Exam in Microeconomics for Phd.

NYU Economics

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Date: October 19th, 2023

Time: 09:00 - 12:00

Instructions: You are required to answer all three questions. Please use a separate notebook for each question. It is an open-book exam and you can use any written source that you wish. It is not permitted to use electronic devices.

Question 1:

Each member of a group N, consisting of n agents, is to choose a point in $X \subseteq R^K$. All of them behave rationally and every agent i has preferences that are strictly convex, continuous and differentiable over X. A dictator wants to achieve the profile of choices $(x^i)_{i \in N}$. He looks for a set Y that fulfills three conditions:

- Achievement of the following goal: For each i, the action x^i maximizes i's preferences over Y.
- Simplicity: Y is convex.
- Maximum freedom: No convex $Z \supsetneq Y$ achieves the goal.

1. Show that if such a set Y exists then Y is an intersection of at most n half-spaces.

2. Prove that if $X = [0,1] \subset \Re$ then the dictator can achieve the goal that all agents will choose the same point $z \in X$.

Question 2: There is a set of presidential candidates $X = N_A \cup N_B$ where N_A is the set of candidates who belong to party A and N_B is the set of candidates who belong to party B. A voter's choice function C is defined over the domain that consists of all sets of two candidates – one from A and one from B.

a. A voter has in mind that the candidates are lined up on a spectrum between L and R. If the two candidates are "close'," then he chooses the A candidate and if not then he chooses the B candidate. Formalize this choice function. Can all such voters be rationalized?

b. Let C be a voter's choice function. Define the binary relation $x \triangleright y$ if $C(\{x, y\}) = x$. Show that if $\triangleright$ does not have a cycle of length 4 then C can be rationalized.

c. An economist (probably a supporter of party A) claims that a voter who satisfies the condition in part (b) must be an A supporter since it can be shown that he behaves as if he assigns a value number $v(x)$ to each candidate x and adds to x a fixed bonus $b > 0$ if $x \in A$. Do you agree with this argument?

Question 3. Let X be a set of K agents and let $P = \{1,..,K\}$ be a set of K positions. Assume that $K \geq 3$. Each position is to be occupied by one agent. An assignment is a one-to-one function from P to X. (For example if $X = \{a,b,c\}$ and $K = 3$ then the assignment (b,c,a) assigns agent b to position 1, agent c to position 2 and agent a to position 3.)

Each of n referees submits a recommendation in the form of an assignment. A decision rule (DR) attaches an assignment to each possible profile of recommendations.

The following are two properties of DRs:

C: If all referees recommend that x be assigned to position k, then x is indeed assigned to k.

I : Any two profiles of recommendations that coincide with respect to their recommendations for position k assign the same agent to k.

A DR is dictatorial if there is a referee whose view is always accepted.

a. Give (no proof is required) three examples of DRs, one that satisfies C but not I, one that satisfies I but not C and one for the case of $K = 2$ that satisfies both C and I and is not dictatorial .

b. Prove that if a DR satisfies C and I, then it is dictatorial. The claim is true for all n but you only need to prove it for $n = 2$.