Solution for problem set 10

1. (Example of mixed and behavioral strategies)

At the initial history choose A and B each with probability 1/2; at the second information set choose l.

2. (Mixed and behavioral strategies and imperfect recall)

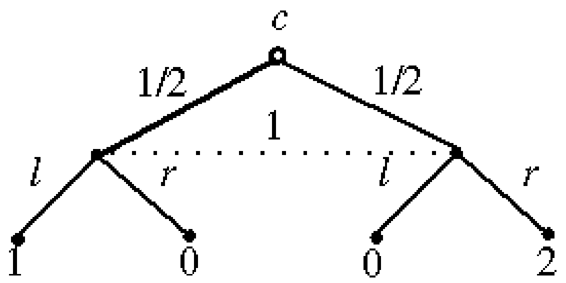
If player 1 uses the mixed strategy that assigns probability 1/2 to ll and probability 1/2 to rr then she obtains the payoff of 1 with probability 1/2 regardless of player 2's strategy. If she uses a behavioral strategy that assigns probability p to l at the start of the game and probability q to l at her second information set then she obtains the payoff $pqt + (1-p)(1-q)(1-t)$, where t is the probability with which player 2 chooses his left action. Thus by such a strategy she guarantees a payoff of only $\min\{pq, (1-p)(1-q)\}$, which is at most 1/4 for any values of p and q.

3. (Splitting information sets)

Suppose that the information set I^* of player 1 in the game Γ_2 is split into the two information sets I' and I'' in Γ_1. Let σ^* be a pure strategy Nash equilibrium of Γ_2 and define a profile σ' of pure strategies in Γ_1 by $\sigma_i' = \sigma_i^*$ for $i \neq 1$, $\sigma'_1(I') = \sigma_1'(I'') = \sigma^*(I^*)$, and $\sigma'_1(I) = \sigma^*_1(I)$ for every other information set I of player 1.

We claim that σ' is a Nash equilibrium of Γ_1. Clearly the strategy σ'_j of every player other than 1 is a best response to σ'_{-j} in Γ_1. As for player 1, any pure strategy in Γ_1 results in at most one of the information sets I' and I'' being reached, so that given σ'_{-1} any outcome that can be achieved by a pure strategy in Γ_1 can be achieved by a pure strategy in Γ_2; thus player 1's strategy σ'_1 is a best response to σ'_{-1}.

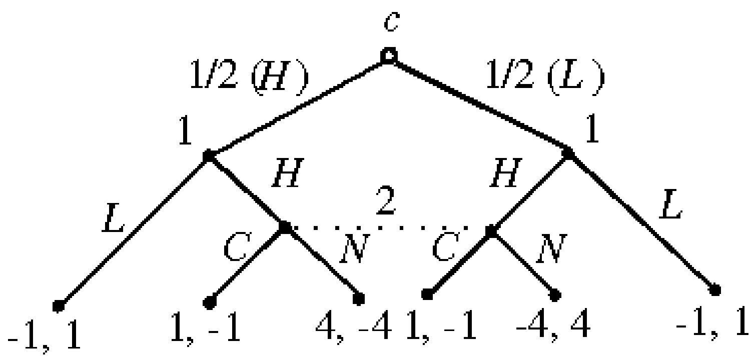
If Γ_2 contains moves of chance then the result does not hold: in the game



the unique Nash equilibrium is for the player to choose *r*. However, if the information set is split into two then the unique Nash equilibrium call for the player to choose *l* if chance chooses the left action and *r* if chance chooses the right action.

4. (Parlor game)

This (zerosum) extensive game is



The strategic form of this game is

	C	N
LH	0, 0	−5/2, 5/2
LL	−1, 1	−1, 1
HL	0, 0	3/2, −3/2
HH	1, −1	0, 0

First note that the strategies *LH* and *LL* are strictly dominated by *HH* and *HL* respectively. (i.e. if player 1 gets the high card she is better off not conceding.) Now, there is a unique Nash equilibrium, in which the mixed strategy of player 1 assigns probability 2/5 to *HL* and probability 3/5 to *HH* and player 2 concedes with probability 3/5. (In behavioral strategies this equilibrium is: player 1 chooses *H* when her card is *H* and chooses *H* with probability 3/5 and *L* with probability 2/5 when her card is *L*; player 2 concedes with probability 3/5.)

5. (absent-minded driver)

Was solved in class.