Course: **Microeconomics, New York University**
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Exam Solutions: **Mid-term, October 2010**

Question 1

A producer in a world with two commodities defines a relation D over the set of technologies satisfying the classical assumptions. ZDZ' if for any price vector p, $max_{x \in Z} px \geq max_{x \in Z'} px$.

(a) Explain the logic behind the definition.

The producer prefers a production technology that will maximize profits no matter what the prices turn out to be. Thus, he prefers one technology to another iff for any possible price vector, the first technology gives higher profits than the other one.

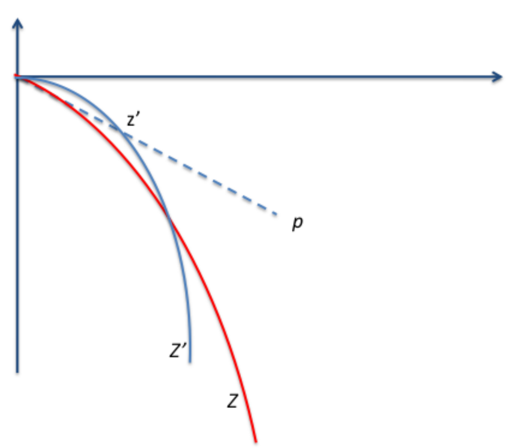
(b) Show that ZDZ' iff $Z \supseteq Z'$.

Assume $Z \supseteq Z'$, then clearly for any price vector p, $max_{x \in Z} px \geq max_{x \in Z'} px$, implying ZDZ'.

Now assume ZDZ', but $Z \not\supseteq Z'$ for contradiction. Then there exists a $z \in Z'$, but $z \notin Z$. Since Z and Z' are convex, by the separating hyperplane theorem, we know there exists a p st $pz > max_{x \in Z} px$.

(c) Define another relation E which will be similar to D and will fit to the assumption that the producer maximizes the production of good 1 subject to non-negative profits and determine if the condition in part (b) is satisfied with this relation.

ZEZ' if for any price vector p, $max_{x \in Z} x_1$ st. $[px \geq 0] \geq max_{x \in Z'} x_1$ st. $[px \geq 0]$



Now we show that ZEZ' iff $Z \supseteq Z'$ (in the 4th Quadrant):
If $Z \supseteq Z'$, then by the same argument above ZEZ'.
Now assume ZEZ', but $Z \not\supseteq Z'$ for contradiction. Then there exists a $z' \in Z'$, but $z' \notin Z$. Now consider the price vector defined by the line connecting z to the origin. Since Z is convex, Z cannot intersect this line to the right of z'. Note that for this price Z' produces strictly higher amount of good 1 with non-negative profits.

Question 2

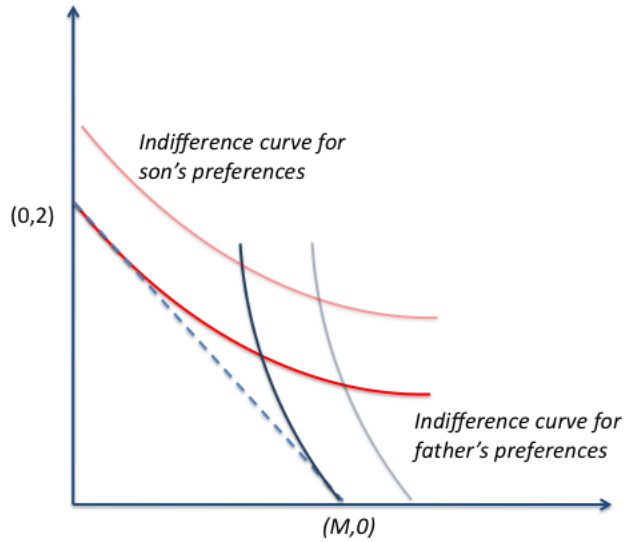
A consumer in a two commodity world, operates in the following way:
The consumer holds a preference relation $\succsim_S$ on the space of his consumption bundles. His father holds a preference relation $\succsim_F$ on the space of his son's consumption bundles. Both relations satisfy strong monotonicity, continuity and strict convexity. The father does not allow his son to purchase any bundle which is not as good (from his perspective) as the bundle $(M, 0)$. The son, when he has to choose from a budget set, maximizes his own preferences subject to the constraint imposed by his father. In case he cannot satisfy his father's wish, he feels free to maximize his own preferences.

(a) Show that the behavior of the son is rationalizable.
Define $\succ$ as follows: $a \succ b$ iff $a \succsim_F (M, 0)$ and $(M, 0) \succ_F b$, or $a \succ_S b$.
$\succ$ can easily be shown to be complete and transitive.

(b) Show that the preferences which rationalize this kind of behavior are monotonic, but not necessarily continuous or convex (this part you can demonstrate diagrammatically).

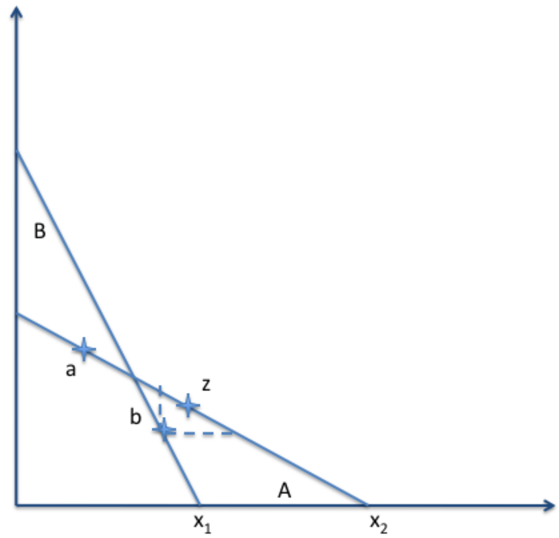
Monotonicity: Take any x, y st. $x_1 \geq y_1$ and $x_2 \geq y_2$. Since $\succsim_S, \succsim_F$ are monotonic, $x \succsim_S y$ and $x \succsim_F y$. Thus by construction of $\succ$, $x \succsim y$.

To see possible violation of continuity and convexity consider the example below.



Note that $(M, 0) \succ (0, 2)$ since $(0, 2)$ is below the father's indifference curve passing through $(M, 0)$. However we can see from the son's preferences that for any $\alpha \in (0, 1)$, $(0, 2) \succ \alpha(0, 2) + (1 - \alpha)(M, 0)$ violating convexity and continuity.

(c) Assume that the father's instruction is that given the budget set (p, w) the son will not purchase any bundle which is $\succsim_F$ worse than $(w/p_1, 0)$. The son's behavior is to maximize his preferences subject to satisfying his father's wish. Show that the son's behavior satisfies the Weak Axiom of Revealed Preferences.



Assume there is a violation of the WARP. Then there must be two overlapping budget sets as shown above such that a is chosen from set A and b is chosen from set B.
By monotonicity, $x_2 \succsim_F x_1$
Thus if $a \succsim_F x_2$ then $a \succsim_F x_1$
Since b is chosen over a in set B, $b \succ_S a$
By monotonicity, there exists an $z \in B$ st $z \succsim_S b \succ_S a$
Also by convexity $z \succsim_F x_2$, contracting a being optimal in set A.

(d) ***This part is not for the exam, for you to think at home.*** *Can you write down a preference relation that rationalizes the son's behavior?*

Question 3

Let Z be a finite set of prizes and $L(Z)$ be the space of all lotteries with prizes in Z. An individual has an ordering $\succsim$ on the set Z. His mental attitude towards any pair of lotteries p and q is obtained by the following procedure: he samples ***once*** *p and* ***once*** *q, and compares between the outcomes he gets. If the realization of p was better than of q he prefers p to q. (He takes a fresh sample of p when he compares p and r, but always looks at the same sample when comparing p and q.) Thus, the binary relation which describes his mental attitude towards the alternatives is summarized by a binary relation $\succsim$ (which obviously extends the ordering on Z).*

(a) Discuss the following properties of the relation: completeness, anti-symmetry, transitivity.

Completeness: Take any lottery p and q and compare realized outcomes. Since $\succsim$ on the set Z is complete, either p is preferred to q or q is preferred to p.

Anti-Symmetry: When comparing any lottery p and q, we always look at the same realization. We can only have the realization of p strictly better than the realization of q, or the realization q strictly better than the realization of p.

Transitivity: Assume p, q, r have support z_1, z_2 such that $z_1 \succ z_2$.

- Assume when comparing p to q, z_1 is realized for p and z_2 is realized for q, implying p is preferred to q.
- Assume when comparing q to r, z_1 is realized for q and z_2 is realized for r, implying q is preferred to r.
- Assume when comparing r to p, z_1 is realized for r and z_2 is realized for p, implying q is preferred to r.

Transitivity is violated.

(b)Give an example with three lotteries p, q, r such that the probability that there is a violation of transitivity is 1/4.

Assume that $z_1 \succ z_2 \succ z_3 \succ z_4$
p gives z_1 with probability 1/2 and z_4 with probability 1/2.
q is a sure lottery giving z_2 and r is a sure lottery giving z_3.
$p \succ q \succ r \succ p$ will be observed in the event that p realizes z_1 when compared against q, but realizes z_4 when compared against r. The probability of this event is equal to $p(z_1)p(z_4) = 1/4$.

(c) Given the ordering on Z, $\succsim$, define a partial relation
pDq if for every z $\sum_{x \succsim z} p(x) \geq \sum_{x \succsim z} q(x)$.
Show that if pDq then the probability that the individual ranks p above q is at least 0.5.

Assume there is no indifferences while comparing p to q.
Probability that p is ranked above q is $\sum_{z \in Z} p(z) \sum_{x \precsim z} q(x)$.
By first order stochastic dominance, $\sum_{x \precsim z} q(x) \geq \sum_{x \precsim z} p(x)$ for all x.
Thus, $\sum_{z \in Z} p(z) \sum_{x \precsim z} q(x) \geq \sum_{z \in Z} p(z) \sum_{x \precsim z} p(x)$.
Note that $\sum_{z \in Z} p(z) \sum_{x \precsim z} p(x) \geq \frac{1}{2} \sum_{z \in Z} p(z) \sum_{z \in Z} p(z) = \frac{1}{2}$.