Eco 501, Solutions for the Midterm (November 2001)

1. (a) The utility function is $\alpha v(x) + (1-\alpha)w(x)$. It is concave as a convex combination of concave functions, thus it is quasi-concave so the induced preference is convex.

   (b) Let $x(p,w)$ denote the demand and f, f_v and f_w the indirect utility functions. Then, $f(p,w) = \alpha v(x(p,w)) + (1-\alpha)w(x(p,w)) \leq \alpha f_v(p,w) + (1-\alpha) f_w(p,w)$.

   (c) Suppose that v and w are continuous and monotonic so that f, f_v and f_w are strictly increasing and continuous in w. Then for given (p,w), by 1(b), there is a unique $\beta^*(p,w)$ that satisfies:

$$f(p,w) = \alpha f_v\left(p, w - \beta^*(p,w)\right) + (1-\alpha) f_w\left(p, w - \beta^*(p,w)\right).$$

   The demand for this good is 1 if its price is below $\beta^*(p,w)$ and 0 otherwise.

2. (a) Let us say that $\succsim_1$ is close to $\succsim_0$ more than $\succsim_2$ is close to $\succsim_0$ if $x \succsim_2 [\succ_2]\, y$ and $x \succsim_0 [\succ_0]\, y$ implies $x \succsim_1 [\succ_1]\, y$.

   (b) Let $0 \leq t \leq 1$. If $U_2(x) \geq [>]\, U_2(y)$ and $U_0(x) \geq [>]\, U_0(y)$, then $U_1(x) = tU_2(x) + (1-t)U_0(x) \geq [>]\, tU_2(y) + (1-t)U_0(y) = U_1(y)$. So the preference induced by U_1 is close to the preference induced by U_0 more than the preference induced by U_2 is close to the preference induced by U_0.

   (c) No, let $K = 3$, $U_0(x) = \min\{x_1, x_2\}$ and $U_2(x) = \min\{x_2, x_3\}$ and let U_1 be defined as in 2(b) with $t = \frac{1}{2}$. Then for $p = (1,1,1)$ and $w = 1$, the demands associated with U_0, U_1 and U_2 are $x^0 = (\frac{1}{2}, \frac{1}{2}, 0)$, $x^1 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $x^2 = (0, \frac{1}{2}, \frac{1}{2})$ respectively. By 2(b), the preference induced by U_1 is close to the preference induced by U_0 more than the preference induced by U_2 is close to the preference induced by U_0, but $|x_2^1 - x_2^0| = \frac{1}{6} > 0 = |x_2^2 - x_2^0|$.

3. (a) No, let $X = \{x, y, z, w\}$ with $x \succ y$, $z \succ w$ and $class(x) = class(y) \neq class(z) = class(w)$. Then, $C(\{x,y,z\}) = x$ whereas $C(\{x,z,w\}) = z$ violating condition $(*)$.

   (b) No, let us say that a choice function C satisfies the property γ, if $x \notin A$ and $x = C(\{x\} \cup A)$ implies $x = C\left(\{x\} \cup A \setminus \{C(A)\}\right)$. Now assume that C is the choice function induced by the procedure explained above, $x \notin A$ and $x = C(\{x\} \cup A)$. Let $y = C(A)$. Since $y = C(A)$, $class(y)$ is the most populated class with the highest class number in A. So if $class(x) = class(y)$, then this is the most populated class with the highest class number in $\{x\} \cup A \setminus \{y\}$. Since $x = C(\{x\} \cup A)$, $class(x)$ is the most populated class with the highest class number in $\{x\} \cup A$. So if $class(x) \neq class(y)$, then $class(x)$ is the most populated class with the highest class number in $\{x\} \cup A \setminus \{y\}$. In either case, $class(x)$ is the most populated class with the highest class number in $\{x\} \cup A \setminus \{y\}$. So $x = C(\{x\} \cup A)$ implies $x = C\left(\{x\} \cup A \setminus \{y\}\right)$. Thus, C satisfies property γ which is clearly not satisfied by all choice functions.