Course: **Microeconomics, New York University**
Lecturer: **Ariel Rubinstein**
Exam: **Mid-term, October 2008**
Time: **3 hours (no extensions)**
Instructions: Answer the following three questions (each question in a separate exam booklet)

Question 1

A decision maker has a preference relation over the pairs (x_{me}, x_{him}) with the interpretation that x_{me} is the amount of money he will receive and x_{him} is the amount of money that another person will receive. Assume that

(i) for all (a,b) such that $a > b$ the decision maker strictly prefers (a,b) over (b,a).

(ii) if $a' > a$ then $(a',b) \succ (a,b)$.

The decision maker is to allocate M between himself and another person.

1. Show that these assumptions guarantee that he will never allocate more to the other person than to himself

2. Assume (i), (ii) and

(iii) The decision maker is indifferent between (a,a) and $(a-\varepsilon, a+4\varepsilon)$ for all a and $\varepsilon > 0$.
Show that nevertheless he might allocate the money equally.

3. Assume (i), (ii), (iii) and

(iv) The decision maker's preferences are also differentiable (according to the definition given in class).

Show that in this case, he will allocate (strictly) more to himself than to the other.

Question 2 (Kfir Eliaz and Ariel Rubinstein)

Let X be a (finite) set of alternatives. Given any choice problem A (where $|A| \geq 2$), the decision maker chooses a set $D(A) \subseteq A$ of **two** alternatives which he wants to examine more carefully before making a final decision.

Following are two properties of D:

A1: If $a \in D(A)$ and $a \in B \subset A$ then $a \in D(B)$.

A2: If $D(A) = \{x,y\}$ and $a \in D(A - \{x\})$ for some a different from x and y, then

$a \in D(A - \{y\})$.

Answer the following four questions. A full proof is required only for the last question:

1. Find an example of a D function which satisfies both A1 and A2.
2. Find a function D which satisfies A1 but not A2 .
3. Find a function D which satisfies A2 but not A1.
4. Characterize the set of D functions which satisfy both axioms .

Question 3

An economic agent is to choose from among a group of projects. The outcome of each project is uncertain and can be either a failure or one of K “types of success”. Thus, each project z can be described by a vector of K non-negative numbers, $(z_1, \ldots, z_K)$ where z_k is the probability that the project will yield a type k success.

Let $Z \subset \Re_+^K$ be the set of feasible projects. Assume Z is compact, convex and satisfies "free disposal".

The decision maker is an expected utility maximizer.

Denote by u_k the vNM utility from the k–th type of success and attach 0 to failure. Thus the decision maker chooses a project (vector) $z \in Z$ in order to maximize $\sum z_k u_k$.

1. First, formalize the decision maker’s problem. Then, formalize (and prove) the following claim: If the decision maker suddenly attributes a higher value to type k success than previously, he will choose a project assigning a (weakly) higher probability to k.

2. Apparently, the decision maker now realizes that there is an additional source of uncertainty: The world may go "one way or another". With probability α the vNM utility of the k’th type of success will be u_k and with probability $1 - \alpha$ will be v_k. Failure remains 0 in both contingencies.

   First, formalize the decision maker’s new problem. Then, formalize (and prove) the following claim: Even if the decision maker would have obtained the same expected utility, if he had known in advance the state of the world, the existence of uncertainty makes him (at least weakly) less happy.