Solution for Problem set 11

229.1 (Non-ordered information sets)

The three sequential equilibria are:

- Strategies: $\beta_1(s) = 1$, $\beta_2(d) = 1$, $\beta_3(s) = 1$.

  Beliefs: $\mu_1(a) = 1$, $\mu_2(a, c) = \mu_2(b, e) = 1/2$, $\mu_3(b) = 1$.

- Strategies: $\beta_1(c) = 1$, $\beta_2(l) = 1$, $\beta_3(e) = 1$.

  Beliefs: $\mu_1(a) = 1$, $\mu_2(a, c) = \mu_2(b, e) = 1/2$, $\mu_3(b) = 1$.

- Strategies: $\beta_1(c) = 1$, $\beta_2(r) = 1$, $\beta_3(e) = 1$.

  Beliefs: $\mu_1(a) = 1$, $\mu_2(a, c) = \mu_2(b, e) = 1/2$, $\mu_3(b) = 1$.

It is straightforward to check that each of these assessments satisfies sequential rationality and consistency.

The first equilibrium has the following undesirable feature. Player 2's strategy *d* is optimal only if he believes that each of the two histories in his information set occurs with probability 1/2. If he derives such a belief from beliefs about the behavior of players 1 and 3 then he must believe that player 1 chooses *c* with positive probability and player 3 chooses *e* with positive probability. But then it is no longer optimal for him to choose *d*: *l* and *r* both yield him 2, while *d* yields less than 2. That is, any alternative strategy profile that rationalizes player 2's belief in the sense of structural consistency makes player 2's action in the sequential equilibrium suboptimal.

Nevertheless, player 2's strategy can be rationalized by another explanation of the reason for reaching the information set. Assume that player 2 believes that players 1 and 3 attempted to adhere to their behavioral strategies but made errors in carrying out these strategies. Then the fact that he believes that there is an equal probability that each of them made a mistake does not mean that he has to assign a positive probability to a mistake in the future.

236.1

In any sequential equilibrium of the game in Figure 236.1

- player 1 chooses C after the history r
- player 1 chooses X after the history (r,C,C)
- player 2 chooses C at the information set I^1
- player 2 chooses X with probability at least 4/5 at his information set I^2 (otherwise player 1 chooses C after the history l and (l,C,C), so that player 2 assigns probability 1 to the history (l,C,C,C) at his information set I^2, making C inferior to X)
- player 1 chooses X after the history l.

Thus player 2's belief at I^1 assigns probability 1 to the history r, while his belief at I^2 assigns positive probability to chance having chose l (otherwise C is better than X).

237.1 (Bargaining under imperfect information)

Refer to the type of player 1 whose valuation is v as *type* v. It is straightforward to check that the following assessment is a sequential equilibrium: type 0 always offers the price of 2 and type 3 always offers the price of 5. In both periods player 2 accepts any price at most equal to 2 and rejects all other prices (regardless of the history). If player 2 observes a price different from 5 in either period then he believes that he certainly faces type 0. (Thus having rejected a price of 5 in the first period, which he believed certainly came from type 3, he concludes, in the event that he observes a price different from 5 in the second period, that he certainly faces type 0.)

Comment There are other sequential equilibria, in which both types offer a price between 3 and 3.5, which player 2 immediately accepts.

246.2 (Pre-trial negotiation)

The signaling game is the Bayesian extensive game with observable actions $\langle \Gamma, (\Theta_i), (p_i), (u_i) \rangle$ in which Γ is a two-player game form in which player 1 first chooses either

3 or 5 and then player 2 chooses either *Accept* or *Reject*; $\Theta_1 = \{Negligent, Not\}$, Θ_2 is a singleton, and $u_i(\theta, h)$ takes the values described in the problem.

The game has no sequential equilibrium in which the types of player 1 make different offers. To see this, suppose that the negligent type offers 3 and the non-negligent type offers 5. Then the offer of 3 is rejected and the offer of 5 is accepted, so the negligent player 1 would be better off if she offered 5. Now suppose that the negligent type offers 5 and the non-negligent type offers 3. Then both offers are accepted and the negligent type would be better off if she offered 3.

The only sequential equilibria in which the two types of player 1 make the same offer are as follows.

- If $p_1(Not) \geq 2/5$ then the following assessment is a sequential equilibrium. Both types of player 1 offer the compensation of 3 and player 2 accepts any offer. If the compensation of 3 is offered then player 2 believes that player 1 is not negligent with probability $p_1(Not)$; if the compensation 5 is offered then player 2 may hold any belief about player 1. The condition $p_1(Not) \geq 2/5$ is required in order for it to be optimal for player 2 to accept when offered the compensation 3.)
- For any value of $p_1(Not)$ the following assessment is a sequential equilibrium. Both types of player 1 offer the compensation 5; player 2 accepts an offer of 5 and rejects an offer of 3. If player 2 observes the offer 3 then he believes that player 1 is not negligent with probability at most 2/5.

Consider the case in which $p_1(Not) > 2/5$. The second type of equilibrium involves the possibility that if player 1 offers only 3 then the probability assigned by player 2 to her being negligent is increasing. A general principle that excludes such a possibility emerges from the assumption that whenever it is optimal for a negligent player 1 to offer the compensation 3 it is also optimal for a non-negligent player 1 to do so. Thus if the out-of-equilibrium offer 3 is observed a reasonable restriction on the belief is that the relative probability of player 1 being non-negligent should increase and thus exceed 2/5. However, if player 2 holds such a belief then his planned rejection is no longer optimal.