# Solution of Problem Set 5

## 1.

The actions of player 1 that are rationalizable are $a_1$, $a_2$, and $a_3$; those of player 2 are $b_1$, $b_2$, and $b_3$. The actions $a_2$ and $b_2$ are rationalizable since $(a_2,b_2)$ is a Nash equilibrium. Since $a_1$ is a best response to $b_3$, $b_3$ is a best response to $a_3$, $a_3$ is a best response to $b_1$, and $b_1$ is a best response to $a_1$ the actions $a_1$, $a_3$, $b_1$, and $b_3$ are rationalizable. The action $b_4$ is not rationalizable since if the probability that player 2's belief assigns to $a_4$ exceeds 1/2 then $b_3$ yields a payoff higher than does $b_4$, while if this probability is at most 1/2 then $b_2$ yields a payoff higher than does $b_4$. The action $a_4$ is not rationalizable since without $b_4$ in the support of player 1's belief, $a_4$ is dominated by $a_2$.

*Comment* That $b_4$ is not rationalizable also follows from the fact that $b_4$ is strictly dominated by the mixed strategy that assigns the probability 1/3 to $b_1$, $b_2$, and $b_3$.

## 2. (Cournot duopoly)

Player $i$'s best response function is $B_i(a_j) = (1 - a_j)/2$; hence the only Nash equilibrium is (1/3, 1/3).

Since the game is symmetric, the set of rationalizable actions is the same for both players; denote it by $Z$. Let $m = \inf Z$ and $M = \sup Z$. Any best response of player $i$ to a belief of player $j$ whose support is a subset of $Z$ maximizes $E[a_i(1 - a_i - a_j)] = a_i(1 - a_i - E[a_j])$, and thus is equal to $B_i(E[a_j]) \in [B_j(M), B_j(m)] = [(1 - M)/2, (1 - m)/2]$. Hence (using the second definition of rationalizability), we need $(1-M)/2 \leq m$ and $M \leq (1-m)/2$, so that $M = m = 1/3$: 1/3 is the only rationalizable action of each player.

## 3. (Guess the average)

The action 0 is rationalizable since it is a Nash Equilibrium.

Since the game is symmetric, the set of rationalizable actions is the same, say $Z$, for all players. Note that if an action is a best response to some belief, its expected payoff must be positive according to that belief. Let $k^* > 0$ be the largest number in $Z$. $k^*$ must be a best response to some belief over the action of the other players in the support of Z. Then $k^* < \frac{3}{2}k^* = \left[\sum_{i \in N} \frac{a_i}{n}\right] \leq \sum_{i \in N} \frac{a_i}{n}$. But then for some player i, $a_i > k^*$ contradicting $k^*$ being maximal. By this argument the action $k^*$ is a best response to a belief whose support is a subset of $Z$ only if $k^* = 0$.

4.

At the first stage of elimination positions 1 and 7 are erased since position 1 is strictly dominated by position 2 and position 7 is strictly dominated by position 6 (see matrix payoff below). Then positions 2 and 6 are eliminated since they are strictly dominated by positions 3 and 5 respectively. And finally positions 3 and 5 are eliminated since they are strictly dominated by position 4.

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| **1** | 7,7 | 2,12 | 3,11 | 4,10 | 5,9 | 6,8 | 7,7 |
| **2** | 12,2 | 7,7 | 4,10 | 5,9 | 6,8 | 7,7 | 8,6 |
| **3** | 11,3 | 10,4 | 7,7 | 6,8 | 7,7 | 8,6 | 9,5 |
| **4** | 10,4 | 9,5 | 8,6 | 7,7 | 8,6 | 9,5 | 10,4 |
| **5** | 9,5 | 8,6 | 7,7 | 6,8 | 7,7 | 10,4 | 11,3 |
| **6** | 8,6 | 7,7 | 6,8 | 5,9 | 4,10 | 7,7 | 12,3 |
| **7** | 7,7 | 6,8 | 5,9 | 4,10 | 3,11 | 2,12 | 7,7 |

* divide all payoffs by 14 in order to get the share of each seller

**5.**

At the first round every action $a_i \leq 50$ of each player $i$ is weakly dominated by $a_i + 1$. No other action is weakly dominated, since 100 is a strict best response to 0 and every other action $a_i \geq 51$ is a best response to $a_i + 1$. At every subsequent round up to 50 one action is eliminated for each player: at the second round this action is 100, at the third round it is 99, and so on. After round 50 the single action pair (51, 51) remains, with payoffs of (50, 50).