flow price." This is the only price that can be supported with a stock equilibrium when the flow of agents leaving the market is equalized by the flow of agents entering the market. As before, Gale assumes the take-it-or-leave-it bargaining procedure; the search technology is linear and a matched agent has complete information about his opponent's reservation value.

The primitives of the standard economic models are the individuals in the market and those of their characteristics relevant to their behavior in the market. A description of the strategic behavior of individuals in the framework of a game requires that information about the players operating in the game be common knowledge. This is a very strong assumption in a world where even econometricians find it hard to get a rough estimate of supply and demand. It seems that a shopkeeper bases his price strategy more on the frequency with which shoppers enter his shop than on the size of the population of the world or his town.

In contrast, the model of Section 5.3 - as well as the models of Diamond, Mortensen, and Gale - take the primitives to be the stochastic processes of arrivals of new opportunities. It is this arbitrary assumption about the search technology that makes it seem as if the Diamond–Mortensen–Gale results refer to the standard demand and supply structure. I believe that it may often be useful to analyze economic environments using information about the streams of personal opportunities, rather than confining attention to data about the flows of potential entrants to the market.

At present, I do not think that this question is settled: Under what conditions will the strategic bargaining approach generate the competitive outcome? The works surveyed above provide new insights for understanding the competitive assumption. It seems that the extensive form game-theoretic approach provides tools for analyzing exciting issues that could not be studied by other means - money, inflation, and unemployment [see Shaked and Sutton (1984a)]; as well as trading processes [see Rubinstein and Wolinsky (in press)]. The economic insights derivable from such such market models are the chief goal of sequential strategic models.

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