Micro Theory Exam January 2010

Problem 1:

A basketball coach considers buying players from a set A. Given a budget w and a price vector $(p_a)_{a \in A}$ the coach can purchase any set such that the total cost of the players in it is not greater than w. Discuss the rationality of each of the following choice procedures, defined for any budget level w and price vector P:

(P1) The consumer has in mind a fixed list of the players in A.: $a_1, \ldots, a_n$. Starting at the beginning of the list, when he arrives to the $i's$ player he adds him to the team if his budget allows him to after his past decisions, and then continues to the next player on the list with his remaining budget. This continues until he runs out of budget or has gone through the entire list.

Let $\{1, \ldots n\}$ be the list of players. Identify a set B with a vector $x(B)$ of 0's and 1's such that $x(B)_i = 1$ if $i \in B$ and $x(B)_i = 0$ if $i \notin B$. The consumer's choice is rationalized by the preferences: $B \succsim C$ if $x(B) \succsim_L x(C)$, where $\succsim_L$ are the standard lexicographic preferences on $\mathbb{R}^n$.

(P2) He purchases the combination of players that minimize the excess budget he is left with.

The procedure is NOT rationalizable since it is even does not induce a choice from a choice set:

The choice set is $\{\{a_1\}, \{a_2\}\}$ for both sets of parameters:

$p_1 = 2$, $p_2 = 3$ and $w = 3$

and

$p_1 = 3$, $p_2 = 2$ and $w = 3$.

Nut in one case $\{a_1\}$ is chosen and in the other $\{a_2\}$.

Problem 2:

A decision maker has a preference relation over $\mathbb{R}^n_+$. A vector (x_1, x_2) is interpreted as an income combination where x_i is the dollar amount the

decision maker receives at period i.

Let P be the set of all preference relations satisfying:

(i) Strong Monotonicity (SM) in x_1 and x_2 .

(ii) Present preference (PP): $(x_1+\varepsilon, x_2-\varepsilon) \succsim (x_1,x_2)$ **for all** $\varepsilon > 0$**.**

Define $(x_1,x_2)D(y_1,y_2)$ **if** $(x_1,x_2) \succsim (y_1,y_2)$ **for all** $\succsim \in P$**.**

(1) Interpret the relation D. Is it a preference relation?

D is a domination relation: x dominates y if for every monotonic present biased preference relation, x is considered at least as good as y.

D is not a preference relation: although it is transitive, it is not complete.

(2) Is it true that $(1,4)D(3,3)$**? What about** $(3,3)D(1,4)$**?**

$(1,4)D(3,3)$ is false: consider the preference relation represented by the utility function $u(x_1,x_2) = x_1+x_2$. It satisfies the two properties, but $(1,4) \prec (3,3)$.

$(3,3)D(1,4)$ is true:.for any preference relation which satisfies the two properties, by PP $(3,3) \succsim (1.5,4.5)$ and by SM, $(1.5,4.5) \succ (1,4)$

(3) Find and prove a proposition of the following type:

$(x_1,x_2)D(y_1,y_2)$ **if and only if [put here a condition on** (x_1,x_2) **and** (y_1,y_2)**].**

Proposition: $(x_1,x_2)D(y_1,y_2)$ Iff $x_1 \geq y_1$ and $(x_1+x_2) \geq (y_1+y_2)$.

Proof:

$\Leftarrow$ Assume $x_1 \geq y_1$ and $(x_1+x_2) \geq (y_1+y_2)$.

Let $\succsim \in P$, i.e. a preference relation satisfying SM and PP. If $x_2 \geq y_2$, by SM $x \succsim y$. If $y_2 > x_2$ let $\epsilon = y_2 - x_2$.

It follows that $x_1 \geq y_1+\epsilon$. By SM $(x_1,x_2) \succsim (y_1+\epsilon, x_2)$ and by PP $(y_1+\epsilon,x_2) = (y_1+\epsilon, y_2-\epsilon) \succsim (y_1,y_2)$.

$\Rightarrow$ Assume $(x_1,x_2)D(y_1,y_2)$.

The condition $x_1 \geq y_1$ is necessary: let a and b be two vectors such that aDb and $a_1 < b_1$. Consider the preferences $\succsim_t$ represented by $u(x_1,x_2) = tx_1+x_2$ where $t > 1$. Obviously they satisfy PP and SM. For t large enough $ta_1+a_2 < tb_1+b_2$ and thus $b \succ_t a$.

The condition $x_1+x_2 \geq y_1+y_2$ is necessary: let a and b be two vectors such that aDb and $a_1+a_2 < b_1+b_2$. Then $b \succ_1 a$.

Problem 3:

Let $\succsim$ be a preference relation on R^n satisfying the following two properties:

Weak Pareto (WP): If $x_i \geq y_i$ for all i, then $x=(x_1,..,x_n) \succsim y=(y_1,..,y_n)$ and if $x_i > y_i$ for all i, then $(x_1,..,x_n) \succ (y_1,..,y_n)$.

Independence (IIA): Let $a,b,c,d \in \mathbb{R}^n$ be vectors such that in any coordinate $a_i > b_i$, $a_i = b_i$or $a_i < b_i$ if and only if $c_i > d_i$, $c_i = d_i$or $c_i < d$, accordingly. Then, $a \succsim b$ iff $c \succsim d$.

(1) Find a preference relation different from those represented by $u_i(x_1,\ldots,x_n)=x_i$ which satisfies the two properties.

Lexicographic preferences such as: $x \succ y$ iff $x_{i^*} > y_{i^*}$ where $i^* = \min\{i | x_i \neq y_i\}$.

(2) Show, for $n=2$, that there is an i such that $a_i > b_i$ implies $a \succ b$.

Assume that $(4,2) \succ (2,4)$. By Pareto $(4,2) \succ (2,0)$.

Also $(4,2) \succ (2,2)$ since by by Pareto $(2,4) \succsim (2,2)$.

Now, consider two vectors $a=(a_1,a_2)$ and $b=(b_1,b_2)$ such that $a_1 > b_1$. By IIA the preference between a and b when $a_2 < b_2$, $a_2 = b_2$ or $a_2 > b_2$ is the same as between $(4,2)$ and $(2,4)$, $(2,2)$ or $(2,0)$ respectively, namely $a \succ b$.

(3) Provide a "social choice" interpretation for the result in (2). Explain how it differs from Arrow's Impossibility Theorem.

We can interpret a point in R^n as an allocation of a desirable good between n individuals. The preferences of all individuals are fixed (each wants as much as possible). The independence property expresses a requirement that the social preference between any two alteratives a and b is a function of only the n comparisons between a_i and b_i.

(4) Expand (2) for any n.

Let $A \subseteq \{1, \ldots, n\}$. We say that A is *decisive* if whenever for all $i \in A$, $x_i > y_i$ then $x \succ y$.

Let $A \subseteq \{1, \ldots, n\}$. We say that A is *almost decisive* if whenever for all $i \in A$, $x_i > y_i$ and for all $i \notin A$, $y_i > x_i$ then $x \succ y$

First, if A is almost decisive then it is decisive: By the independence it is enough to look at two vectors a and b such that $a_i = 3$ and $b_i = 1$ if $i \in A$, and all other a_j and b_j are either 1 or 3.

Let $c_i = 3$ if $i \in A$ and $c_i = 1$ otherwise and let $d_i = 1$ if $i \in A$ and $d_i = 3$ otherwise.

By the almost decisiveness of A, $c \succ d$. By Pareto $a \succsim c$ and $d \succsim b$, thus $a \succ b$.

Now let A be a decisive set. and let A_1 and A_2 be a partition of A. We will see that either A_1 or A_2 is almost decisive.

Assume not. Consider the three vectors:

	A_1	A_2	$N - A$
a	1	3	5
b	5	1	3
c	3	5	1

By A's decisiveness, $c \succ a$. If A_1 is not almost decisive then $a \succsim b$ and if A_2 is not almost decisive then $b \succsim c$. A contradiction.

Thus, there is i such that $\{i\}$ is decisive.