### MIDDLEMEN\*

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We study a model of a market with three types of agents: sellers, buyers, and middlemen. Buyers and sellers can trade directly or indirectly through the middlemen. The analysis focuses on steady state situations in which the numbers of agents of the different types and hence the trading opportunities are constant over time. The paper provides a framework for analyzing the activity of middlemen and the endogenous determination of the extent of that activity. It highlights the relations between the trading procedure and the distribution of the gains from trade.

## I. INTRODUCTION

This paper presents a simple market model that captures explicitly the role of middlemen in the trading process. Despite the important role played by intermediation in most markets, it is largely ignored by the standard theoretical literature. This is because a study of intermediation requires a basic model that describes explicitly the trade frictions that give rise to the function of intermediation. But this is missing from the standard market models, where the actual process of trading is left unmodeled. The purpose of this paper is to suggest a simple framework that captures these missing elements and hence enables one to think in a formal manner about some of the issues related to intermediation.

We introduce intermediation into a model of pairwise meetings in which the imperfection takes the form of a time-consuming matching/trade process (see Diamond and Maskin [1979], Diamond [1982], and Mortensen [1982] for detailed descriptions of such models). The model features three types of agents: sellers, buyers, and middlemen. Each seller has a unit of the good for sale, and each buyer seeks to buy a unit. Transactions can take place directly between buyers and sellers or indirectly through the middlemen, who buy from sellers in order to sell to buyers. There is a timeconsuming process that stochastically brings together sellers, buyers, and middlemen pairwise. When a pair of agents who have a mutual interest in carrying out a transaction are brought together, they negotiate the price instantaneously. The negotiated price is assumed to be such that the net surplus associated with the match is split equally between the parties. What makes the middlemen's activity possible is the time-consuming nature of the trade, which enables the middlemen to extract surplus in return for short-

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ening the time period that sellers and buyers have to wait for a transaction.

The model is highly stylized. It attempts to capture in a simple manner some elements that are common to markets in which intermediation appears to be a time-saving institution, but it does not aspire to describe closely any specific market. One of the markets that can be pointed out as being roughly described by this model is the housing market. Although it seems that, contrary to the model, in that market a seller or a buyer can find a broker immediately, it should be noted that this market consists of different specialized submarkets, and it may involve some search to find a broker who deals with the type of housing or clientele in which a particular buyer or seller may be interested.

Perhaps the main contribution of this paper is that it provides a framework for analyzing the activity of the middlemen and the endogenous determination of the extent of that activity. In addition, the paper contains more specific insights into the functioning of intermediation, such as the observations made in Section IV concerning the relations between the nature of the trading procedure and the distribution of the gains from trade. It is shown that a trading procedure in which the middleman has to assume ownership of the good in the process of trade biases the distribution of the gains in favor of the buyers. In contrast, when the middlemen trade by consignment, that bias disappears, and the gains to sellers and buyers are distributed symmetrically.

We are aware of a number of articles that presented a formal model of intermediation: Kurz and Wilson [1974], Garman [1976], Glosten and Milgrom [1985], and Bhattacharya and Hagerty [1984]. We shall not attempt to survey this literature here, since it is not related directly to our work. The distinguishing features of the present model are that here the process of matching between middlemen and their customers is modeled explicitly; the transaction costs are determined endogenously; and the trade through middlemen coexists with direct trade between sellers and buyers.

#### II. THE MODEL

We consider a market for an indivisible good. There are three types of agents: sellers, buyers, and middlemen. Each seller has one unit, and each buyer seeks to buy one unit. The consumption values (in monetary terms) of a unit for a seller, a middleman, and a buyer are normalized at zero, zero, and one, respectively.

Buyers and sellers can trade directly with each other or with middlemen. Trade takes place at an infinite sequence of dates. At each date there is a matching stage in which an agent either will meet no one or will meet exactly one agent of another type (e.g., a seller may meet a buyer or a middleman who seeks to buy the good). Over the period matched agents negotiate the price and decide whether or not to transact.

A seller or a buyer who has transacted leaves the market, while a middleman stays in the market perpetually and continues in the process of searching for potential partners and transacting. It is assumed that a middleman cannot store more than one unit, so that he will not buy another unit before he will have sold the one he owns at present.

Let the indices B, S, N, and M refer to buyers, sellers, middlemen without the good, and middlemen with the good, respectively. Let  $L_B$ ,  $L_S$ ,  $L_M$ , and  $L_N$  denote the measures of agents of the different types who are active in the market at a given time. The model will deal with steady state situations in which these numbers are constant over time. It is assumed that there are exogenous and constant flows of arrival at the rates of e new buyers and e new sellers per period of time. These rates must be equal to ensure existence of a steady state. It is also assumed that there is a measure  $\overline{K}$  of potential middlemen, and that a potential middleman will enter if the expected profit is positive and only if it is nonnegative.

## The Matching Process

The probability that any agent of type i has of meeting an agent of type j at a given period depends only on the numbers  $L_i$ ,  $L_j$  of those present at that period. The matching process is then described by the probabilities  $\alpha_B(L_B,L_S)$ ,  $\alpha_S(L_S,L_B)$ ,  $\beta_B(L_B,L_M)$ ,  $\beta_S(L_S,L_N)$ ,  $\gamma_N(L_N,L_S)$ , and  $\gamma_M(L_M,L_B)$ . The probability  $\alpha_B(\alpha_S)$  is the probability that a particular buyer (seller) will meet a seller (buyer) at a given period;  $\beta_B(\beta_S)$  is the probability that a particular buyer (seller) will meet a middleman at a given period; and  $\gamma_N(\gamma_M)$  is the probability that a particular middleman who is interested in buying (selling) will meet a seller (buyer) at a given period. It is assumed that the total number of meetings between agents of types i and j is increasing in  $L_i$ . E.g., the function  $\gamma_N(L_N,L_S)L_N$  is increasing in  $L_N$ .

At the steady state these probabilities are constant over time. Assuming that all meetings are concluded by transactions, the steady state conditions can be written in terms of the matching

probabilities as

(1) 
$$\alpha_B(L_B, L_S)L_B + \gamma_M(L_M, L_B)L_M = e$$

(2) 
$$\alpha_S(L_S, L_B)L_S + \gamma_N(L_N, L_S)L_N = e$$

(3) 
$$\gamma_M (L_M, L_B) L_M = \gamma_N (L_N, L_S) L_N.$$

Observe that if  $L_S = L_B$ , then steady state conditions imply that  $\alpha_S = \alpha_B$ . Since  $\gamma_M L_M = \beta_B L_B$  and  $\gamma_N L_N = \beta_S L_S$ , then  $L_S = L_B$  also implies that  $\beta_S = \beta_B$ . Further, if the matching between buyers and middlemen who seek to sell is governed by the same rules as the matching between sellers and middlemen who seek to buy (i.e., the functions  $\gamma_M$  and  $\gamma_N$  are the same), then  $L_B = L_S$  and (3) imply that  $\gamma_M = \gamma_N$  as well.

It turns out that these symmetry assumptions make later derivations more tractable and therefore attention is confined to symmetric situations in the above sense. The configuration is then summarized by the two numbers  $L=L_S=L_B$  and  $K=L_M=L_N$ , and the three probabilities  $\alpha=\alpha_S=\alpha_B, \beta=\beta_S=\beta_B$ , and  $\gamma=\gamma_M=\gamma_N$ , and so the steady state equations (1)–(3) are reduced to the single equation,

(4) 
$$\alpha(L,L)L + \gamma(K,L)K = e.$$

## Preferences

A typical seller will leave the market after receiving a price p some t periods after entering the market. The seller's utility of the outcome (p,t) is  $\delta^t p$ , and the buyer's utility is  $\delta^t (1-p)$ , where  $\delta \subset (0,1)$  is a common discount factor. It is assumed that both sellers and buyers are maximizers of expected utility. A typical middleman carries out a sequence of transactions in which he buys a unit at time  $t_{2n}$  for the price  $p_{2n}$  and sells it at  $t_{2n+1}$  for the price  $p_{2n+1}$ , where  $n=0,1,2,\ldots$  It is assumed that the middleman seeks to maximize the expected value of the discounted stream of his profits,

$$\sum_{n=0}^{\infty} \left[ \delta^{t_{2n+1}} p_{2n+1} - \delta^{t_{2n}} p_{2n} \right].$$

## The Bilateral Bargaining

When two agents are matched, they bargain instantaneously over the terms of the transaction. The process of the bargaining will not be specified. Instead we shall assume that the outcome is either an agreement on price that divides equally the net surplus associated with the match, or a disagreement upon which the two agents return to the pool of the unmatched.

Let  $V_i$ , i=B, S, M, N, denote the expected utility for an agent of type i from being in the pool of the unmatched, and let  $Z_{ij}$  denote the sum of the benefits that will accrue to i and j in the event that they transact. For the three relevant pairs SB, SN, and MB, we have  $Z_{SB}=1$ ,  $Z_{SN}=V_M$ , and  $Z_{MB}=1+V_N$ . The latter expression, for example, follows from the observation that when a middleman with a unit M sells to a buyer B, there is the buyer's utility from having a unit, 1, plus the middleman's expected utility of being without a unit and unmatched,  $V_N$ . The assumption concerning the outcome of the bargaining is that if  $Z_{ij} \geq V_i + V_j$ , then the parties agree on a price that gives party i the utility  $V_i + \frac{1}{2}(Z_{ij} - V_i - V_j)$ , and if  $Z_{ij} < V_i + V_j$ , the bargaining terminates with disagreement.

Thus, the bilateral bargaining component is modeled as in the above cited work by Diamond, Maskin, and Mortensen. This contrasts with our earlier work [Rubinstein and Wolinsky, 1985], in which the bargaining component was modeled as a sequential game of the type introduced by Rubinstein [1982]. The reason that we abandon the strategic approach here is that it would greatly complicate the exposition without adding insights that are relevant for the issues that interest us here.

## The Equilibrium

Consider a symmetric steady state configuration, where  $L = L_S = L_B$ ,  $K = L_M = L_N$ . Let the triple  $P = (P_{SB}, P_{MB}, P_{SN})$  describe the prevailing agreements, where each  $P_{ij}$  is either the price paid by agent of type i to agent of type i, or it is the symbol d, which stands for disagreement. Let  $V_i(P)$  denote the maximal expected utility of an agent of type i who operates in a market where transactions are carried out according to P, so that whenever the agent meets an agent of type j, he may either agree on  $P_{ij}$  or reject the transaction altogether.

DEFINITION. A market equilibrium (M.E.) is a symmetric configuration and a triple  $P=(P_{SB},P_{SN},P_{MB})$  such that

$$(i) \quad P_{ij} = \begin{cases} V_i(P) + \frac{1}{2}[Z_{ij} - V_i(P) - V_j(P)] \\ \text{if } V_i(P) + V_j(P) \leq Z_{ij}, & ij = SB, SN \\ \frac{1}{2}[1 + V_M(P) - V_B(P) - V_N(P)] \\ \text{if } V_M(P) + V_B \leq Z_{MB}, & ij = MB \\ d \text{ if } V_i(P) + V_j(P) > Z_{ij}, \end{cases}$$

where  $Z_{ij}$  is the sum of gross benefits associated with the match.

(ii) 
$$\alpha(L,L)L + \gamma(K,L)K = e.$$

(iii) Either 
$$V_N(P) = 0$$
, and  $2K \le \overline{K}$ :

or 
$$V_N(P) > 0$$
, and  $2K = \overline{K}$ .

Thus, the equilibrium is a symmetric steady state configuration such that the agreements-disagreements in all meetings are prescribed by P. Condition (i) requires that the outcomes of the bilateral bargaining will reflect the positions of the parties as captured by the maximized values  $V_i(P)$ . Condition (ii) is the symmetric steady state condition. Condition (iii) is the entry condition to the middlemen's trade: either the number of middlemen is such that there is no incentive for further entry or exit, or all  $\overline{K}$  potential middlemen are active.

### III. MARKET EQUILIBRIUM

Associated with a symmetric market equilibrium (M.E.) are the steady state numbers,  $L=L_S=L_B$  and  $K=L_M=L_N$ , and the equilibrium values of the matching probabilities,  $\alpha$ ,  $\beta$ , and  $\gamma$ . The following proposition relates the equilibrium to the values assumed by these probabilities.

#### PROPOSITION.

- (i) If there exists a M.E. in which the middlemen are not active (K=0), then  $\gamma(0,L) \leq \alpha(L,L)$  and the equilibrium agreements are  $P_{SB}=\frac{1}{2}$ .
- (ii) If there exists a M.E. in which the middlemen are active (K>0), then  $\gamma(K,L) \geq \alpha(L,L)$ , and the equilibrium agreements are

$$P_{SB} = (1 + V_S - V_B)/2,$$
  $P_{SN} = (V_M + V_S - V_N)/2,$   $P_{MB} = (1 - V_N - V_B + V_M)/2,$ 

where the values  $V_i$  are the unique solution to the system,

(5) 
$$V_S = \delta[\alpha[(1 + V_S - V_B)/2] + \beta[(V_M + V_S - V_N)/2] + (1 - \alpha - \beta)V_S]$$

(6) 
$$V_B = \delta[\alpha[(1 - V_S + V_B)/2] + \beta[(1 + V_N + V_B - V_M)/2] + (1 - \alpha - \beta)V_B]$$

(7) 
$$V_M = \delta[\gamma[(1 + V_N - V_B + V_M)/2] + (1 - \gamma)V_M]$$

(8) 
$$V_N = \delta[\gamma[(V_M - V_S + V_N)/2] + (1 - \gamma)V_N].$$

The proof is quite straightforward and will therefore be omitted (the reader is referred to the working paper version [Rubinstein and Wolinsky, 1985b]). The central observation is that, when the middlemen are active and all possible exchanges  $(P_{SB}, P_{SN}, P_{MB})$  take place, the different values are given by (5)–(8). To see this, consider, for example, equation (5). If the seller is going to trade with the first agent (buyer or middleman) he meets, then the seller's utility of being unmatched  $V_S$  is the discounted expected value of the meeting a buyer and receiving  $P_{SB}$  (which will occur with probability  $\alpha$ ), of meeting a middleman and receiving  $P_{SN}$  (which will occur with probability  $\beta$ ), and of remaining unmatched (which will occur with probability  $1 - \alpha - \beta$ ). Thus,  $V_S = \delta[\alpha P_{SB} + \beta P_{SN} + (1 - \alpha - \beta)V_S]$ , which is exactly equation (5). The other equations can be explained similarly.

Now, if a solution for (5)–(8) is indeed an equilibrium, it will also satisfy the individual rationality constraints that all agents want to participate  $V_B \geq 0$ ,  $V_S \geq 0$ ,  $V_N \geq 0$  and that all agents want to take part in all exchanges,  $P_{SB} \geq V_S$ ,  $1-P_{SB} \geq V_B$ ,  $P_{SN} \geq V_S$ ,  $1-P_{MB} \geq V_B$ . It is a matter of routine calculation to verify that the unique solution for (5)–(8) satisfies these additional constraints only if  $\gamma \geq \alpha$ . The interpretation of this requirement is that for intermediation to be viable at equilibrium the middleman must be efficient enough (in fact, at least as efficient as regular buyers and sellers) in making contacts.

The other type of equilibrium is such that middlemen do not participate. The equilibrium's values satisfy a degenerate version of (5)–(8), and an additional condition that it is unprofitable for a middleman to become active. A calculation shows that this condition will be satisfied only if  $\gamma(0,L) \leq \alpha(L,L)$ . That is, the existence of equilibrium without intermediation requires that the potential middlemen are relatively inefficient in making contacts.

Finally, the existence of equilibrium amounts to the existence of nonnegative numbers L and K that satisfy the steady state condition (4) and an additional condition that determines the extent of entry to the middlemen's trade. The latter condition is either  $\alpha(L,L) < \gamma(K,L)$  and  $2K = \overline{K}$ , or  $\alpha(L,L) \ge \gamma(K,L)$  with strict inequality only if K=0, according to whether or not the middlemen are active at equilibrium. Under natural and quite mild assumptions on the functions  $\alpha$  and  $\gamma$  at

least one of the above pairs of conditions has a solution, and hence there exists a market equilibrium.

#### IV. DISCUSSION

The discussion will focus on the equilibria in which the middlemen participate actively. Upon solving system (5)–(8), the equilibrium prices  $P=(P_{SB},P_{SN},P_{MB})$  and values  $V_i$  can be expressed in terms of the equilibrium matching probabilities  $\alpha$ ,  $\beta$ ,  $\gamma$ , and the discount factor  $\delta$ . The magnitudes of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  depend on the length of a single time period in the model. Let  $\Delta$  denote the length of a time period, and write  $\alpha(\Delta)=\alpha\Delta$ ,  $\beta(\Delta)=\beta\Delta$ ,  $\gamma(\Delta)=\gamma\Delta$ , and  $\delta(\Delta)=e^{-r\Delta}$ , where  $\alpha$ ,  $\beta$ , and  $\gamma$  are the different instantaneous rates of meeting and r is the instantaneous rate of time preference.

Since we would like to think of the process of matching and transacting as taking place continuously, we shall be interested in the limiting equilibrium magnitudes, as the length of a period  $\Delta$  is made arbitrarily small. These limiting magnitudes do not describe the outcomes in a "frictionless" market, since the source of friction is the passage of time, which could be of great significance if r is relatively large. In order to capture the outcomes in an approximately frictionless market, we shall also consider the limiting equilibrium magnitudes under the assumption that r is negligible relative to the rates of meeting  $\alpha$ ,  $\beta$ , and  $\gamma$ .

The Asymmetry Between Sellers and Buyers

Upon computing the equilibrium magnitudes (the equilibrium with active middlemen), we get that  $V_S < V_B$  and  $P_{SB} < \frac{1}{2}$  both when there are frictions and when the market is approximately frictionless. E.g.,

$$\lim_{\Delta \to 0} P_{SB} = (r + \gamma)/(2r + 2\gamma + \beta)$$

and

$$\lim_{r\to 0}\lim_{\Delta\to 0}\,V_S=\gamma/(2\gamma\,+\,\beta)<(\gamma\,+\,\beta)/(2\gamma\,+\,\beta)=\lim_{r\to 0}\lim_{\Delta\to 0}\,V_B.$$

Thus, although it seems that the assumptions of the model treat the sellers and the buyers symmetrically, the equilibrium outcome is biased in favor of the buyers.

This asymmetry owes of course to the participation of middlemen, since in the absence of middlemen the equilibrium outcome is symmetric between sellers and buyers (see part (i) and the proposition). Furthermore, as

$$\lim_{r\to 0}\lim_{\Delta\to 0}(V_B-V_S)=\beta/(2\gamma+\beta),$$

we may conclude that the extent of the asymmetry depends on the extent to which the trade is conducted through middlemen. That is, the larger is  $\beta$  and hence the larger is the share of the trade that passes through middlemen, the larger will be the difference  $V_B-V_S$ , which reflects the asymmetry.

The qualitative reason for this asymmetry will be explained below after we demonstrate how a change in the procedure of the trade removes the asymmetry.

### Consignment

The above described asymmetry will disappear if the trading procedure takes the form of consignment. That is, a middleman gets the good without actually buying it and only upon selling it to a buyer does he pay the seller-owner a predetermined price  $\pi$ .

Consider the modified model in which the middlemen trade by consignment. The counterpart of the proposition will state that if there exists an equilibrium with active middlemen, then  $\gamma \geq \alpha/2$ , and the equilibrium magnitudes are given by the counterpart of system (5)–(8), which consists of (5) and the following:

$$\begin{split} V_B &= \delta[\alpha[(1-V_S+V_B)/2] + \beta[(1+V_N-V_M-\pi+V_B)/2] \\ &+ (1-\alpha-\beta)V_B] \\ V_M &= \delta[\gamma[(1+V_N-\pi+V_M-V_B)/2] + (1-\gamma)V_M] \\ V_N &= \delta[\gamma V_M + (1-\gamma)V_N] \\ &- \pi \delta \gamma/(1-\delta(1-\gamma)) = V_S + \frac{1}{2}(V_M-V_N-V_S). \end{split}$$

The variable  $\pi$  is the predetermined price paid by a middleman to a seller after the middleman has sold the unit consigned to him by the seller. At the time of the agreement between a seller and a middleman, the expected discounted value of the price  $\pi$  is  $\pi \delta \gamma/[1-\delta(1-\gamma)]$ . Thus, the last equation states that the expected discounted value of  $\pi$  is such that the net surplus associated with a seller-middleman transaction is divided equally between the parties. The main difference between these equations and systems (5)–(8) is that, since  $\pi$  is paid to the seller only after the middleman finds a buyer, the total value associated with a middleman-buyer match is only  $1+V_N-\pi$ .

Upon solving the above equations and deriving the equilibrium

magnitudes, we have that

$$\lim_{r\to 0}\lim_{\Delta\to 0}\ V_B=\lim_{r\to 0}\lim_{\Delta\to 0}\ V_S=\frac{1}{2}.$$

Thus, when the middlemen trade by consignment, the asymmetry found in the main version is removed.

Both the asymmetry in the main version and the symmetry of the last case can be explained by a simple example with one seller, one buyer, and one middleman, whose valuations for the unit are 0, 0, and 1, respectively. Suppose that in this example the seller can pass the good to the buyer only through the middleman and that the price paid in each of the two exchanges is determined by Nash's bargaining solution. Consider first the case where, as in the main version of the model, the middleman has to buy the good from the seller. In this case, once the middleman obtained the good, the price he paid to the seller is sunk. Therefore, the total surplus to be divided in the middleman-buyer exchange is 1. Since the disagreement point is (0,0), the bargained price is 1/2, and the buyer's surplus is 1/2. Consequently, in the seller-middleman exchange the total value to be divided is just 1/2, and since the disagreement point is (0,0), the price received by the seller is 1/4. This example is reminiscent of Harsanyi's joint bargaining paradox (see Harsanyi [1977, p. 209]), where similar asymmetry arises in a three-person bargaining problem when two of the parties bargain jointly.

In contrast, if the middleman gets the good by consignment promising to pay  $\pi$ , then in the middleman-buyer exchange the price  $\pi$  is not sunk, and the value to be divided is just  $1-\pi$ . The price paid by the buyer is then  $\frac{1}{2}(1+\pi)$ , and this is also the value to be divided in the seller-middleman exchange. Therefore,  $\pi=\frac{1}{4}(1+\pi)$  implying that  $\pi=\frac{1}{3}$  so that the surplus is divided symmetrically among the three participants.

The insight gained by the example is that the symmetry-asymmetry of the distribution of the gains from trade depend on whether or not the price paid by the middleman to the seller is already sunk when the middleman deals with the buyer. The model of the paper incorporates these simple examples into a more complicated market framework. When the trading procedure is such that the middleman has to buy the good, the price paid by the middleman is already sunk when he meets the buyer, and the bargaining with the buyer is over the division of the full surplus  $1 + V_N$ . In contrast, the consignment procedure strengthens the middleman's position against the buyer since the price  $\pi$ , which is not sunk, is excluded from the bargaining, and the total value to be

divided is just  $1 + V_N - \pi$ . The consignment also strengthens the position of the seller against the middleman, since price concessions made by the middleman will partly be rolled over to the buyer by diminishing  $1 + V_N - \pi$ . This is opposed to the main version of the model in which such price concessions are fully absorbed by the middleman, and hence the middleman is less compromising.

## Middlemen's Profitability

The middlemen's profit opportunity is explained by the time-consuming nature of the trade in this market, which makes it possible to extract some of the sellers' and buyers' surplus in return for shortening their waiting time. Indeed, the middleman's markup,  $P_{\mathit{MB}} - P_{\mathit{SN}}$ , depends positively on the rate of impatience r,

$$\lim_{\Delta\to 0} (P_{MB} - P_{SN}) = r/(2r + 2\alpha + \beta).$$

Notice that  $\lim_{M \to \infty} (P_{MB} - P_{SN}) = \lim_{M \to \infty} \frac{1}{2} (1 - V_S - V_B)$ . If the middle-man had the power to set prices, the markup would be  $P_{MB} - P_{SN} = 1 - V_B - V_S$ , but since he has to bargain over prices, the limiting value of the markup is only  $\lim_{M \to \infty} \frac{1}{2} (1 - V_B - V_S)$ .

The markup  $P_{MB}-P_{SN}$  or rather the difference  $1-V_B-V_S$  represents the profit opportunity for a middleman. The profit itself depends also on the discount factor  $\delta$  and the rate  $\gamma$  at which a middleman can contact partners. As noted above, the middleman's expected discounted profit  $V_N$  is positive only if the equilibrium values are such that  $\gamma > \alpha$ . This is not surprising, since in this model a middleman can profit only by exploiting the impatience of buyers and sellers. But since the middlemen are assumed to be as impatient as the others, they can profit from the other agents' impatience only if they are more efficient in creating contacts.

In contrast, in the consignment case described above we have

$$\lim_{r\to 0}\lim_{\Delta\to 0}V_N=(2\gamma-\alpha)/(4(2\alpha+\beta)),$$

which means that the middleman's activity is profitable even for  $\gamma < \alpha$ . The reason is that in this case the middlemen do not profit just from speeding the trade, but also from their improved position against the buyers, which owes to the fact that they do not possess the good. To see this, consider an equilibrium with no middlemen. In this case  $P_{SB} = \frac{1}{2}$ , and

$$\lim_{\Delta\to 0} V_B = \lim_{\Delta\to 0} V_S = \alpha/(2(\alpha+r)).$$

Suppose now that a middleman who operates by consignment

enters this market. If the middleman is as efficient as the sellers  $(\gamma=\alpha)$ , he can still make positive profit by agreeing with some seller on a price  $\pi$  between  $\frac{1}{2}$  and  $1=V_B$  to be paid upon the completion of a transaction, and then agreeing with a buyer on some price between  $\pi$  and  $1-V_B$ . It can be verified that both the seller and the buyer will indeed agree to some such prices. This description will remain true even when the middleman is somewhat less efficient;  $\gamma<\alpha$ . However, the maximum price that the middlemen can extract from a buyer is bounded by  $1-V_B$  and when the middleman is too inefficient,  $\gamma<\alpha/2$ , even  $\pi=1-V_B$  is unprofitable for the seller, since the present value of getting it through the middleman,  $\gamma(1-V_B)/(\gamma+r)$ , is smaller than  $V_S=\alpha/2(\alpha+r)$ .

## The Gains to Sellers and Buyers

A natural question in the context of the present model is whether the intermediation improves the well-being of buyers and sellers. To address this question, let us ignore the distribution of the gains from trade, and compare  $V_B + V_S$  in a steady state equilibrium with active middlemen to its value in a steady state equilibrium in which the participation of middlemen is forbidden. It should be emphasized that this comparison of two alternative steady states does not tell us whether adding middlemen to a steady state market without them would improve the total welfare of buyers and sellers, since to address this question we would have to also take into account the transition time before the new steady state was reached.

Suppose that there exists a unique market equilibrium and that it is such that the middlemen participate in the trade. It follows from solving (5)–(8) that at this equilibrium

$$\lim_{\Delta\to 0} (V_B + V_S) = (2\alpha + \beta)/(2r + 2\alpha + \beta).$$

If the participation of middlemen is prevented, then at the resulting steady state equilibrium

$$\lim_{\Delta\to 0} (V_B + V_S) = 2\alpha'/(2r + 2\alpha'),$$

where  $\alpha'$  is the matching probability that corresponds to the latter steady state situation. Observe that  $\lim_{N \to \infty} (V_B + V_S)$  is greater or smaller with intermediation according to whether  $2\alpha + \beta$  is greater or smaller than  $2\alpha'$ . However, there is no necessary relationship between  $2\alpha + \beta$  and  $2\alpha'$ . Therefore, without specifying the proper-

ties of the matching probabilities in more detail, there is no general conclusion regarding the effect of the intermediation on the combined benefits of sellers and buyers.

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