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RESEARCH ARTICLE

Choosing the two finalists

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Abstract This paper studies a decision maker who for each choice set selects a subset of (at most) two alternatives. We axiomatize three types of procedures: (i) The top two: the decision maker has in mind an ordering and chooses the two maximal alternatives. (ii) The two extremes: the decision maker has in mind an ordering and chooses the maximal and the minimal alternatives. (iii) The top and the top: the decision maker has in mind two orderings and he chooses the maximal element from each.

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1 Introduction

Choice theory aims to provide a framework in which observations about a decision maker's behavior may be organized conceptually. A decision maker is described by

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the possible choices he would make in all feasible situations that the decision maker might confront within a certain context. These choices are represented by a choice function or a choice correspondence. The typical interpretation of both is that the decision-maker will ultimately “consume” only a *single* alternative from the available set. Thus, if the choice correspondence selects more than one alternative, we interpret that to mean that the single element, which the decision-make will ultimately consume, may be any of the selected elements (not that the selected elements form a bundle, which the decision-maker will consume all at once). We have in mind a situation where a decision maker is observed choosing small sets, out of any possible subset of alternatives. Here, we focus on the case that the subsets consist of at most two elements. We will comment on how our analysis would change if we were to assume that the decision maker cannot choose more than some arbitrary fixed number of elements. We offer two interpretations for the decision-maker’s choice of more than one element:

(i) *Consideration sets.* When the decision maker chooses a set in our framework he actually selects a *consideration set*. That is, the decision maker employs a decision process in which he first selects a small number of alternatives that draw his attention, or that he finds deserving a closer look at the second and final stage (see Eliaz and Spiegler 2009). When the decision maker is an individual, this may reflect an attempt to minimize the number of considered elements as deliberating on all of them is too “costly”. When the decision maker is an organization, the choice procedure may involve two distinct agents, one who narrows down the list of possible options to a small set of contenders, and a top manager who decides on the final choice (see García-Sanz 2008).

(ii) *A team.* The chosen set is not an input for another decision problem, but is itself the final decision. This interpretation fits situations in which the decision maker must choose a team with two members. Examples of such situations include the choice of candidates for the position of president and vice president in an election, selecting a two-pilot crew for an airplane, selecting a pair of tennis players for a doubles match, or choosing a pair of instructors for teaching the microeconomics and macroeconomics courses in a graduate program.

Formally, let X be a (finite) set of alternatives. A choice correspondence D is taken in this paper to be a function which assigns to every non-empty $A \subseteq X$ a non-empty subset of A of size at most two.

With regards to our first interpretation, the framework we study here is in the spirit of most of the decision-theoretic literature on preferences over menus (e.g., Kreps 1979) which visualizes the decision process as a two stage process and axiomatizes only the first stage which contains the selection of the second-stage choice problem, ignoring the actual choice in the second stage. An alternative framework, would describe a decision maker by a function that assigns to every choice set A, a subset $B \subseteq A$ and an element $a \in B$ with the interpretations that B is the set of the finalists and a is the final chosen element.

Our objective is to axiomatize the following three types of choice correspondences (note that we use the term “ordering” to denote a strict total order):

I. *The top two.* The decision maker has in mind an ordering and he chooses the two best elements according to this ordering. Under the consideration set interpretation, this procedure fits situations in which the decision maker proceeds in two steps. He first narrows the set of options to only a few contenders by using some initial criterion represented by an ordering, which does not necessarily coincide with his true preferences (for example, the simplest two alternatives). The elements chosen in the initial stage (which constitute the decision-maker's consideration set) are examined more closely at a later stage, where the decision-maker then applies his true preferences. Under the team selection interpretation, this procedure means that the two individuals are chosen independently using the same preference criterion.

II. *The two extremes.* The decision maker orders the alternatives along one dimension and chooses the two most extreme options. Under the consideration set interpretation, this procedure fits situations where a decision maker thinks that his choice is actually between two different "directions". He therefore selects a consideration set consisting of the two extremes with the aim of choosing the direction in the second stage. We may also interpret the choice of the two extremes as the choice of two "attention-grabbers". Under this interpretation, the options are ordered along some dimension (small to big, cheap to expensive, ugly to handsome), and the consideration set consists of the two extremes, the two elements that draw the most attention. Under the team selection interpretation, the choice of the two extremes fits a desire for the most extreme variety.

III. *The top-and-the-top.* The decision maker has in mind two considerations. He selects the (one or two) alternatives that are deemed the best by at least one of the considerations. Of course, any two-extremes procedure may be thought of as a top-and-the-top procedure where the two rationales are the ordering of the two-extremes and its inverse. Note that if a top-and-the-top procedure assigns a couple to every set, then each of its rationales must be the inverse of the other, and hence, it may be described as a two-extremes procedure.

One may also interpret the different considerations in the top-and-the-top procedure as the orderings of two distinct individuals (e.g., a married couple) who need to choose from a set of options. The top-and-the-top selects the (at most) two extreme Pareto efficient alternatives. Of course, the set of Pareto efficient alternatives may be bigger (for a characterization of the Pareto efficient correspondence see Sprumont 2000).

Under the team interpretation, this procedure may be viewed as a way of preparing for two possible scenarios that the decision maker may face: for each scenario, he picks the best alternative to handle that contingency (if there is one alternative that is best in both contingencies, he picks it).

Note that our paper differs from some recent papers, which have analyzed choice functions that are consistent with two-stage choice procedures, in which the decision maker first selects a subset of options to consider, and then chooses from that subset. See, Manzini and Mariotti (2006), Manzini and Mariotti (2007), Masatlioglu et al. (2008), and Manzini and Mariotti (2009). Contrary to us, in these papers, the final choice is observed and the decision-maker's "shortlist" or "attention-grabbers" are inferred from the final choice. For a work on rationalization of a choice correspondence by any number of multiple rationales, see (Aizerman and Malishevski 1981).

In the rest of this short paper we provide axiomatizations of the three choice correspondences discussed above.

2 Three axiomatizations

2.1 The top two

We will use three axioms in order to characterize the "top two" choice correspondence:

A0: $\forall A \subseteq X$ such that $|A| \geq 2$, $|D(A)| = 2$.
A1: If $a \in D(A)$ and $a \in B \subset A$ then $a \in D(B)$.

$A1$ is commonly referred to in the literature as Sen's α Axiom. If an element is chosen from a set, and it is also a member of a subset, then it is chosen from that subset as well.

A2: For every set A and $a \notin D(A) = \{x, y\}$, if $a \in D(A \backslash \{x\})$ then $a \in D(A \backslash \{y\})$.

$A2$ states that if an alternative is added to the set of selected alternatives when one of the chosen elements is removed, then it is also selected when the other chosen element is removed.

Proposition *(The top two). A choice correspondence D satisfies A0, A1 and A2 iff there exists an ordering $\succ$ on X such that D(A) consists of the two top elements in A according to $\succ$.*

Proof It is trivial to verify that a top two choice correspondence satisfies the axioms A0, A1, and A2.

Let D be a choice correspondence satisfying the three axioms. We will prove the proposition by induction on the size of X. For the inductive step, let $D(X) = \{x, y\}$ (well defined by A0).

By the inductive hypothesis, there exists an ordering $\succ'$ on $X - \{y\}$ such that $D(A)$ consists of the $\succ'$-top two elements for any $A \subseteq X - \{y\}$. By A1, $x \in D(X - \{y\})$ and thus, must be one of the two $\succ'$-top elements. Without loss of generality we can assume x is at the top of $\succ'$. Extend $\succ'$ on $X - \{y\}$ to $\succ$ on X by putting y on top of $\succ$ and letting $\succ$ be equal to $\succ'$ otherwise.

We need to show that for every A that contains y, $D(A)$ consists of the $\succ$-top two elements in A. By A1, $y \in D(A)$ and indeed it is one of the $\succ$-top elements in A. Let z be the other element in $D(A)$. If $x \in A$ then by A1, $z = x$ is the second top element in A. If not, then by A2, $z \in D(A - \{y\} \cup \{x\})$ (z was selected after x was removed from $A \cup \{x\}$, hence, it should also be selected when y is removed instead of x). By the inductive step, z is the second $\succ'$-top element in $A - \{y\} \cup \{x\}$ and thus, also the second $\succ$-top element in this set. It follows that z is also the second $\succ$-top element in A (where y is replaced by x). □

Note that the three axioms are independent.

The choice correspondence that selects only the top element clearly satisfies $A1$ and vacuously $A2$.

The choice correspondence that selects the second and third elements from the top according to some fixed ordering $\succ$ satisfies $A0$ and $A2$ but not $A1$.

The two extremes procedure satisfies $A0$ and $A1$ but not $A2$.

Comment: The above proposition could be extended as follows. Let D satisfy A1 and A2 and a modified version of A0, where $D(A)$ is required to contain M elements (unless A contains less than M elements, a case where $D(A)$ is required to be equal to A). Then there is an ordering $\succ$ on X such that whenever A contains more than M elements, $D(A)$ contains the top M elements in A. The proof (by induction on the size of X) is similar to the one of the above proposition and we provide it here for the sake of completeness.

For the inductive step, let $y \in D(X)$. By the inductive hypothesis, there exists an ordering $\succ'$ on $X - \{y\}$ such that $D(A)$ consists of the $\succ'$-top M elements for any $A \subseteq X - \{y\}$. By A1, the other $M - 1$ elements of $D(X)$ are in $D(X - \{y\})$ and thus, must be one of the M $\succ'$-top elements in X. Without loss of generality we can assume that they consist of the top $M - 1$ elements in $\succ'$. Extend $\succ'$ on $X - \{y\}$ to $\succ$ on X by putting y on top of $\succ$ and letting $\succ$ be equal to $\succ'$ otherwise.

We need to show that for every A that contains y, $D(A)$ consists of the $\succ$-top M elements in A. Let $z \in D(A)$. If $z \in D(X)$ then by A1, z is one of the M $\succ$-top elements in A. If not, then by A1, it must be that there is an element $x \in D(X)$ which is not in A. By A2, $z \in D(A - \{y\} \cup \{x\})$ (z was selected after x was removed from $A \cup \{x\}$, hence, it should also be selected when y is removed instead of x). By the inductive step, z is one of the $\succ'$-M top elements in $A - \{y\} \cup \{x\}$ and thus, one of the $\succ$-M top elements in this set. It follows that z is also one of the M $\succ$ -top element in A (where y is replaced by x).

2.2 The two extremes

To axiomatize the "two extremes" choice correspondence we use $A0$ and a new axiom that states that if a is selected when the alternative x is added to a set A and also when the alternative y is added to A, then a is selected after both x and y are added.

A3: $\forall A \subseteq X$ such that $|A| \geq 2$ and $x, y \notin A$, if $a \in D(A \cup \{x\})$ and $a \in D(A \cup \{y\})$, then $a \in D(A \cup \{x, y\})$.

Lemma *A choice correspondence D that satisfies $A0$ and $A3$ also satisfies $A1$.*

Proof Let D be a choice correspondence that satisfies $A0$ and $A3$ but not $A1$. Take A to be a minimal subset of the ground set X such that D violates $A1$ on A. Denote $D(A) = \{a, z\}$. The minimality of A means that there is some $x \neq a$ such that $a \notin D(A - x)$ (or similarly with z) and that $A1$ holds for all smaller subsets of A. Notice that if for some $b \in X$, $b \in D(A - x)$ and $b \in D(A - y)$, then by $A3$, $b \in D(A)$ and hence, $b \in \{a, z\}$.

Case (i): $a \notin D(A - z)$. Let $D(A - z) = \{c, d\}$ and $D(A - a) = \{e, f\}$. One of the elements in $\{e, f\}$, say e, is not z. Now e cannot be c or d because otherwise, it would be a member of $D(A)$ though it is neither a nor z. By the fact that $A1$ holds for $A-a$ and $A-z$ the three distinct elements c, d, e are all members of $D(A-a-z)$, violating $A0$.

Case (ii): $a \in D(A-z)$ and $a \notin D(A-c)$ for some $c \notin \{a, z\}$. As before, $c \notin D(A-a)$ or $c \notin D(A-z)$. If $c \notin D(A-a)$, then since $A1$ holds for $A-a$ and $A-c$, we have $D(A-c) = D(A-a-c) = D(A-a)$. But by $A3$ and $A0$, $D(A-a) = D(A)$, a contradiction. Suppose $c \notin D(A-z) = \{a, d\}$. As before, at least one of the elements of $D(A-c)$, say e, is not z. Hence, the three distinct elements, a, d and e, are all members of $D(A-z-c)$, violating $A0$. □

Proposition *(The two extremes): A choice correspondence D satisfies $A0$ and $A3$ iff there exists an ordering $\succ$ on X such that $D(A)$ consists of the $\succ$-maximal and $\succ$-minimal alternatives in A.*

Proof Any two-extremes choice correspondence satisfies $A0$. To see that it satisfies $A3$, let A be a set such that $|A| \geq 2$ and let a be a member of both $D(A \cup \{x\})$ and $D(A \cup \{y\})$. WLOG let $a = \max(A \cup \{x\}, \succ)$. Then $a = \max(A, \succ)$. It follows that $a \neq \min(A \cup \{y\}, \succ)$, hence $a = \max(A \cup \{y\}, \succ)$. Therefore, $a = \max(A \cup \{x, y\}, \succ)$ and thus, $a \in D(A \cup \{x, y\})$.

Let D be a choice correspondence that satisfies $A0$ and $A3$. By the lemma, D also satisfies $A1$. Let $D(X) = \{L, R\}$.

For any two distinct elements a and b, we say that a "is to the left of" b, and denote $a \to b$, if $D(\{a, b, R\}) = \{a, R\}$ and $D(\{L, a, b\}) = \{L, b\}$. By definition, $\to$ is an asymmetric relation.

We first verify that the relation "to the left of" is total. Notice that by $A1$, $L \to a$ for any $a \neq L$ and $a \to R$ for any $a \neq R$. Consider a, b distinct from L, R. By $A1$, $R \in D(\{a, b, R\})$ and $L \in D(\{L, a, b\})$. If $D(\{a, b, R\}) = \{a, R\}$, then $D(\{L, a, b\}) = \{b, L\}$ (and $a \to b$), because if $D(\{L, a, b\}) = \{L, a\}$, then by $A3$, $a \in D(\{L, a, b, R\})$ but by $A1$, $D(\{L, a, b, R\}) = \{L, R\}$.

We now show that the relation "to the left of" is also transitive. Assume $a \to b$ and $b \to c$. That is, $D(\{a, b, R\}) = \{a, R\}$, $D(\{L, a, b\}) = \{L, b\}$, $D(\{b, c, R\}) = \{b, R\}$ and $D(\{L, b, c\}) = \{L, c\}$. By $A0$ and $A1$, $D(\{a, b, c, R\}) = \{a, R\}$ and by $A1$, $D(\{a, c, R\}) = \{a, R\}$. Similarly, $D(\{L, a, b, c\}) = \{L, c\}$ and $D(\{L, a, c\}) = \{L, c\}$. Hence, $a \to c$.

Finally, let $A \subseteq X$ and denote by l and r the maximal and the minimal element of the "to the left of" relation. We will show that $D(A) = \{l, r\}$. Assume $D(A)$ does not contain l. The set $D(A \cup \{R\})$ contains R (by $A1$ and $D(X) = \{L, R\}$) but not l (since if it were, then by $A1$, it would also be included in $D(A)$). Thus, $D(A \cup \{R\}) = \{a, R\}$ for some $a \neq l$. By $A1$, $D(\{l, a, R\}) = \{a, R\}$ contradicting the definition of l as being to the left of a. Similarly, $D(A)$ contains r. □

The two axioms used above are independent.

The choice correspondence that chooses the single top element satisfies $A3$ and $A1$ but not $A0$.

The top-two choice correspondence satisfies $A0$ and $A1$ but not $A3$ (if the third best element in X is the top in A, then it is selected when either the best or second best element in X is added, but not when both are added).

2.3 The top and the top

In order to characterize the top-and-the-top procedure, we use $A1$, a variation of $A0$ and two additional axioms.

A0': $\forall A \subseteq X,\ |D(A)| \leq 2$.

$A0$' modifies $A0$ to allow for the possibility that the two rationales agree on the maximal element on some choice set.

A4: If $D(A) = \{a\}$ and $a \in B \subseteq A$, then $D(B) = D(A)$.

$A4$ requires that if $D(A)$ is a singleton, which is also a member of a subset B, then $D(B)$ is this singleton. It is a weakening of the traditional Independence of Irrelevant Alternatives, $D(A) \subseteq B \subseteq A \Rightarrow D(B) = D(A)$. Note that $A1$ would guarantee that a is a member of $D(B)$ but not that $D(B) = \{a\}$. $A1$ and $A4$ imply the traditional Independence of Irrelevant Alternatives in the presence of $A0$' (or $A0$).

A5: If a is a member of each of the sets, $D(A_1)$, $D(A_2)$ and $D(A_1 \cap A_2)$, and also $D(A_1 \cap A_2)$ contains two elements, then $a \in D(A_1 \cup A_2)$.

$A5$ captures situations where the decision maker is choosing at most two alternatives from every set using two "reasons" R_1 and R_2. Interpret aR_ib to mean that R_i is a reason for choosing a and not b. An element a is chosen from a set A if there is a reason i for which aR_ix for *all* x in A. A decision maker who follows this procedure satisfies the following: if both a and b are chosen from $A_1 \cap A_2$, then it must be that for one of the two reasons, say R_2, bR_2a. Thus, it must be that a is chosen from A_1 and A_2 by applying the *same* reason R_1. Thus, aR_1x for any x in either A_1 *or* in A_2, and thus, it is chosen also from $A_1 \cup A_2$.

Proposition *(The top and the top) A choice correspondence D satisfies Axioms A0', A1, A4, and A5 iff there are orderings* $\succ_1, \succ_2$*(possibly identical) such that* $D(A) = \{\max(A, \succ_1), \max(A, \succ_2)\}$.

Proof The top and the top procedure clearly satisfies $A0$', $A1$ and $A4$. As to A5, assume $x \in D(A_1)$, $D(A_2)$ and $D(A_1 \cap A_2) = \{x, y\}$, where $y \neq x$. The element y maximizes one of the orderings, say $\succ_1$, in $A_1 \cap A_2$ so that $y \succ_1 x$. Thus, x must be the $\succ_2$-maximal element in both A_1 and A_2 and therefore in $A_1 \cup A_2$, which implies $x \in D(A_1 \cup A_2)$.

Let n be the number of elements in X and let D be a choice correspondence, satisfying the four axioms. We construct inductively two rankings $a_1 \succ_1 \cdots \succ_1 a_n$, and, $b_1 \succ_2 \cdots \succ_2 b_n$ and then show that $D(A) = \{\max(A, \succ_1), \max(A, \succ_2)\}$. For ease of notation, for $i \leq j$ let us denote $A_{i,j} \equiv \{a_i, \ldots, a_j\}$ and $B_{i,j} \equiv \{b_i, \ldots, b_j\}$. For convenience, denote $A_{1,0} \equiv B_{1,0} \equiv \emptyset$.

Assume that we have deduced the first $m \geq 0$ elements of both orderings, then the algorithm finds the $m + 1^{st}$ element of each ordering by considering the following three cases:

Case 1: $|D(X \backslash A_{1,m})| = 1$. Then we assign a_{m+1} to be the element chosen by D from $X \backslash A_{1,m}$.

Case 2: $|D(X\backslash A_{1,m})| = 2$ and $B_{1,m} = A_{1,m}$. Then we set a_{m+1}, b_{m+1} to be the two members of $D(X\backslash A_{1,m})$.

Case 3: $|D(X\backslash A_{1,m})| = 2$ and $B_{1,m} \neq A_{1,m}$. Then there is an element $b_i \in X\backslash A_{1,m}$ where $i \leq m$. Let b_{i^*} be the element of $X\backslash A_{1,m}$ with the minimal index. By construction, $b_{i^*} \in D(X\backslash B_{1,i^*-1})$ and $X\backslash A_{1,m} \subset X\backslash B_{1,i^*-1}$. By $A1$, $b_{i^*} \in D(X\backslash A_{1,m})$. Define a_{m+1} to be the other element in $D(X\backslash A_{1,m})$.

To determine b_{m+1}, apply the above algorithm to $D(X\backslash B_{1,m})$. Note that when $|D(X\backslash A_{1,m})| = 2$ and $B_{1,m} = A_{1,m}$, then $D(X\backslash B_{1,m}) = D(X\backslash A_{1,m})$, and applying the above algorithm to either $D(X\backslash A_{1,m})$ or $D(X\backslash B_{1,m})$ yields the same pair, (a_{m+1}, b_{m+1}).

Consider an arbitrary set A. We have constructed $\succ_1, \succ_2$ and need to show that $D(A) = \{\max(A, \succ_1), \max(A, \succ_2)\}$. Let $a_i = \max(A, \succ_1)$ and $b_j = \max(A, \succ_2)$. Note that $A \subseteq A_{i,n}$ and $A \subseteq B_{j,n}$, and hence, $a_i, b_j \in D(A)$ by $A1$. If $a_i \neq b_j$, then by $A0'$ we have $D(A) = \{a_i, b_j\}$.

It remains to verify that $D(A) = \{x\}$ whenever $a_i = b_j = x$. If $|D(A_{i,n})| = 1$ (or if $|D(B_{j,n})| = 1$), then by $A4$ and the fact that $A \subseteq A_{i,n}$ (respectively, $A \subseteq B_{j,n}$), we have that $D(A) = D(A_{i,n}) = \{x\}$. Suppose $|D(A_{i,n})| = |D(B_{j,n})| = 2$. Denote $C_\cap \equiv A_{i,n} \cap B_{j,n}$ and $C_\cup \equiv A_{i,n} \cup B_{j,n}$. Since $A \subseteq C_\cap$, then by $A4$ all we need to verify is that $D(C_\cap) = \{x\}$. By construction, $x = a_i \in D(A_{i,n})$ and $x = b_j \in D(B_{j,n})$, thus by $A5$, it is sufficient to show that $x \notin D(C_\cup)$. We achieve this by proving that $D(C_\cup)$ contains two elements that are distinct from x. This would imply that one of the conditions of $A5$ is violated, and since $x \in D(A_{i,n}), D(B_{j,n})$, it must be that $D(C_\cap) = \{x\}$.

From our construction of the two orderings, $\succ_1$ and $\succ_2$, and from our assumption that $|D(A_{i,n})| = 2$, it follows that $a_i \neq \max(A_{i,n}, \succ_2) \equiv b_l$ and similarly, $b_j \neq \max(B_{j,n}, \succ_1) \equiv a_k$. Moreover, x, a_k, and b_l are all distinct because $a_k \succ_1 b_j = x = a_i \succ_1 b_l$. In addition, note that by the definition of a_k, and by the observation that $a_k \succ_1 a_i$, it follows that $A_{i,n} \subset A_{k,n}$ and also $B_{j,n} \subset A_{k,n}$. Similarly, $B_{j,n} \subset B_{l,n}$ and also $A_{i,n} \subset B_{l,n}$. Therefore, $C_\cup$ is contained in both $A_{k,n}$ and $B_{l,n}$. By construction, $a_k \in D(A_{k,n})$ and $b_l \in D(B_{l,n})$. Hence, by $A1$ and $A0'$, $D(C_\cup)$ must contain both a_k and b_l, which are distinct from x. □

The four axioms used for the proposition above are independent:

(i). The correspondence $D(A) \equiv A$ satisfies all axioms except $A0'$.

(ii). Consider the following variant of the top-and-the-top procedure. Let $\succ_1, \succ_2$ be a pair of linear orderings on X. For all $A \subseteq X$ with $|A| \geq 3$, let

$$D(A) = \{\max(A, \succ_1), \max(A, \succ_2)\}$$

and for all pairs $\{a, b\} \subset X$, let

$$D(\{a, b\}) = \{\max(\{a, b\}, \succ_1)\}$$

This satisfies all axioms except $A1$.

(iii). Let $\succ$ be a linear ordering on X. For all A with three or more elements, let $D(A) = \{\max(A, \succ)\}$, while for all pairs A, let $D(A) = A$. This satisfies all axioms except $A4$. To verify that $A5$ is satisfied, note that $D(A \cap B)$ contains two elements only if $A \cap B$ is a pair. If $A \subseteq B$ (or $B \subseteq A$), then $A5$ is vacuous. Otherwise, we have that $|A|, |B| \geq 3$ and $A5$ is again vacuously satisfied because $\{a\} \in D(A), D(B)$ implies $a = \max(A, \succ) = \max(B, \succ)$ and then $a = \max(A \cup B, \succ)$ which implies $D(A \cup B) = \{a\}$.

(iv). Let D be the top-two procedure. This procedure satisfies $A0'$, $A1$, and $A4$ (vacuously). To see that $A5$ is violated when $|X| \geq 4$, denote the top four elements of X by a_1, a_2, a_3, a_4. Then, $a_3 \in D(X \backslash a_2) \cap D(X \backslash a_1)$ and $\{a_3, a_4\} = D(X \backslash a_1, a_2)$. Therefore, we should have that $a_3 \in D(X \backslash a_1 \cup X \backslash a_2) = D(X)$, but this is false.

Comment: An open question remains of how this procedure may be extended to a procedure that selects the top of $M > 2$ orderings.

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