ON THE QUESTION "WHO IS A J?"*
A Social Choice Approach

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Abstract
The determination of "who is a J" within a society is treated as an aggregation of the views of the members of the society regarding this question. Methods, similar to those used in Social Choice theory are applied to axiomatize three criteria for determining who is a J:
1) a J is whoever defines oneself to be a J.
2) a J is whoever a "dictator" determines is a J.
3) a J is whoever an "oligarchy" of individuals agrees is a J.

* We wish to thanks Dubi Samet for his useful comments on an earlier version of this paper.

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1. Introduction

Each person belongs to collectives of various kinds, such as a family, a guild or a nation. Some of these collectives have well-defined extensions whereas other do not. Consider, for example, a fundamental concept like "family". One definition of that term, according to the *Oxford English Dictionary*, is "the group of persons consisting of the parents and their children, whether actually living together or not". By this definition, a person's family consists either of one's parents and siblings, if there are any, or of one's spouse and descendants, if there are any. But a more problematic meaning of "family" is "those descended or claiming descent from a common ancestor". In this sense, the term family depends on the views held by people about descent from a common ancestor. Under ordinary circumstances, there is no room for a collective decision on an issue of the type "who is a J". However, under certain circumstances, a decision has to be made. If the Sikhs are to be the legal guardians of certain temples, then it should be determined in advance "who is a Sikh?" (McLeod 1989). Or, if the Jews naturalize in Israel under a special "Law of Return", then the extension of the collective of the Jews has to be determined for the law to be enactable (Kasher, 1985).

Kasher (1990) presents the collective identity problem, as an *aggregator*: each of n individuals in a society holds a view with respect to every individual, including oneself, whether the latter is a J. The collective identity of J is determined by the individual views of "who is a J". The method of determining who is a J is viewed as a function which assigns a meaning to "who is a J" for each profile of all the individual views.

Kasher (1990) looks for an aggregation method which satisfies a principle of *fairness* (in the sense of Rawls (1971)): At the starting point of a decision procedure, all given views of "who is a J" should be treated on a par with each other, none enjoys any privilege or suffers from any prejudice. In Kasher (1990), such a "fair" method has been introduced and discussed (see Section 3 for more details).

The present paper springs from Kasher (1990) and links it to the formal theory of aggregators which has been developed mainly in economic theory (see e.g., Rubinstein

and Fishburn (1986)). Within the latter theory, an aggregator is a function that maps every n-tuple of elements of a given set X into the set X itself. Intuitively speaking, such a function is interpreted as being a systematic "averaging" of the "collective perception" of members of N of the aggregated object. The most famous sphere of problems and theories of aggregation is that of social choice theory (see e.g. Arrow (1963) and Sen (1970)) which deals with methods of aggregating the preferences held by members of a society.

The present paper applies formal results from the theory of aggregation to issues of collective identity. We will use the *axiomatic approach*. A typical investigation along the lines of the axiomatic approach involves two steps: First, a presentation of a list of constraints (axioms) imposed on a class of aggregators and second, a formal characterization of the set of all aggregators that satisfy those constraints.

In the sequel we present three axiomatizations characterizing three aggregators (actually the aggregators refer to three settings which are slightly different; we will explain this point later):
(A1) The *"Liberal*" aggregator: An individual is a J if and only if one defines oneself to be a J.
(A2) The "*Dictatorship*" aggregator: A pre-designated member of the given society determines who is a J.
(A3) The "*Oligarchical*" aggregator: Two members of the given society belong to the same group if and only if they are both considered to have the same collective identity by all members of a pre-designated subgroup in the society.

The characterization of the Liberal aggregator is new whereas the other two characterizations provide new interpretations for previous results.

2. "A J is whoever considers oneself to be a J"

We start with the basic model. Let $N=\{1,..,n\}$ be the set of individuals in a given society. Each $i \in N$ perceives the members in the subset $J_i \subseteq N$ to be Js. A profile is an n-tuple of vectors $(J_1, ...,J_n)$ where $J_i \subseteq N$. A Collective Identity Function (CIF) is a

function which assigns to each profile (J_1 ,....,J_n) a subset of N**,** $J(J_1,....,J_n)$. To simplify notation we often write J instead of $J(J_1$,....,$J_n)$.

We will discuss, now, several axioms which are used for our axiomatization in this section: All the axioms refer to CIF's. The first three axioms are close in spirit to axioms familiar in the social choice literature. The first, the Consensus axiom, requires that if there is an agreement among all individuals that a certain member is a J (or, alternatively, that he is not a J), then the CIF determines that this member is a J (or, alternatively, a non-J).

Consensus (C): If $j \in J_i$ for all i, then $j \in J$; if $j \notin J_i$ for all i, then $j \notin J$.

The next axiom, the Symmetry axiom, requires that the aggregator does not discriminate between any two members of the society on any basis other than that embedded in the profile of views. Here, we employ a weak version of this requirement: We will simple require that if individuals j and k are symmetric in a particular profile, then the CIF either determines both to be Js or determines both to be non-Js.

Symmetry (SYM): We will say that j and k are symmetric in a profile (J_1...,J_n) if

(i) they have the same views about all other members (J_j-{j,k}=J_k-{j,k}),

(ii) all other members have the same views about j and k (for all $i \in N$-{j,k}, $j \in J_i$ iff $k \in J_i$)

(iii) j considers himself a J if and only if k considers himself a J ($j \in J_j$ iff $k \in J_k$)

(iv) j's view of k is the same as k's view of j ($j \in J_k$ iff $k \in J_j$)

Then, $j \in J$ if and only if $k \in J$.

The next axiom requires that if one of the individuals in the society, k, has viewed i as a non-J (analogously a J), and he changed his view in favor of i being a J (analogously, a non-J), then if i has been recognized before the change to be a J (a non-J), the change in k's view does not exclude i from being a J (a non-J).

Monotonicity (MON): Assume that $i \in J(J_1,...,J_n)$. Let $(J_1',...,J_n')$ be a profile identical to $(J_1,...,J_n)$ except that there are individuals, i and k, so that $i \notin J_k$ and $i \in J_k'$; then $i \in J(J_1',...,J_n')$. (Analogously, if $i \notin J(J_1,...,J_n)$ and if $(J_1',...,J_n')$ is identical to $(J_1,...,J_n)$, except that there is a k, $i \in J_k$ and $i \notin J_k'$, then $i \notin J(J_1',...,J_n')$).

Note that MON has been defined for a single change. The axiom does not exclude the possibility that the change in k's view about i will affect another player, j.

Most axiomatizations contain an axiom that determines the elements that determine whether i is a J. Here we require that the question whether i is a J depends only on the views about i (including i's view about oneself) and the other members' identity as members of J.

Independence (I): Consider two profiles $(J_1,...,J_n)$ and $(J_1',..., J_n')$ and let i be a member of N. If for every $k \neq i$, $k \in J$ if and only if $k \in J'$, and for all k (i inclusive) $i \in J_k$ if and only if $i \in J_k'$, then $i \in J$ if and only if $i \in J'$.

We now move to an axiom which does not have a clear analogy in Social Choice Theory and is special to the present context of collective identity. The following Liberal Principle states that it is impossible that no one will be determined to be a J, though there is an i who considers oneself to be a J. Similarly, it is impossible that everyone will be considered a J, though there is an i who considers oneself to be a non-J.

The Liberal Principle (L): If there is an i such that $i \in J_i$, then $J(J_1,...,J_n) \neq \varnothing$, and if there is an i such that $i \notin J_i$, then $J(J_1,...,J_n) \neq N$.

The "Liberal Principle" captures a "liberal" view under some seemingly extreme conditions: If no one is considered to be a J or everyone is considered to be a J, then a member's view of oneself should be held decisive.

Notice that the axioms C, SYM and L refer to the way that the aggregator operates on a certain profile in isolation from the way it is defined on other profiles. In contrast,

axioms MON and I impose constraints on the way the aggregator is defined on various profiles.

We are ready for the axiomatization of the strong liberal CIF defined by $J(J_1,...,J_n)=\{i|\ i\in J_i\}$.

Theorem 1(a): The strong liberal CIF is the only CIF that satisfies axioms C, SYM, MON, L and I.

Proof: Obviously, the strong liberal CIF satisfies these five axioms. Consider an arbitrary CIF that satisfies the five axioms. Assume that there is a profile P_1 in which $i\in J_i$ but $i\notin J(P_1)$. By applying MON several times, we arrive at a profile P_2 that is identical to P_1 with the possible exception that for all $k\neq i$, $i\notin J_k$ so that $i\notin J(P_2)$. Denote $J(P_2)=M$. Let P_3 be the profile where $J_j=\{j\}$ for each $j\in M\cup\{i\}$ and $J_j=\varnothing$ for any $j\notin M$. By C, $J(P_3)$ does not contain any of the members of N-M-{i}. By SYM, the aggregator classifies all members of $M\cup\{i\}$ identically. It is impossible that $J(P_3)=\varnothing$ because this results in a contradiction to L. Thus, $J(P_3)=M\cup\{i\}$. Finally, we get a contradiction to I because $J(P_2)$ and $J(P_3)$ are identical with the exception of member i, and member i is treated equally by all members of N (including itself); nevertheless, $i\in J(P_3)$ and $i\notin J(P_2)$. By analogous arguments, if $i\notin J_i$, then $i\notin J$. ÿ

Next, we prove that all axioms used in theorem 1 are necessary for the characterization of the aggregator.

Theorem 1(b): The strong liberal CIF is not the only CIF that satisfies some but not all of the axioms C, SYM, MON, L and I.

Proof: The proof consists of 5 examples, each satisfies four of the five axioms but not the fifth.

(1) Let n be an odd number. Consider the aggregator defined by $J(J_1,...,J_n)=\{i|i\in J_i\}$ if the cardinality of $\{i|i\in J_i\}$ is odd and $J(J_1,...,J_n)= \{i|i\notin J_i\}$ otherwise. This aggregator does not satisfy C (member 3 is not a J for the profile where $J_i=\{1,2\}$ for all i, although no one considers 3 a J). All other axioms are satisfied; following are some hints to verify this. Of course, SYM is satisfied by J. L is satisfied because if $J(J_1,...,J_n)=N$, then it must be that $i\in J_i$ for all i. MON is satisfied because if $i\in J(J_1,...,J_n)$, then any change of some k's view about i does not change i's status, and if $i\notin J_i$ and $i\in J'_i$, then the cardinality of $\{i|i\in J_i\}$ must be even and the cardinality of $\{i|i\in J'_i\}$is odd; thus $i\in J(J_1',...,J_n')$. I is satisfied because for any two profiles, $(J_1,...,J_n)$ and $(J'_1,...,J'_n)$, which have the same set of Js (other than i), and the same view of i on itself, the cardinality of $\{i|i\in J_i\}$ is the same as of $\{i|i\in J'_i\}$ and thus $i\in J(J_1,...,J_n)$ iff $i\in J(J'_1,...,J'_n)$.

(2) Consider the aggregator which assigns to $J(J_1,\ldots,J_n)$ any i for which $i\in J_i$ with the exception of member 1, who will be considered to be a J if (i) he is the only i for whom $i\in J_i$ or (ii) he is considered to be a J by all members of N. This aggregator satisfies all axioms but SYM. To verify that it satisfies I, note that for all $i\neq1$, i being a J depends only in his view on himself. As for member 1, being a J depends on how many other members are Js and how the whole group views member 1.

(3) Consider $J=\{i|\ J_i=\{i\}\}$, that is, a J is anyone who considers only oneself to be a J. This aggregator satisfies all axioms but MON.

(4) Consider the aggregator which classifies a member to be a J if and only if all members of N agrees that member is a J. This aggregator satisfies all axioms but L.

(5) Let J(0) be the set of all individuals for which there is a consensus that they are Js (possibly an empty set). Expand the set inductively by adding, at the t-th stage, those members of N who consider themselves as Js and for whom there is a consensus among J(t-1) that they are Js. This procedure satisfies all axioms but does not satisfy I, as can be seen by considering the following two profiles: Let P_1 be the profile $J_1=\{1,2,3\}$, $J_2=\{1,2\}$, and $J_3=\{1\}$; for P_1 the procedure determines all members of N to be J's ($J(1)=\{1\}$ and $J=J(2)=N$). Let P_2 be the profile $J_1=\{1,2,3\}$, $J_2=\{1,2$, and $J_3=\{1,2\}$.

Now, the procedure starts and ends with J=J(1)=J(2)={1,2}. Although members 1 and 2 are determined in the two profiles to be Js and although the attitude to member 3 is unchanged in the two profiles, $3\in J(P_1)$ but $3\notin J(P_2)$. Thus, I is not satisfied.

3. A discussion of Kasher's method

The result of the last section sheds light on the aggregation method suggested in Kasher (1993). Kasher (1993) suggests a recursive procedure: Start by J(0), the set of all members of the given society for whom there is an absolute consensus within the society that they are Js. Add to J(0) all members that at least one member of J(0) considers a J. Call the new set J(1) and continue inductively until you cannot expand the set any further. Formally, $J(t)=J(t-1)\cup\{k\in J_i \text{ for some } i\in J(t-1)\}$ and let J=J(t)=J(t+1).

Kasher's method satisfies all the axioms which we employed in the previous section with the exception of L! This axiom is not adopted by Kasher for he attempts to derive an aggregation method from pure considerations of fairness and he does not consider L as derivable from fairness considerations only. For some questions of collective identity (like political collectives) it seems that fairness requires application of some self-determination principle, but on other occasions (like professional collectives) fairness does not require adherence to that liberal principle.

Note that Kasher's method treats asymmetrically "being a J" and "being a non-J". In contrast, the axioms in the previous section treats "being a J" and "being a non-J" symmetrically.

The axiomatization of Kasher's method remains to be completed. Note that the difficulty in finding a suitable axiomatization is due to the difficulty of justifying why the recursive process starts with the set $\{i|\ i\in J_j \text{ for all } j\}$ and not with another set, such as $\{i|i\in J_i\}$, for example.

5. The dictatorship

In this section we use results from aggregation theory in order to axiomatize a dictatorship method: A J is whoever i* (the dictator) perceives to be a J (that is, there is i* so that $j \in J$ if $j \in J_{i^*}$.).

The axiomatization will be carried out by using a slightly modified version of the notion of the Collective Identity Function. In this section, we assume that there is a consensus in the society that the set of Js is a proper subset of N; that is, all agree that there is someone who is a J and someone who is a non-J. A CIF* is a function which assigns a proper subset of N to every profile of proper subsets of N.

The axiomatization employs two axioms, C, which is familiar from the previous section, and a more stringent version of the independence axiom, I*. By this axiom, whether i is a J or not depends only on how the individuals view i, independently of how the other members are viewed.

Independence (I*): Consider two profiles, $(J_1,...,J_n)$ and $(J_1',...,J_n')$, satisfying that for all k, $i \in J_k$ if and only if $i \in J_k'$. Then $i \in J(J_1,...,J_n)$ if and only if $i \in J(J_1',...,J_n')$.

The following theorem (taken from Rubinstein and Fishburn (1986)) is related to Arrow's celebrated impossibility theorem:

Theorem 2: The only CIF*s that satisfy C and I* are the dictatorships.

To get some intuitive grasp of the result, consider the following aggregators:

(1) The majority rule aggregator determines i to be a J if a majority of individuals consider i to be a J. This aggregator is not a CIF* because it may assign to a profile of views a non-proper subset of N. For example, consider the profile, $J_1=\{1,2\}$, $J_2=\{1,3\}$, and $J_3=\{2,3\}$. By the majority rule aggregator, $J=\{1,2,3\}$.

(2) An aggregator which assigns the same proper subset of N to all profiles does not, of course, satisfy C though it does satisfy I*.

(3) Given a profile $(J_1,...,J_n)$, denote by N(i) the number of Js who consider i to be a J. Let $W=\{i|\ N(i)\geq N(j)$ for all $j\}$ be the set of "most popular Js". If W=N, define J=J* where J* is a fixed proper subset of N. If $W\neq N$, define J=W. For all i, the cardinality of J_i is strictly between 0 and n. Thus, a member i with N(i)=0 cannot be in W, and a member i with N(i)=n is necessarily in W. Thus the CIF* satisfies C. On the other hand, clearly, the aggregator does not satisfy I*.

6. Oligarchy

In this section the question "who is a J" is considered as part of a task partitioning all members of the society into an unlimited number of classes (and not only to Js and non-Js). Each individual in the society has a view about the partition of N and an aggregator is required to determine the partition of N as a function of the individuals' views.

Formally, each individual, $i\in N$, specifies an equivalence relation on N, $\sim_i$, with the interpretation that if i considers j and k to be equivalent ($j\sim_i k$), then he views j and k as belonging to the same class. A CIF** is a function which assigns to each profile of equivalence relations $(\sim_1,\ldots,\sim_n)$, an equivalence relation $\sim(\sim_1,\ldots,\sim_n)$. To simplify notation, we will sometime refer to $\sim(\sim_1,\ldots,\sim_n)$ as $\sim$.

Note that in this formalization, the classes in the partition induced from $\sim_i$ do not have names: That is, the model does not distinguish between the case that i classifies 1 and 2 to be Js and the rest of the society to be non-Js, and the case in which individual i considers 1 and 2 to be the only non-Js.

Once again we will employ a consensus axiom:

C** (consensus) : If all individuals consider j and k to be in the same equivalence class (for all i, $j\sim_i k$), then the aggregator classifies i and j in the same class ($i\sim j$).

Note that according to C** , the fact that there is a consensus that i and j are in the same class, does not mean that all agree on the name of that identity. It might be that some members think that j and k are Js while some other members think that they are both actually fake Js. Yet, an aggregator satisfying C** must classify j and k in the same group.

The independence axiom, which we employ here, requires that the question whether j and k are in the same class will be determined by the members' opinions about whether j and k are in the same class independently of their views on other couples of individuals.

I** (independence): Consider two profiles of equivalence relations, $(\sim_1,...,\sim_n)$ and $(\sim_1',...,\sim_n')$, in which for every i, j, and k, $i\sim_k j$ if and only if $i\sim_k' j$. Then $i\sim(\sim_1,...,\sim_n)j$ if and only if, $i\sim(\sim_1',...,\sim_n')j$.

The following theorem, proved in Berthelemy, Leclerc and Monjardet (1986) (see also Fishburn and Rubinstein (1986)), characterizes the oligarchical aggregators. An oligarchical CIF** is one for which there is a non-empty subset of individuals M so that $i\sim(\sim_1,...,\sim_n)j$ if only if $i\sim_k j$ for all $k\in M$.

Theorem 3: The only CIF**s that satisfy C** and I** are oligarchical.

In order to get some intuitive grasp of the result, consider the following aggregators:

(1) The aggregator which determines i and j to be equivalent if and only if a majority of individuals consider them to be equivalent, does not necessarily define an equivalence relation. For example, consider the profile P_1 where individual 1 perceives all members to be in the same group, 2's partition of society is {{1,2},{3}}, and individual 3's partition is {{1},{2,3}}. By the majority rule, 1~2 and 2~3 but not 1~3.

(2) The aggregator which determines every i and j to be equivalent, independently of the profile of individuals' opinions, satisfies I* but, of course, does not satisfy C**.

(3) The transitive closure of the majority aggregator satisfies C** but does not satisfy I**. To see this note that when applied to the profile P_1 (the profile used in (1)), 1~3. On the other hand, when applied to the profile P_2 (where 1's partition is {{1,2,3}} and individuals 2 and 3 partition N into {{1},{2},{3}}), the aggregator determines that $1 \not\sim 3$ although all individuals have the same opinion in P_1 and P_2 regarding members 1 and 3 as being in the same class.

7. Conclusion

In this paper we presented the collective identity problem as an aggregation problem using methods taken from social choice theory. Admittedly, one of our motivations in working on the project was the fascination with the connection between a non-formal problem like collective identity and formal models like those of social choice theory.

Let us stress the point that the discussion here is not meant to express our views about the question "who is a J?" in any of its concrete real-life versions. Our analysis here is, of course, a purely logical exercise. Arrow's impossibility theorem does not support dictatorship and, by analogy, Proposition 1 in this paper does not necessarily support the view that a J is whoever defines oneself to be a J. If anyone does not approve of that criterion in a concrete context, he has now a tool to examine his intuition by pointing out an axiom which he does not agree with.

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