Micro Theory Exam, Tel Aviv, January 2009.

Problem 1:

Inspired by: Mandler Michal, Paula Manzini and Marco Mariotti, "A Million Answers to Twenty Questions: Choosing by Checklist", 2008.

Consider a decision maker who is choosing an alternative from subsets of a finite set X using the following procedure:

Following a fixed list of properties (a checklist), he examines one property at a time and deletes from the set all the alternatives that do not satisfy this property. When only one alternative remains, he chooses it.

a. Show that if this procedure induces a choice function, then it is consistent with the rational man model.

Solution:

Let $a \in B \subset A$ such that a is the chosen alternative from A. Any other alternative in A, and in B in particular, fails to satisfy at least one of the first m properties that a does satisfy. Therefore, a is the surviving alternative also in B and condition α holds.

b. Show that any rational decision maker can be described *as if* he follows this procedure.

Solution:

Order all the alternatives in X in ascending order. For each alternative $x \in X$, define the property "not being x" and order the properties according to the same order. Clearly, the only alternative for which "not being x" does not hold is x itself.

Let $A \subset X$ be the set from which the individual chooses. In the first stage, he deletes from A the worst element in X, if it belongs to A, and does nothing otherwise. Similarly, at any subsequent stage, if the alternative is not in A he continues to the next stage and if it does belong to A he deletes it. Hence, at each stage he deletes the worst alternative from his choice set. This process continues until he is left with the best alternative in A.

Problem 2:

Inspired by: Real Life.

There are N men and N women designated for matchmaking. Each man has a strict ordering on the women and each woman has a strict ordering on the men (there are no indifferences). In a market equilibrium, a numerical value is attached to each man and each woman. Each man chooses the best woman, according to his ordering, from the set of women whose value is not larger than his own and every woman chooses the best man, according to her ordering, from the set of men whose value is not larger than her own. In equilibrium, every man chooses the woman who chooses him.

a. Show a necessary condition for the existence of a market equilibrium and explain why such an equilibrium usually does not exist.

Solution:

In equilibrium, each man chooses the woman he prefers the most from those whose value is not greater than his own. This implies that in any match, the woman's value is smaller or equal to the man's. Similarly, since each woman chooses the man she prefers the most from those whose value is not greater than her own, in any match the man's value is smaller or equal to the woman's. Therefore, the values of the man and the woman are the same in every match.

Focus on the couple with the highest value (if there is more than one, choose one of them). The man in this couple has a value which is equal to that of the woman with the highest value and therefore the man's value is equal to, or higher than, the value of all the women in the set. This man could have chosen any partner. Therefore, since he chose this specific woman, one can conclude that she is the first one in his ordering. Similarly, the man in this couple is the first in the woman's ordering.

Hence, if an equilibrium exists, then there is at least one couple in which each person prefers his partner to any other. Since such a couple does not necessarily exist, there is usually no such equilibrium.

Assume that because of this market failure, we allow the use of force. The stronger sex (assume the men, just for simplicity) is ordered in a "strength" relation. A Strength Equilibrium is a matching between the men and the women such that there are no two couples, M_1 matched to F_1, and M_2 matched to F_2, such that:

1. M_1 is stronger than M_2.
2. M_1 prefers F_2 to F_1.
3. F_2 prefers M_1 to M_2

b. Explain why for each preference profile there exists a Strength Equilibrium.

Solution:

We will use the algorithm for a Jungle Equilibrium presented in class. Order the men from strongest to weakest. In turn, each man chooses his preferred woman from those who haven't been chosen yet. Clearly, there's no M_1 stronger than M_2 who prefers F_2 to F_1 (since M_1 chose F_1 even though he could have chosen F_2).

Therefore, conditions 1 and 2 cannot hold simultaneously and therefore this is a Strength Equilibrium.

c. Show that there is a preference profile for which at least two strength equilibria exist.

Solution:

Assume that all the women have the same preference relation: the weaker the better. In this case, any matching is a strength equilibrium since conditions 1 and 3 never hold for the same couple: if a man, M_1, prefers a woman, F_2, who is matched to a man weaker than him, M_2, then this woman will not prefer him to the man she is already matched to.

Problem 3:

Inspired by: Alan Miller, "The Reasonable Man and Other Legal Standards", 2007.

Lately we have been using the term a "reasonable reaction" quite frequently. In this problem we assume that this term is defined according to the opinions of the individuals in the society with regard to the question:"What is a reasonable reaction?".

Assume that in a certain situation, the possible set of reactions is X and the set of individuals in the society is N.

A "reasonability perception" is a non-empty set of possible reactions that are perceived as reasonable.

The social reasonability perception is determined by a function f which attaches a reasonability perception (a non-empty subset of X) to any profile of the individuals' reasonability perception (a vector of non-empty subsets of X).

a. Formalize the following proposition:

Assume that the number of reactions in X is larger than the number of individuals in the society and that f satisfies the following four properties:

A. If in a certain profile all the individuals do not perceive a certain reaction as reasonable, then neither does the society.

B. All the individuals have the same status.

C All the reactions have the same status.

D. Consider two profiles that are different only in one individual's reasonability perception. Any reaction that f determines to be reasonable in the first profile, and regarding which the individual did not change his opinion from reasonable to unreasonable in the second profile, remains reasonable.

Then f determines that a reaction is socially reasonable if and only if at least one of the individuals perceives it as reasonable.

Solution:

Denote by S_i the reasonability perception of individual i.

Proposition:

Assume $|X| > |N|$. Let f be a function that satisfies:

A. $\forall i \in N . x \notin S_i \Longrightarrow x \notin f(\{S_i\}_{i\in N})$.

B. Let σ be a permutation of N. If $\{S_i\}_{i\in N}$ and $\{S_i^{'}\}_{i\in N}$ are two reasonability perception profiles such that for every i: $S_i^{'} = S_{\sigma(i)}$ then $f(\{S_i\}_{i\in N}) = f(\{S_i^{'}\}_{i\in N})$.

C. Let $\sigma^{'}$ be a permutation of X. If $\{S_i\}_{i\in N}$ and $\{S_i^{'}\}_{i\in N}$ are two reasonability perception profiles such that for every x and for every i it holds that $x \in S_i \iff \sigma^{'}(x) \in S_i^{'}$, then $x \in f(\{S_i\}_{i\in N}) \iff \sigma^{'}(x) \in f(\{S_i^{'}\}_{i\in N})$.

D. Let $\{S_i\}_{i \in N}$ and $\{S_i^{'}\}_{i \in N}$ be two reasonability perception profiles such $S_i^{'} = S_i$ that for any $i \neq j$. Let $x \in f(\{S_i\}_{i \in N})$. If $x \neq S_j$ or $x \in S_j^{'}$, then $x \in f(\{S_i^{'}\}_{i \in N})$.

Then, $x \in f(\{S_i\}_{i \in N}) \iff \exists i \textit{ such that } x \in S_i$.

b. Show that all four properties are necessary for the proposition.

Solution:

1. The fixed function $f(\cdot) = X$ satisfies B, C and D but not A.

2. The function $f(\{S_i\}_{i \in N}) = S_i$ for some fixed i (a dictatorship) satisfies A, C and D but not B.

3. A function which determines that a reaction is reasonable if and only if at least one of the individuals perceives it as such, except for one specific reaction for which it is necessary that two individuals perceive it as reasonable, satisfies A, B and D but not C.

4. A function which determines as reasonable the reaction(s) that the largest number of individuals perceive as reasonable (the most popular reaction(s)), satisfies A, B and C but not D.

c. Prove the proposition.

Solution:

$\Rightarrow$

Let f be a function satisfying A, B, C and D. Let $\{S_i\}_{i \in N}$ be a reasonability perception profile. Let y be a reaction that is perceived as reasonable by at least one individual, denoted by j.

Define the profile $\{T_i\}_{i \in N}$ by arbitrarily assigning one alternative to each individual, with no repetitions (that is $T_i = \{x_i\}$ and $x_i \neq x_k$ for any $i \neq k$), such that $x_j = y$. This is possible since there are more alternatives than individuals.

Claim: In the profile $\{T_i\}_{i \in N}$, all reactions are determined to be socially reasonable.

Proof: $f(\{T_i\}_{i \in N})$ is non-empty and therefore for some x, $x \in f(\{T_i\}_{i \in N})$. By property A, there is an individual k such that $x = x_k \in T_k$. Let $i \neq k$ and let σ be a permutation of N that switches between i and k. Now $\{x_k\} = T_i$ and $\{x_i\} = T_k$ and by property B, $x_k \in f(\{T_{\sigma(i)}\}_{i \in N})$. Let $\sigma^{'}$ be a permutation of X that switches between x_i and x_k. By property C, x_i is now socially reasonable, but in fact we are back to the original profile $\{T_i\}_{i \in N}$. Therefore, for every i, it holds that $x_i \in f(\{T_i\}_{i \in N})$.

The above claim implies that $y \in f(\{T_i\}_{i \in N})$.

Define $\{R_i\}_{i \in N}$ such that for every i, $R_i = S_i \cup T_i$. One can transform $\{T_i\}_{i \in N}$ into $\{R_i\}_{i \in N}$ by adding one reaction to one individual at a time. By property D, in each of these stages, the reaction y remains socially reasonable and thus $y \in f(\{R_i\}_{i \in N})$.

If for some individual i the reaction that we chose arbitrarily in T_i is not in the original reasonability perception set S_i, then subtract it from R_i. After a finite number of subtractions, we will obtain the original profile $\{S_i\}_{i \in N}$. Since at no step did we change the status of y in any reasonability perception set, $y \in f(\{S_i\}_{i \in N})$ by D.

$\Leftarrow$
Trivial.