Two Comments on the Principle of Revealed Preference

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Abstract.

We present two arguments suggesting that the principle of revealed preference facilitates the introduction of procedural and psychological aspects of choice to economic models.

First, some choice procedures cannot be described as the outcome of maximizing a preference relation. However, they can be characterized and differentiated based on a simple revealed preference argument, i.e. based on simple properties of choice.

Second, even if a choice procedure corresponds to maximizing a preference relation, there may still be a revealed preference justification to study the psychology of the procedure. The information concerning the available set of alternatives is often coupled with other information pertinent to the psychology of choice. This latter information can shed light on aspects of choice not fully captured by a preference relation, and hence should be part of the revealed preference analysis.

Keywords. Revealed preference, choice function, choice correspondence, rationality.

JEL classification. D00, D01.
1. Introduction

The revealed preference approach states that economic analysis should be based only on entities observed by the economist. Gul and Pesendorfer (2005) have recently reopened the discussion on the role of revealed preference in economic analysis. Without entering the essence of the discussion (see Rubinstein (2006)\textsuperscript{1}), we argue in this short note that the revealed preference approach facilitates the introduction of procedural and psychological aspects of choice to economic models.\textsuperscript{2}

We make two comments to support our argument.

Comment 1. Some choice procedures cannot be described as the outcome of maximizing a preference relation. However, they can be characterized and differentiated based on a simple revealed preference argument, i.e. based on simple properties of choice.

For example, Manzini and Mariotti (2004) introduce a two stage procedure in which the decision maker first selects the set of elements he will seriously consider and then applies a standard preference relation in order to make a choice. This procedure has simple properties of revealed preference which differ from the standard axioms of choice.

Comment 2. Some choice procedures are indistinguishable from standard choice correspondences that can be described as the outcome of maximizing a preference relation. One can therefore claim that the psychological considerations involved in these procedures are not anchored in behavior. However, in many cases, additional information relevant to the psychology of choice is available (in the same sense that the set of alternatives and the chosen alternative are available). The revealed preference approach does not imply that one should ignore this information, but rather that one should use a model of choice that takes this information into account instead of using a standard choice correspondence.

For example, a choice problem is often presented in the form of a list. A decision maker who uses a systematic method to choose from lists may choose differently from two lists that induce the same set of alternatives. A standard choice correspondence would attach to every choice problem the set of all elements chosen for some listing of the alternatives. Under certain conditions on the method of choice from lists, this choice correspondence can be explained as the result of maximizing a preference relation (see Rubinstein and

\textsuperscript{1}Rubinstein (2006) argues that there is no escape from including unobserved components of choice in welfare analysis. For example, if a decision maker maximizes the function $-v$, where $v$ represents his own perceived interests, it would be wrong to use the function $-v$ as a component in welfare analysis.

\textsuperscript{2}Our argument does not conflict with Gul and Pesendorfer (2005) but is a critique of some of its possible interpretations.
Salant (2006)). The possible conclusion that the analysis of choice from lists is not part of the economist’s toolkit is false. The listing of the alternatives is often available and can be used to describe how actual choices are made — something which a standard correspondence may not accomplish.

2. Model

Let $X$ be a finite set of alternatives. The standard model of choice assumes that a choice problem is a non-empty subset of $X$. Let $D$ be the collection of standard choice problems. A choice function $c$ attaches to every choice problem $A \in D$ a single element $c(A) \in A$. A choice correspondence $C$ attaches to every $A \in D$ a non-empty subset of $A$.

In real life situations, a choice problem often appears with a frame. A frame is additional information associated with the choice problem that may serve as a component of the choice procedure though it may not convey information relevant to the assessment of the alternatives. A frame may be the outcome of exogenous manipulation such as the order in which vacation packages are listed in a brochure. A frame can also be purely internal, as in the case of a decision maker who mentally enumerates the elements of the set.

Formally, an extended choice problem is a pair $(A, f)$ where $A \in D$ and $f$ is an abstract object called a frame. Let $D^*$ be the collection of extended choice problems. An extended choice function $c^*$ assigns an element of $A$ to every $(A, f) \in D^*$. An extended choice function $c^*$ with domain $D^*$ induces a standard choice correspondence $C_{c^*}$ with domain $D$, where $C_{c^*}(A)$ is the set of elements chosen from the set $A$ for some frame $f$. In other words,

$$C_{c^*}(A) = \{ x \mid c^*(A, f) = x \text{ for some } (A, f) \in D^* \}.$$

When several frames are associated with a given choice problem $A$, the cardinality of $C_{c^*}(A)$ may reflect the tendency of the decision maker to be influenced by the frame. The smaller $C_{c^*}(A)$ the less the decision maker is influenced by manipulating the frame.

Following are several examples of extended choice problems that we discuss throughout the paper.

1. Leading Considerations. The description or the content of the choice problem triggers the decision maker to think primarily about a particular consideration. An extended choice problem is a pair $(A, \succ)$ where $\succ$ is an ordering that reflects the consideration used by the decision maker to evaluate the elements of the set $A$.

2. Focus on Relevant Elements. The decision maker identifies a subset of elements
in the choice problem as the relevant alternatives and chooses from among them. An extended choice problem is a pair \((A, B)\) where \(B \subseteq A\) is the set of relevant elements in \(A\).

3. **List.** The decision maker evaluates the elements of the set as a list. An extended choice problem is a pair \((A, >)\) where \(>\) is an ordering of the elements of \(A\) from first to last.

4. **Number of Appearances.** An alternative may appear more than once in the choice menu. An extended choice problem is a pair \((A, i)\) where \(i\) is a function that assigns to every \(a \in A\) the number \(i(a)\) of times \(a\) appears in the menu.

5. **Default Alternative.** One of the alternatives is designated as the default alternative. An extended choice problem is a pair \((A, x)\) where \(x \in A\) is a default alternative.

The first two examples appear most often in contexts in which we do not observe the frame associated with the choice problem. We usually do not observe the leading consideration triggered by the content of the choice problem (example 1) or the set of alternatives which are seriously considered (example 2). The rest of the examples are often (though, of course, not always) observed with the frame. We often observe the order of the elements in a set (example 3), the number of appearances of an alternative within a menu (example 4) or the default alternative (example 5).

### 3. First Comment: Non-Standard Choice

In this section we examine two contexts in which certain assumptions on a frame-sensitive choice procedure are equivalent to non-standard restrictions on choice correspondences. These examples demonstrate how procedural aspects of choice can be differentiated based on a standard revealed preference argument, i.e. based on actual choices from standard choice problems.

#### 3.1. Triggered Rationality

The choice procedure we have in mind in this subsection is one in which the most salient element in the choice problem induces the decision maker to use a particular rationale when making a choice. For example, when choosing among vacation packages, one may either maximize the entertainment value or the historic significance of the trip depending on whether Las Vegas appears among the available options.

Formally, an extended choice problem is a pair \((A, \succ)\) where \(\succ\) is an ordering that reflects the consideration the decision maker uses when choosing from \(A\). We say that an
extended choice function \( c^* \) satisfies \textit{Triggered Rationality} if there is an array of orderings \( \{\succ_a\}_{a \in X} \) (not necessarily distinct) and a saliency ordering \( R \) over \( X \) such that:

(i) the set \( D^* \) contains all the pairs \((A, \succ)\) where \( \succ = \succ_{a^*} \) for \( a^* \) which is the \( R \)-maximal element in \( A \).

(ii) \( c^*(A, \succ) \) is the \( \succ \)-maximal element in \( A \).

Of course, there are natural choice procedures that use more general attributes of the choice problem in order to determine which consideration to use. For example, a choice procedure which uses a particular rationale when the set is symmetric\(^3\) and a different one when the set is asymmetric does not fall within the category of Triggered Rationality.

From the point of view of standard choice, the above procedure is characterized by the property that for every choice problem \( A \), there exists \( a \in A \) such that the standard \textit{Independence of Irrelevant Alternatives}\(^4\) (IIA) property holds for subsets of \( A \) that contain \( a \).

Formally, a standard choice function \( c \) satisfies the \textit{Reference Point} property if for every set \( A \), there exists \( a \in A \) such that if \( a \in A'' \subset A' \subset A \) and \( c(A') \in A'' \), then \( c(A'') = c(A') \).

\textbf{Proposition.} A standard choice function \( c \) satisfies the Reference Point property if and only if there is an extended choice function \( c^* \) satisfying Triggered Rationality such that \( c = C_{c^*} \).

\textbf{Proof.} Assume that \( c \) satisfies the Reference Point property. We construct the function \( c^* \) which satisfies Triggered Rationality recursively. Consider the set \( X \). By the Reference Point property there exists an element \( a \) such that for all subsets of \( X \) that contain \( a \) the standard IIA property holds. Thus, there exists a preference relation \( \succ_a \) such that its maximization describes the choices of \( c \) whenever \( a \) is available. Let \( a \) be the \( R \)-maximal element in \( X \). Continue recursively with the set \( X \setminus \{a\} \).

In the other direction, assume \( c^* \) satisfies Triggered Rationality with respect to a preference array \( \{\succ_a\}_{a \in X} \) and a saliency ordering \( R \). Let \( a \) be the \( R \)-maximal element in \( A \). Assume that \( a \in A'' \subset A' \subset A \) and \( c(A') \in A'' \). The element \( a \) is also \( R \)-maximal in both \( A' \) and \( A'' \) and thus \( c(A') \), the \( \succ_a \)-maximal element of \( A' \), is also the \( \succ_{a^*} \)-maximal element of \( A'' \). Consequently, \( c(A'') = c(A') \). \( \blacksquare \)

\(^3\)A set \( A \) is symmetric if \( x \in A \) implies \( y \in A \) for any \( y \sim x \) where \( \sim \) is a symmetric binary relation over \( X \).

\(^4\)A choice function \( c \) satisfies the standard Independence of Irrelevant Alternatives property if \( c(A) \in B \subset A \) implies that \( c(B) = c(A) \).
3.2 Post-Dominance Rationality

The choice procedure we have in mind in this subsection is one in which the decision maker first eliminates any alternative which he deems dominated in some sense by another alternative. He then chooses the best alternative from among the non-dominated alternatives.

Formally, an extended choice problem is a pair \((A, B)\) where \(A\) is a choice problem and \(B \subseteq A\) is a non-empty subset of relevant elements. We say that an extended choice function \(c^*\) satisfies Post-Dominance Rationality if:

(i) There exists an acyclic binary relation \(R\) such that \(D^*\) consists of all the pairs \((A, B)\) where \(B = \{b \mid \text{there is no } a \in A \text{ such that } aRb\}\). That is, \(aRb\) means that the presence of \(a\) in the choice problem excludes \(b\) from the set of relevant elements.

(ii) There exists a binary relation \(\succ\), which is transitive whenever restricted to sets of elements that do not dominate one another, such that \(c^*(A, B)\) is the \(\succ\)-maximal element in \(B\).

Comment. The relation \(\succ\) need not be transitive for all triples of alternatives. Intransitivity is possible for triples in which one of the alternatives is excluded by \(R\). For example, let \(X = \{a, b, c, d\}\) and let \(R\) be the relation for which \(x \not\in R y\) except for \(a Rc\) and \(b Rd\). Let \(\succ\) be the order relation \(a \succ b \succ c \succ d\) except that \(d \succ a\) and not \(a \succ d\). Define \(c^*\) accordingly. Then \(c^*\) is a non-empty function that satisfies Post-Dominance Rationality though \(\succ\) is not transitive.

The notion of Post-Dominance Rationality was suggested by Manzini and Mariotti (2004) who characterize this procedure in terms of properties of choice from sets. We establish a different connection between Post-Dominance Rationality and standard choice.

Exclusion Consistency. A standard choice function \(c\) satisfies Exclusion Consistency if for every set \(A\) and for every \(a \in X\), if \(c(A \cup \{a\}) \notin \{c(A), a\}\) then there is no set \(A'\) which contains \(a\) such that \(c(A') = c(A)\).

Proposition. A standard choice function \(c\) satisfies Exclusion Consistency if and only if there exists an extended choice function \(c^*\) which satisfies Post-Dominance Rationality such that \(c = c^*\).

Proof. Assume \(c\) satisfies Exclusion Consistency. We define two binary relations \(R\) and \(\succ\) as follows:

(i) \(aRb\) if there is a set \(A\) such that \(c(A) = b\) and \(c(A \cup \{a\}) \notin \{a, b\}\).

(ii) \(a \succ b\) if \(c(\{a, b\}) = a\).
The relation $R$ is acyclic. If there were a cycle, then by Exclusion Consistency, no element could be chosen from the set of all elements in the cycle.

The relation $\succ$ is asymmetric and complete. The relation $\succ$ is transitive whenever restricted to sets of elements that do not relate to one another by $R$. Otherwise, assume that $a \succ b$, $b \succ c$ and $c \succ a$ and that $a, b, c$ are not related by $R$. Without loss of generality assume that $c(\{a, b, c\}) = b$. Then, since $c(\{a, b\}) = a$ we should have $cRa$, a contradiction.

For every set $A$, define $B$ to be the set of $R$-maximal elements in $A$, and $c^*(A, B)$ to be the $\succ$-maximal element in $B$. Then, $c^*(A, B)$ satisfies Post-Dominance Rationality and is non-empty for every set $A$. We need to show that $c(A) = C_c^*(A)$.

Let $A_0 = B$ be the set of $R$-maximal elements in $A$, and denote its cardinality by $K$. We first show that $c(A_0) = C_c^*(A_0)$. Since the elements of $A_0$ are not related by $R$, $C_c^*(A_0)$ is the $\succ$-maximal element in $A_0$. If $c(A_0) = a$ is not the $\succ$-maximal element in $A_0$ then there exists $b \in A_0$ such that $b \succ a$. By definition $c(\{a, b\}) = b$. Enumerate the set $A_0$: $a_1 = a, a_2 = b, ..., a_K$. Let $k^* = \max\{k \mid c(\{a_1, a_2, ..., a_k\}) \neq a\}$. Then $2 \leq k^* < K$ and $a_{k^*+1} \succ cRa\{a_1, ..., a_{k^*}\}$ which contradicts the definition of $A_0$.

Inductively, construct a sequence of sets $A_k$ starting with $A_0$. Let $A_{k+1} = A_k \cup \{b\}$ where $b$ is an $R$-maximal element in $A - A_k$. Then $b$ is $R$-dominated by an element in $A_k$ and hence by Exclusion Consistency $c(A_k \cup \{b\}) \neq b$. By construction, $b$ does not dominate any element in $A_k$ including $c(A_k)$ and hence $c(A_k \cup \{b\}) = c(A_k)$. Consequently, $c(A) = c(A_0)$. Thus, $c(A)$ is the $\succ$-maximal element among the $R$-maximal elements in $A$, which implies that $c(A) = C_c^*(A)$.

In the other direction, suppose that $c^*$ satisfies Post-Dominance Rationality with the relations $R$ and $\succ$. Then $C_c^*(A)$ is the $\succ$-maximal element among the $R$-maximal elements in $A$. We now need to show that $c = C_c^*$ satisfies Exclusion Consistency. Assume that $c(A) = C_c^*(A) = a$ and $c(A \cup b) = C_c^*(A \cup b) \notin \{a, b\}$. It must be that $bRa$ and, by the definition of $c^*$, the element $a$ is never chosen from a set in which $b$ appears. Consequently, $c(A') \neq a$ whenever $b \in A'$.

4. Second Comment: Standard Choice and Framing

In this section we examine three contexts in which properties of a frame-sensitive choice procedure imply that the induced choice correspondence satisfies standard axioms of choice. Thus, one might argue that there is no revealed preference basis for integrating these procedures into economic models. We would argue otherwise. It is often natural
to assume that an observer sees not just the choice problem and the chosen element
but the frame as well. This is especially true when the frame is manipulated by an
exogenous device, such as when a marketer arranges the elements according to some order
or highlights one element as the default. In such cases, the revealed preference approach
does not exclude the interest in frame-sensitive choice procedures even if they imply only
standard assumptions on choice. The information conveyed by the frame may provide
important insights into choice, particularly in cases in which the induced correspondence
specifies more than one element as a possible choice.

4.1 Choice from Lists

In this subsection, an extended choice problem is a pair \((A, >)\) where \(A\) is a choice
problem and \(>\) is an ordering of \(A\). In other words, the decision maker chooses from lists.
We assume that any ordering of the elements of \(A\) is possible.

In Rubinstein and Salant (2006) we studied the following property of an extended
choice function \(c^*\):

**List Independence of Irrelevant alternatives (LI\(\text{IIA}\)).** If \(c^*(A, >) = a\), then
\(c(A - \{b\}, >_{|A-(b)} = a\) for every \(b \neq a\).

We showed that any extended choice function which satisfies LI\(\text{IIA}\) induces a choice
correspondence which satisfies the standard Weak Axiom\(^5\) (WA). We provide a simpler
proof here.

**Proposition (Rubinstein and Salant (2006)).**

(i) If an extended choice function \(c^*\) satisfies LI\(\text{IIA}\), then \(C_{c^*}\) satisfies WA.

(ii) If \(C\) is a choice correspondence that satisfies WA, then there exists an extended
choice function \(c^*\) satisfying LI\(\text{IIA}\) such that \(C = C_{c^*}\).

**Proof.** (i) Assume \(a, b \in A \cap B, a \in C_{c^*}(A)\) and \(b \in C_{c^*}(B)\). Then there exist
\(>_1\) and \(>_2\) such that \(c^*(A, >_1) = a\) and \(c^*(B, >_2) = b\). Let \(>_3\) be an ordering which is
identical to \(>_2\) except for \(a\) appearing first if \(a >_1 b\) and last if \(b >_1 a\). We now show that
\(c^*(B, >_3) = a\).

It is impossible that \(c^*(B, >_3) = x \notin \{a, b\}\). Otherwise, by LI\(\text{IIA}\) \(c^*(B - \{a\}, >_{3|B-(a)}\)
\(x \neq x\). Since \(c^*(B, >_2) = b\) LI\(\text{IIA}\) implies that \(c^*(B - \{a\}, >_{2|B-(a)}\) = b\). This contradicts
the fact that \(>_2\) and \(>_3\) are identical on \(B - \{a\}\) and thus should induce the same choice.

\(^5\)A choice correspondence \(C\) satisfies the Weak Axiom if \(a, b \in A \cap B, a \in C(A)\) and \(b \in C(B)\) imply
that \(a \in C(B)\).
It is also impossible that $c^*(B, >_3) = b$. Otherwise, by LIIA $c^*(\{a, b\}, >_3|\{a, b\}) = a$ but $>_1$ and $>_3$ are identical on $\{a, b\}$.

Thus, $c^*(B, >_3) = a$ as required.

(ii) If $C$ satisfies WA then there exists a weak preference relation $\succeq$ over $X$ which $C$ maximizes. Define $c^*(A, >)$ to be the first $\succeq$-maximal element in $A$ according to $>$. Then $c^*$ satisfies LIIA and $C = C_{c^*}$. ■

The model of choice from lists most clearly illustrates our assertion regarding revealed preference and the observability of the frame. The information on the set of available alternatives is often supplemented by the order of the alternatives. This is true especially when the list is generated by an exogenous mechanism, e.g., entrees are listed on a menu and products in a brochure. In such cases, the study of choice from lists is valuable, as it analyzes an important observable factor that affects choice and suggests a novel interpretation of choice correspondences.

4.2 Number of Appearances

In this subsection an extended choice problem is a pair $(A, i)$ where $i(a)$ is the number of times the element $a$ appears in the set $A$.

We discuss two properties of extended choice functions:

Subtraction. If $c^*(A, i) = a$ and $i'$ is such that $i'(b) = i(b) - 1 \geq 1$ for $b \neq a$, then $c^*(A, i') = a$. If $b \neq a$ and $i(b) = 1$, then $c^*(A - \{b\}, i|_{A-\{b\}}) = a$.

Additivity. If $c^*(A, i) = a$ and $i'$ is such that for every $b \in B \subseteq A$ $i'(b) = i(b) + 1$, then $c^*(A, i') = a$ if $a \in B$.

In other words, adding one instance of several elements including the chosen element does not alter the choice.

Proposition. (i) If an extended choice function $c^*$ satisfies Subtraction and Additivity then $C_{c^*}$ satisfies WA.

(ii) If a choice correspondence $C$ satisfies WA then there exists $c^*$ satisfying Subtraction and Additivity such that $C = C_{c^*}$.

Proof. (i) Assume that $a, b \in A \cap B$, $a \in C_{c^*}(A)$ and $b \in C_{c^*}(B)$. Then there exists $i$ such that $c^*(A, i) = a$ and $i'$ such that $c^*(B, i') = b$. By Additivity, we can assume
without loss of generality that $i(b) = i'(b)$. By Subtraction, $c^*(\{a, b\}, i_{\{a,b\}}) = a$ and $c^*(\{a, b\}, i'_{\{a,b\}}) = b$, which implies that $i(a) > i'(a)$. Define $i'' = i'$ except for $i''(a) = i(a)$. Then, $c^*(B, i'') \in \{a, b\}$ by Subtraction. Finally, $c^*(B, i'') = c^*(\{a, b\}, i''_{\{a,b\}}) = c^*(\{a, b\}, i'_{\{a,b\}}) = a$ which implies that $a \in C(B)$.

(ii) If $C$ satisfies WA then there exists a weak preference relation $\succsim$ which $C$ maximizes. Define $c^*(A, i)$ to be a $\succsim$-maximal element with the highest number of appearances and resolve ties according to some order relation. Then $c^*$ satisfies Subtraction and Additivity and $C = C_{c^*}$. ■

This model is another case in which the information conveyed by the frame is often observed and the presentation of the choice problem as a standard set excludes available relevant information. Once again, the above proposition implies that one cannot distinguish between a choice correspondence induced by a procedure which takes into account the number of appearances of the alternatives and a choice correspondence which is the outcome of maximizing a preference relation. Nonetheless, even from the point of view of revealed preference, it is a mistake to conclude that there is no place for models of choice in which this frame appears.

4.3 Default Alternative

In this subsection an extended choice problem is a pair $(A, x)$ where $x \in A$ is interpreted as a default alternative. We assume that any element $x \in A$ can serve as a default alternative. A similar framework is discussed in Masatlioglu and Ok (2005) and Zhou (1997).

We study the following two properties of extended choice functions:

**Independence of Irrelevant Alternatives (IIA**$^*$**).** If $x \in A \subset B$ and $c^*(B, x) \in A$, then $c^*(A, x) = c^*(B, x)$.

In other words, if an element is chosen from a set, it is also chosen from all subsets of the set as long as the default does not change. This property is equivalent to the existence of an array of preference relations $\{\succ_a\}_{a \in X}$ such that $c^*(A, x)$ is the $\succ_x$-maximal element in $A$.

**Default Tendency.** If $c^*(A, x) = a$ then $c^*(A, a) = a$.

That is, if an element $a$ is chosen from a set $A$ with some default element, then $a$ is also chosen from $A$ when it becomes the default element.

Given IIA$^*$, the property of Default Tendency is equivalent to the condition whereby
$a \succ_b b$ implies that $a \succ_a b$. Thus, the two properties characterize choice procedures that apply different rationales as a function of the default option and show preference for the default alternative.

The following proposition complements the result of Gul and Pesendorfer (2005).\(^6\)

**Proposition.**

(i) If $c^*$ satisfies IIA* and Default Tendency then there exists a transitive asymmetric binary relation $\succ$ over $X$ such that for every set $A$, the set $C_{c^*}(A)$ contains all the $\succ$-maximal elements in $A$.

(ii) Let $C$ be a choice correspondence that maximizes a transitive asymmetric binary relation $\succ$. Then there exists a choice function $c^*$ that satisfies IIA* and Default Tendency such that $C = C_{c^*}$.

**Proof.** (i) Assume $c^*$ satisfies IIA* and Default Tendency. For any two elements $a$ and $b$, define $a \succ b$ if $c^*(\{a, b\}, b) = a$. By Default Tendency $\succ$ is asymmetric.

Note that if $a \succ b$, then for every set $A$ such that $a, b \in A$, $c^*(A, x) \neq b$. Otherwise, by Default Tendency, $c^*(A, b) = b$ and by IIA*, $c^*(\{a, b\}, b) = b$.

To see that the relation $\succ$ is transitive, assume that $a \succ b$ and $b \succ c$. By the above, $c^*(\{a, b, c\}, c)$ is neither $b$ nor $c$ and therefore it must be $a$. Then, by IIA*, $c^*(\{a, c\}, c) = a$ which implies that $a \succ c$.

It remains to show that $C_{c^*}(A)$ is the set of $\succ$-maximal elements in $A$. By Default Tendency, $x \in C_{c^*}(A)$ if and only if $c^*(A, x) = x$. By IIA*, $c^*(A, x) = x$ if and only if $c^*(\{x, y\}, x) = x$ for every $y \in A$. By the definition of $\succ$, $c^*(\{x, y\}, x) = x$ if and only if $y \not\succ x$. Thus, $x \in C_{c^*}(A)$ if and only if there is no $y \in A$ such that $y \not\succ x$.

(ii) Expand the relation $\succ$ to form a complete order relation $\succ^*$. For every $A \subseteq X$ and $x \in A$, define

$$c^*(A, x) = \begin{cases} x & \text{if } x \in C(A) \\ \text{the } \succ^*-\text{maximal element in } \{y \in A \mid y \succ x\} & \text{if } x \notin C(A). \end{cases}$$

The function $c^*$ is single-valued because $x \notin C(A)$ implies that $\{y \in A \mid y \succ x\} \neq \emptyset$ and because $\succ^*$ is a complete order relation.

The function $c^*$ satisfies IIA*. Assume that $c^*(A, x) = a$. Let $B \subseteq A$ with $a, x \in B$. We need to show that $c^*(B, x) = a$. If $x \in C(A)$ then $x = a$ is $\succ$-maximal in $A$ and therefore in $B$, which implies that $x \in C(B)$ and $c^*(B, x) = x$. If $x \notin C(A)$ then $a$ is

\[^6\]Gul and Pesendorfer's characterization involves a complete but not necessarily transitive binary relation, while ours involves a transitive but not necessarily complete relation.
the $\succ^*$-maximal element in $\{y \in A \mid y \succ x\}$. Since $a \in B \subseteq A$, $x$ is not $\succ$-maximal in $B$ and thus $x \notin C(B)$. The element $a$ continues to be the $\succ^*$-maximal element in $\{y \in B \mid y \succ x\} \subseteq \{y \in A \mid y \succ x\}$ and thus $c^*(B, x) = a$.

The function $c^*$ satisfies Default Tendency. If $c^*(A, x) = a$ and $a \neq x$ then $a$ is $\succ^*$-maximal in $\{y \in A \mid y \succ x\}$. Therefore, $a$ is $\succ$-maximal in $A$, which means that $a \in C(A)$. By definition, $c^*(A, a) = a$.

Finally, $C_{c^*} = C$. If $x \in C(A)$ then $c^*(A, x) = x$ and thus $x \in C_{c^*}(A)$. If $x \notin C(A)$, then by definition $c^*(A, x) \neq x$. Since $c^*$ satisfies Default Tendency, $c^*(A, y) \neq x$ for all $y \in A$ and thus $x \notin C_{c^*}(A)$.

As in the previous two examples, one may interpret the above result as a claim that there is no revealed preference basis for maximization that depends on the default alternative. However, there are contexts (though probably less common than in the previous two examples) in which we observe not only the set of alternatives and the chosen element but also the default alternative. This occurs, for example, when the default is an alternative previously chosen by the decision maker or one chosen by another person. In such cases, we believe that the right model of choice is an extended choice function rather than a standard choice correspondence.

References


