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Notes, Comments, and Letters to the Editor

The curse of wealth and power ☆

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Abstract

We study a model in which being more powerful does not necessarily imply being wealthier.

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1. Introduction

In strategic situations, being wealthy and powerful is considered to be advantageous. However, imagine a world where being powerful means being able to seize control of the assets held and accumulated by others. Then, being wealthy might attract the attention of those who are powerful and be detrimental to one's wealth. So is being powerful, as those who seize control of the wealth of others will in turn become a desirable target for those who are in a position to seize their acquired wealth. In this short paper, we identify a class of situations where one agent is stronger than another and yet ends up being poorer.

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In our model, each agent is endowed with an exogenous amount of initial wealth. Agents are ordered by power. Each agent can wield his power over at most one other agent. The outcome is a hierarchy in which an agent at the root of each of the resulting chains accumulates all the wealth belonging to the members of the chain. The decisions regarding whose wealth to seize are made simultaneously and the solution concept we employ is the mixed strategy Nash equilibrium. We find that if there are at least three players and power is correlated with initial wealth, the second strongest player (who is also the second wealthiest player) will always be worse off than one of the weaker players.

Our result shares some features with several other examples where the notion of mixed strategy equilibrium predicts that weakness can be bliss. For example, in a simultaneous voting game with costly voting, the minority can have better chances of victory than the majority (see [1,2]). In an example of the "truel" proposed by Shubik in 1954 (see [4]), three gunmen having different accuracy are involved in a fight. For some parameters, the most accurate shooter has the smallest probability of winning.

2. The model

The primitives of the model are a set of agents, a power relation, and the initial endowments. The set of agents is $\{1, 2, \dots, N\}$, where $N \geqslant 3$. We assume that the agents are ordered by power: if $i < j$ then i is stronger than j. There is only one good which we will refer to as wealth. Each agent i is initially endowed with $a^i > 0$ units of wealth.

The situation is analyzed as a standard strategic game. An agent can establish at most one link with one weaker agent. The set of actions agent i is $\{0, i+1, i+2, \dots, N\}$, where the action $j > i$ means linking with agent j and action 0 means "no link". A link from i to j results in agent i acquiring the total amount of wealth owned originally and acquired by j. If two agents establish a link with the same agent k only the stronger agent acquires agent k's wealth.

The payoff of an agent is equal to the total wealth he holds at the end of the game minus, if he establishes a link with another agent, a not transferable disutility cost $c > 0$ measured in units of wealth. Thus, for example, if 1 links with 2 who links with 3, agent 1 payoff is $a^1 + a^2 + a^3 - c$, agent 2's payoff is $-c$ and agent 3's payoff is 0.

As we have said, we model this situation as a simultaneous game. An agent decides who to link to before he knows the decisions made by the other agents. This game might not have pure strategy Nash equilibria and we will investigate the mixed strategy Nash equilibria. Let $p_{i,j}$ denote the probability with which agent i establishes a link with agent j and $p_{i,0}$ denote the probability with which agent i establishes no links. Of course, all actions are simultaneous and no player can change his decision after he observes the other players' realized play. The standard criticism of mixed strategy equilibrium (see [3]) applies: it is hard to interpret the mixed strategies as descriptions of courses of actions.

3. The result

For one moment, consider the same model as we describe above with the exception that wealth can only be appropriated directly and no indirect transfers are possible. The analysis of this game is trivial. If $a^1 > a^2 > \cdots > a^N$ and c is small, in any equilibrium agent i links to agent $i+1$ and thus agent 1's final wealth is $a^1 + a^2$, agent 2's final wealth is a^3, and so on. The allocation of wealth resulting from any Nash equilibrium preserves the correlation between wealth and strength: if i is stronger than j, the final wealth of i exceeds the final wealth of j.

We will now show that, if wealth can be transferred via the hierarchy of links, a somewhat surprising phenomenon emerges in *any* equilibrium: if c is small and wealth is correlated with power, there *must* exist a pair of agents i and j such that i is stronger than j but j's expected wealth is strictly above i's expected wealth.

Theorem 1. *Suppose that $a^1 > a^2 > \cdots > a^N$. There exists $\bar{c}$ such that, when $c \in (0, \bar{c})$, in any Nash equilibrium of the game there is an agent i who is weaker than agent 2 and whose expected wealth and payoff are strictly above agent 2's.*

Proof. Take $c < \min_i a^i$. Consider a Nash equilibrium of the game. If $p_{1,2} = 1$, that is, agent 1 links with agent 2 with probability 1, agent 2 does not link with anyone and agent 3 keeps at least his initial wealth. Hence, agent 2's expected wealth and payoff is 0 and lower than agent 3's wealth and payoff. Henceforth, assume that $p_{1,2} < 1$. Consider an agent i for whom $p_{i,0} \neq 1$. Since i maximizes utility, the expected wealth that agent i extracts, directly and indirectly, from any agent in the support of his strategy is the same. Thus, define then G^i to be the expected wealth extracted in equilibrium by an agent i with $p_{i,0} \neq 1$ from an agent in the support of his strategy. Set $G^i = 0$ if $p_{i,0} = 1$. Our claim will follow from the following facts. $\square$

Fact 1. *If $i < j$ and $p_{i,0} \neq 1$, then $G^i \geqslant G^j$.*

If $p_{j,0} = 1$ the claim holds trivially. Suppose that $p_{j,0} \neq 1$ and $G^i < G^j$. Let k be the weakest agent in the support of agent j. Since j maximizes utility, G^j is equal to the expected wealth that agent j appropriates from k. The flow of wealth to k is independent of the actions in the support of agent j's equilibrium strategy since agent j does not link with any agent weaker than k. Hence, if agent i links with k with probability equal to one, he will get from k an expected wealth at least as high as G^j.

Fact 2. (i) $p_{1,0} = 0$; (ii) *if $p_{1,i} > 0$ for $i > 2$, then $p_{i,0} \neq 1$*; (iii) $p_{1,N} = 0$.

(i) Agent 1 always links with some agent as linking with agent 2 is profitable ($a^2 > c$); (ii) If $p_{1,i} > 0$ and $p_{i,0} = 1$ for $i > 2$, then $G^1 = a^i$, which is lower than a^2; (iii) agent N does not link with anybody and thus, from (ii), $p_{1,N} = 0$.

Fact 3. $p_{2,0} > 0$.

Suppose that $p_{2,0} = 0$. By linking with agent 2, agent 1 gets at least $a^2 + G^2$. Let k be the weakest agent in the support of agent 1's strategy. Such an agent exists by Fact 2(i) and $k > 2$ since by hypothesis $p_{1,2} < 1$. By linking with agent k, agent 1 gets $a^k + (1 - p_{k,0})G^k$. By Fact 1, $G^2 \geqslant G^k$. Since $a^2 > a^k$, agent 1 is strictly better off linking with agent 2 only.

Fact 4. *Agent* 1 *links with agent* 2 *with a probability of at least* $\frac{a^N - c}{a^N}$ (*and thus agent* 2's *expected final wealth and payoff converge to* 0 *when* $c \to 0$).

By Fact 2(iii), agent 1 does not link with N and if agent 2 links with N he gets $a^N > c$ with certainty. By Fact 3, it must be that $(1 - p_{1,2})a^N \leqslant c$.

Fact 5. *Let* $\bar{p} = \left(\frac{N-2}{N-1}\right)^{\frac{1}{N-2}}$ *and* $\beta = \frac{a^N - c}{a^N}\bar{p}^{N-3}(1 - \bar{p})$. *There is an agent* $i > 2$ *such that the probability that no agent links with* i *is at least* β (*and thus agent* i's *expected final wealth and payoff is bounded below by* βa_i *when* $c \to 0$).

Let k be the weakest agent in the support of agent 1. Since by hypothesis $p_{1,2} < 1$, Fact 2(i) implies that $k > 2$.

We will show that $p_{j-1,j} < \bar{p}$ for some $2 < j \leqslant k$. Suppose not. Then if agent 1 links with agent 2 he obtains at least

$$
\begin{aligned}
& a^2 + \bar{p}^{k-2}((k-1)a^k + (1 - p_{k,0})G^k) \\
& \geqslant a^2 + \frac{N-2}{N-1}((k-1)a^k + (1 - p_{k,0})G^k) \\
& > a^k + (1 - p_{k,0})G^k
\end{aligned}
$$

since $a^2 > a^k$, $(N-2)a^k \geqslant G^k$ and $k > 2$. A contradiction is obtained since by linking to agent 2 agent 1 extracts strictly more than $a^k + (1 - p_{k,0})G^k$, the expected wealth he extracts from k.

Let j be the strongest agent such that $p_{j-1,j} < \bar{p}$. Then, using Fact 4, the probability that no agent links with j is at least $\frac{a^N - c}{a^N}\bar{p}^{j-3}(1 - \bar{p}) \geqslant \beta$.

The theorem now follows from Facts 4 and 5. $\square$

The analysis of the game in the case of $N = 3$ is particularly simple. In the unique equilibrium agent 1 links with agent 2 with probability 1 and agent 2 does not link with agent 3. The equilibrium payoff of agent 2 is then 0 whereas the equilibrium payoff of agent 3 is positive.

If the agents have the same initial wealth but the cost of linking is *decreasing* with power, a result analogous to Theorem 1 can be proved using similar arguments.

Theorem 1 does not extend to the case of equal a^i's (or equal costs of linking). Take $N = 5$ and suppose that agent 1 links with agents 2 and 3 with probabilities α and $1 - \alpha$, respectively, 2 links with 4 with certainty, and 3 links with 5 with certainty. For c sufficiently small, there is an equilibrium with $\alpha < 0.5$ and agent 2's expected payoff is larger than agent 3's expected payoff. To see this, note that if agent 2 links with agent 3, agent 2's payoff is equal to zero.

Also, if the second wealthiest agent is not the second most powerful, the curse can be defeated. Take $N = 5$ and suppose that $a^1 > a^3 > a^2 > a^4 > a^5$, and $a^3 + a^5 = a^4 + a^2$. For c small, there is an equilibrium where 1 links with 2 and 3 with probabilities α and $1 - \alpha$, respectively, and $\alpha > 0.5$, 2 links with 4 with certainty, and 3 links with 5 with certainty. In this equilibrium, agent 3 has the second largest expected wealth.

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