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False Probabilistic Arguments v. Faulty Intuition

by

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FALSE PROBABILISTIC ARGUMENTS v. FAULTY INTUITION

The State of Israel v. *Eliyahu Tzion*
(brought before the District Court, Beer Sheba)

1. *Introduction*

There was something daring, or at least unconventional, in the decision of the District Court in Beer Sheba, which rejected one of the major arguments put forward by the defence on probabilistic grounds.

The facts of the case were as follows:

A man and his wife were attending a funeral at a cemetary. They noticed a wallet on the ground, and handed it to the accused who was present in his official capacity as a policeman. Shortly after, an acquaintance of theirs told them he had lost his wallet. They referred him to the accused, who handed over to the owner a wallet, which contained one pound only.

The policeman was charged with stealing by agent under sec. 276(b) of the Criminal Ordinance, 1936, for taking 740 pounds.

The prosecution brought two main witnesses: the wallet's owner, who convinced the court that his wallet had indeed contained 740 pounds, and the couple who had first found the wallet, and who claimed that they saw banknotes protruding from it.

In defence, the accused claimed that when he had opened the wallet it contained one pound only.

When assessing the credibility of the witnesses, the judge stated that the prosecution's witnesses "had made a good impression". He added that it was not conceivable that had the couple themselves stolen the money they would have presented themselves before a policeman and disclosed their identity.

The defence claimed that even if the couple did not lie intentionally there nevertheless exists a possibility that they had erred concerning what appeared to them to be banknotes whereas the money was stolen by a pickpocket who threw away the wallet after removing its contents.

It is best here to cite verbatim, the judge's summing up on this point (translation by the author): "I cannot accept this argument, because the 'chance' of both these possibilities occurring together, and I emphasize together, is extremely small. Let me elucidate. Let us assume that the chance that the wallet's owner was pickpocketed is $\frac{1}{10}$.(I believe that the chance is in fact much smaller); and let us assume that the chance that the prosecution's witnesses erred when they said that they saw money in the wallet is also $\frac{1}{10}$. For the purpose of discussing the defence's argument I am ignoring for the moment the fact that the prosecution witnesses insist unreservedly that they indeed saw the money with their own eyes. Even so, the chance that both possibilities transpired is not $\frac{1}{10} + \frac{1}{10} = \frac{2}{10}$ that is 20%, but $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$, that is 1%. In other words, the chance that both situations occurred — as the defence claims — that is, that the wallet was stolen from its owner, thrown away, was discovered by the prosecution witnesses who brought it to the accused, peeped inside and thought, mistakenly, that it contained money — this chance, according to the theory of chance as explained above is so small that it cannot form a basis for reasonable doubt".

The accused was found guilty, sentenced to 3 months in prison and ordered to make restitution to the wallet's owner. Thus came to an end his 21 years of service in the police force.

The court's judgment certainly arouses many questions in the mind of the reader. Where did the judge take the figures which formed the basis for his reasoning? What are the arithmetic computations which he carried out? Does the probability of 0.01 which he calculated have any absolute significance which enables him to assert that "it cannot form a basis for reasonable doubt"?

Similar theoretical questions have been the subject of detailed debate between the proponents and opponents of the inclusion of probabilistic arguments in the evaluation system of the law courts.[1] Especially well worth reading is Tribe.[2] In spite of being an opponent, he discusses both pro's and con's.

In this short paper, I do not pretend to review and investigate all aspects of this many-sided issue, but restrict myself to analysing the judgment and the drawing of what seems to me to be its chief lesson.

In order to ease the task of readers who are not familiar with probabilistic concepts, I shall open in section 2 with an example of a probabilistic analysis of a gambling situation. This example was designed to pave the way to an easier understanding of the probabilistic calculation of section 3, where the judge's arguments will be analysed. We shall show that he was wrong. Opponents to the use of probabilistic calculations may quote this case as an example of the risk involved in such arguments. But, in section 4, I shall claim that the false probabilistic arguments express faulty intuition which

1 Finkelstein and Fairley, "A Bayesian Approach to Identification Evidence" (1970) 83 Harv. L.R. 489; Tribe, "Trial by Mathematics, Precision and Ritual in the Legal Process" (1971) 84 Harv. L.R. 1329; Finkelstein and Fairley, Comment on "Trial by Mathematics" (1971) 84 Harv. L.R. 1801; Tribe, "A Further Critique of Mathematical Proof" (1971) 84 Harv. L.R. 1810.

2 See preceding note.

can be exposed by such an analysis, and for this reason that probabilistic arguments are desirable even if they are incorrect.

2. *An example from a gambling house*

Reuven is a visitor at Shimeon's gambling house. The club runs a roulette table; the roulette wheel is divided into ten equal sections marked with numbers running from 1 to 10. Shimeon agreed to pay Reuven a certain sum if the number 2 should turn up. When the wheel stopped a controversy arose.

Two witnesses, Levy and Yehuda, gentlemen of impeccable good faith, take up the tale.

Levy: From my position I could only observe numbers 3 to 10. I am convinced that the number was not among them.

Yehuda: The number was 2.

An experiment was conducted, during which it was demonstrated that 10% of the cases when in fact the winning number was 2, Yehuda says that 1 turned up, and 10% of the cases when it was in fact 1, he says that 2 turned up. Levy's evidence on the other hand was accurate, though he could not distinguish between 1 and 2.

What is the probability that 2 turned up? It is known already beforehand, that there might be doubt as to what the real outcome was, because Yehuda may err, and Levy could not distinguish between 1 and 2.

The following is a complete list of the possibilities:

The true result is between 3–10: Levy always gives the true result in this case.

The true result is 1, and Yehuda reads it correctly.

The true result is 1 and Yehuda reads 2.

The true result is 2 and Yehuda reads it correctly.

The true result is 2 and Yehuda reads it as 1.

The figures in table 1 give the probabilities of these various possibilities before the wheel is turned. They are based on the following assumptions: 1) the wheel is fair, and therefore each digit has an equal chance of turning up, namely 0.1.; 2) the experiment gave the true probabilities of Yehuda erring.

TABLE 1
Outcome of the Play

	•	1	2	3	4	5	6	7	8	9	10
Yehuda	1	0.09	0.01	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
will say	2	*0.01	**0.09								

After hearing Yehuda's and Levy's evidence, only two of these possibilities remain for consideration, those starred in the table.[3]

3 For example: the probability that the true number was 1 and Yehuda said it was 2 (the square marked (*)) is obtained by multiplying the probability that

Shimeon's win is equivalent to (*), and therefore its probability equals the conditional probability[4] of (*), conditional on the occurrence of (*) or (**), that is $\frac{0.01}{0.01 + 0.09} = 0.1$.

As has been mentioned, the connection between this example and the original problem is not coincidental. However, one should note that in this example the probability space is clearly well defined, whereas in our case different people would assign different probabilities to the same proposition. In fact we shall deal with subjective probabilities[5] which measure the belief in the truth of a proposition. However, as defined by Savage the same way of calculation is suitable to objective and subjective probabilities.

3. *Criticism of the judge's probabilistic argument*

We will go back to the judge's analysis presenting it within the framework of the example in section 2. (I am aware that the judge did not do it this way, but the systematic exposition of the matter is designed to clarify the argument).

The possibility whose probability is to be evaluated is the policeman's innocence. The judge could have from the outset tried to answer this crucial question directly. Instead he chose to derive his answer from an analysis of

the true number was one — 0.1 — by the probability that Yehuda erred — 0.1 — resulting in 0.01.
The probability that the true outcome was 2, and that Yehuda said it was 2 (the square marked (**)) is obtained by multiplying the probability that the outcome was 2 — 0.1 — by the probability that Yehuda did not err — 0.9 — resulting in 0.09.

4 Suppose two true coins are tossed together; the toss of one coin, "heads" or "tails", will be denoted by "H" and "T".
There are four possibilities: 1) HH — both coins fell on heads, 2) HT — the first fell on heads, the second on tails, 3) TH — the first fell on tails, the second on heads, 4) TT — both coins fell on tails.
To each possibility we assign an equal probability, $\frac{1}{4}$.
If A is an event, we will denote the probability of its truth by p(A). If A and B are two events, then the probability of A when it is already known that A occurred is called the probability of A conditional on B, and is denoted by p(A/B). It is calculated according to the formula $p(A/B) = \frac{p(A \text{ and } B)}{p(B)}$
For example: A — "both coins are heads"
B — "at least one coin is heads"
$p(A/B) = \frac{p(A \text{ and } B)}{p(B)} = \frac{1}{4} / \frac{3}{4} = \frac{1}{3}$ does not equal $\frac{1}{4} = p(A)$

5 For an account of the connection between subjective probability and the probability calculus, see L. Savages, *Foundations of Statistics* (1950). A short explanation also appears in Tribe, *op. cit., supra* n. 1, at 1346.

other questions which he felt he could evaluate intuitively better. These questions were whether the wallet was pickpocketed, and whether the couple innocently erred.

The three possibilities present from the beginning, before evidence was given were:

I. The wallet was pickpocketed, and the witnesses state that there was no money present. In this case the witnesses' evidence is accurate.

II. The wallet was pickpocketed and the witnesses state that money was present. In this case the witnesses' evidence is inaccurate.

III. The wallet was not pickpocketed.

The judge reckons the probability of pickpocketing at 0.1, and the probability of a mistake (in the case that the wallet was empty) at 0.1; hence the following table:

TABLE 2[6]

	wallet was pickpocketed	wallet was not pickpocketed
witnesses testify they saw no money	I (0.09)	III (0.9)
witnesses testify they saw money	II (0.01)	

The couple's evidence rules out possibility I, leaving II and III.

The innocence of the policeman in the case where one of II or III occurred is equivalent to II occurring, from which the judge concluded that the probability of this is 0.01.

•As stated, the possibility whose probability is to be investigated is the possibility of the policeman's innocence. Taking into account the witnesses' evidence, the judge identifies this possibility with possibility II — "the wallet was pickpocketed and the witnesses erred when stating that the wallet contained money". The probability of this occurring is 0.01 *a priori* before knowing the couple's testimony. When this testimony is given, the likelihood of II must be evaluated with respect to the remaining possibilities — namely those possibilities which contain the element that "the witnesses say they saw money". Therefore even according to the judge's assumptions one had

6 0.09 is obtained by multiplying the probabilities of "the wallet was pickpocketed" and "the witnesses erred, the wallet in fact being empty".

0.01 is the product of the probabilities that the wallet was pickpocketed and that the wallet was empty.

0.9 is the probability that the wallet was not pickpocketed.

to evaluate the conditional probability of II relative to the outcomes II and III, which equals $\frac{0.01}{0.01 + 0.9} = \frac{1}{91}$.

In section 2 a comparable error would have implied that Shimeon is not obliged to pay Reuven with the probability of (*), and not with the probability of (*) conditional on (*) or (**).[7]

Since the judge identified the innocence of the accused with II it is clear that he also identified the negation of "the wallet was stolen" with "the policeman stole the money". But with this identification, and by setting the probability of pickpocketing having occurred at 0.1, the judge in fact claims (without, I fear, being aware of it) that *a priori,* before having the couple's testimony, the probability of the policeman stealing the money is 0.9, nine times greater than the probability that the wallet was pickpocketed!

What is the origin of this view? Apparently the judge reckoned that the probability that a wallet found on the ground was pickpocketed is 0.1. But when the starting point is an abandoned wallet, there is an additional *a priori* possibility; not only may the policeman steal the money, but the wallet may be returned to its owners with all its contents! Although the evidence in the trial completely obviates this second possibility, it is utterly wrong not to include it in the model.

The analogous error in section 2 is to say that the probability that Shimeon indeed lost to Reuven (i.e. that 2 occurred) is 0.9, 1 minus the probability that 1 had turned up.

Unfortunately the judge failed to state his assumptions explicitly and there remains the possibility that the judge evaluated the probabilities conditionally on the wallet being returned empty to his owners. But if so his evaluation of the ratio of the probability of the policeman stealing the money to the probability that the wallet was pickpocketed seems to me highly unreasonable.

The judge evaluated the probability of the policeman's innocence without conditioning on the possibilities remaining after evidence was given, so he was not aware that if the wallet was full it was not certain that the couple noticed the money in it. The possibility of them not noticing it must be taken into account.

The possibilities which the judge should have considered are given in table 3 (and this without questioning the absolute credibility assigned to the prosecution witnesses by the judge!).

Assume that the possibility of the couple erring when the wallet is full equals the possibility of error with an empty wallet, and are both 0.1. Assume further that the probability that a wallet found on the ground would be returned to its owner is 0.8, and that the probability that the policeman stole the money equals the probability that the wallet was pickpocketed, both being 0.1, then we obtain the values in the table.

7 See also the difference, *supra* n. 4 between p (A) and p (A/B).

TABLE 3

	wallet was pickpocketed	money was stolen by policeman	wallet was returned full to owner
witnesses testify they saw no money	0.09	0.01	0.8
witnesses testify they saw money	0.01	0.09	

Notice the similarity between section 2 and this analysis:

An outcome of 1 is analogous to the possibility that the wallet was pickpocketed. An outcome of 2 is analogous to the possibility that the policeman stole the money. An outcome of 3–10 is analogous to the possibility that the wallet would be returned full to its owner.

Levy's evidence, which removes the possibility of 3–10, is analogous to the wallet owner's evidence that his wallet was returned empty. Yehuda's evidence is analogous to the couple's evidence. The probability that the policeman is innocent equals the probability that Shimeon should not pay Reuven anything,

$$\frac{0.01}{0.01 + 0.09} = 0.1.$$ [8]

Even now it seems to me that assigning an *a priori* probability of the policeman stealing the wallet equal to the probability that it was pickpocketed is very hard on the policeman. Any increase in the ratio between the probability that the wallet was pickpocketed and the *(a priori)* probability that the policeman stole the money will increase the probability of the policeman's innocence. For example, if one believes the ratio is 3:1 then the chance of his innocence rises to 0.25!

4. *Conclusion*

In section 3 we showed that the court's probabilistic arguments were false. Furthermore, the errors made cannot be considered as "technical" mistakes arising from the judge's ignorance of what he terms "the theory of chance", but are rather mistakes in the construction of the model and in the mode of computation. They are therefore more serious than mere technical errors, for they are evidence of a faulty intuitive grasp of the evaluation problem presented before him:

1) The judge did not understand that the probability of an event should be evaluated conditionally relative to the possibilities remaining after evidence was given.

8 If the probability of the witnesses erring about the presence of money in the wallet is t, the probability of the policeman's innocence

$$\frac{0.01}{0.01 + 0.1\ (1-t)} = \frac{1}{11-10t}$$

Even if $t = 0$, this probability is much greater than that calculated by the judge.

2) The judge identified the possibility that the money was stolen by the policeman, with the event that the wallet was not pickpocketed; but it seems that he evaluated the probability of pickpocketing from an initial standpoint in which the negation of the pickpocketing included another possibility, namely that the money was returned to its owner.

Seemingly this case can only strengthen the views of those who decry the inclusion of probabilistic considerations in the law courts. They will see in this judgment further evidence that, in the present state of affairs at least, a licence to use such arguments will cause mistakes rather than be useful.

Before, however, a final conclusion is drawn, the alternative to probabilistic arguments in this particular case should be investigated. Had the judge not specified his considerations in detail, it is my guess he would have merely said that the evidence of the couple was convincing, and that the policeman's guilt was proven beyond any reasonable doubt. But behind this argument there would have remained the same faulty intuition as before, only we would have had no means by which to uncover it! When the judge gave a detailed probabilistic justification for his decision he made it possible to reveal his mistakes.

We cannot challenge the judge's quantification of the likelihood of the various component factors of the situation; but we can check whether he asked the right questions, and to what extent his conclusions were compatible with his fundamental assumptions.

It appears to me that in cases where judges base their arguments on probabilistic models the Appeal Court might not be required to deviate from its present practice and interfere with the basic assumptions made by the lower courts. Thus for example, in the case considered here, it may be expected that the Appeal Court would not overturn the judge's estimation that the couple is honest, and that the probability of their having made an honest mistake is 0.1. However, the Appeal Court might establish that the judge analysed the case incorrectly and return it for a renewed hearing.

To sum up, we cannot answer the question whether or not the policeman did indeed steal the money. There is nothing in the evidence which makes this impossible. However, it should not be forgotten (although the temptation is ever present) that the criterion for the conviction of a defendant is not whether or not he carried out the crime (we could ascertain that only very rarely), but that the evidence presented before the court makes his innocence an "unreasonable" possibility. The judge's considerations in this case were so much off the mark that it seems correct to say that had they been right, he could very well have reached a different conclusion.

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