

John Nash: The Master of Economic Modeling

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I. Introduction

During the past two decades non-cooperative game theory has become a central topic in economic theory. Many scholars have contributed to this revolution, none more than John Nash. Following the publication of von Neumann and Morgenstern's book, it was Nash's papers in the early fifties which pointed the way for future research in game theory. The notion of Nash equilibrium is indispensable. Nash's formulation of the bargaining problem and the Nash bargaining solution constitute the cornerstone of modern bargaining theory. His insights into the non-cooperative foundations of cooperative game theory initiated an area of research known as the Nash program. Nash's analysis of the demand game in which he uses a perturbation of a game to select an equilibrium inspired the construction of several refinements of the notion of Nash equilibrium.

A scholar's influence does not necessarily qualify him for a Nobel prize. One may argue that such awards are a social institution established to serve social goals. It is legitimate to ask what message the Swedish Academy sends to the scientific community and the rest of the world.

In some cases, prizes are designed to encourage individuals to invest their resources in endeavors that are important to society. Nash's achievements depend more on his genius than on characteristics which can be encouraged by utilitarian incentives.

Sometimes, prizes are expressions of social indebtedness towards members of society who have devoted their life towards advancing a social goal: Nash's active career was intensive but very short and I have not heard that he deliberately sacrificed attractive alternatives in favor of advancing economic knowledge.

Prizes are also intended to promote the recipient's field of research. By now, game theory is well recognized; any graduate student in economics is familiar with the foundations of non-cooperative game theory and I doubt that there is any further need to expand the influence of game theory.

So why am I so excited about the decision to award the prize to Nash? There are two reasons. One I will discuss at the end of this note. The other is my hope that this event will promote the unique characteristics of Nash's style of economic modeling. Nash is *the* master of economic modeling and in the next few pages (written hastily between the announcement of the award and the journal's deadline), I wish to spell out some criteria for good economic modeling such as those found, in such a perfect fashion, in Nash's papers.

II. The Ability to Identify Abstract Structures

Let us imagine what would have happened had the development of game theory been delayed by 45 years and that a bright young Princeton graduate student had just submitted Nash's 1950 paper in which he defines the model of non-cooperative games and proves the existence of Nash equilibrium. It is quite likely that the student would receive a negative referee report which, while praising the mathematical ability of the young scholar, would make the following complaints:

1. The paper lacks economic examples which demonstrate the usefulness of the model. The paper provides only one "example", namely a "Three-man Poker Game", of which the author himself writes: "As an example of the application of our theory to a more or less realistic case we include a simplified poker game given below". One would expect better than a "more or less" realistic example and would require an example with more economic content than poker.
2. The model is unrealistic: it is difficult to think of any strategic interaction in which each player chooses a *single* action and all players move *simultaneously*. Exceptions are mostly from the family of zero-sum games (such as children's game of matching pennies), already analyzed by von Neumann and Morgenstern. Thus, it is difficult to see the value of the extension of zero-sum game to non-zero-sum games.
3. The concept of equilibrium is too weak to be interesting. What can be said about the (Nash) equilibrium concept beyond its existence? In economics we need powerful tools and Nash equilibrium is usually uninformative and thus unlikely to produce interesting results.
4. The notion of mixed strategy which has some appeal in the context of zero-sum games is not realistic in the context of non-zero-sum games. Rational decision-makers are able to give reasons for each action they take; outside Las Vegas players do not spin roulette wheels.

Almost 50 years after Nash's paper, the importance of the Nash framework cannot be emphasized enough despite the perfectly valid complaints. In fact, in some sense we are now more worried about the tendency to accept automatically the concept of Nash equilibrium, especially among applied economists who very rarely pose questions regarding the appropriateness of the solution concept they use. What makes Nash equilibrium such an important concept?

The art of economic theoretical modeling is the identification of simple structures which approximately represent the process by which people reason about a situation. Nash modeled interactive reasoning. We often make arguments which have the following circular structure: "I do it because I believe he will do one of the following things and I am aware of why he will do it...". Nash suggests a reasonable way to model such reasoning.

Furthermore, the concept has an additional interpretation which is particularly useful in economics. The concept is applied to situations which are repeated many times with no strategic links between the repetitions. In such situations a decision by one individual depends on what he knows about the behavior of the other and as the situation is repeated with some regularity, this knowledge is acquired from the decision-maker's experience.

As to the notion of mixed strategy, I fully agree with the critiques. However, Nash's instincts were right: many years later game theorists interpreted the mixed strategy of one player as the beliefs of the other players concerning his behavior and provided a more attractive interpretation of mixed strategy equilibrium in terms of beliefs.

To summarize, an ideal theoretical economic paper should identify a simple frame of reasoning, not necessarily close to "reality" in the physical sense, but close to the way in which people reason about situations. Nash did precisely that!

III. The Question of Appropriate Mathematical Level

Economic theory is formal and the mathematical tools which are used are often not trivial. John Nash is known to be a brilliant mathematician. He could easily have generalized and complicated the models he worked with, but chose not to do so.

Consider, for example, the proof of the existence of Nash equilibrium. Nash gives two proofs of the theorem. The first proof utilizes Kakutani's fixed point theorem. Of the second proof, he writes: "The proof given here is a considerable improvement over the earlier version and is based directly on the Brouwer theorem". It is likely that Nash could generalize the

existence theorem to more general strategy spaces. However, he did not do so. He follows a tradition of searching for the most elementary proof and avoids generalizations which are not absolutely necessary to evaluate the concept he is defining.

The same is true of his work on the bargaining problem. It is hard to believe that Nash did not see generalizations of his theory to more than two players or to more general sets of agreements. However, he maintains simplicity, confining himself to the most simple case in which he can express his main ideas.

Nash demonstrates a clear principle: generalize the model only as long as the basic logic of the arguments is not lost. Express the idea in a way that reveals the basic argument without trivialization of the proposition.

IV. The Relationship between Economic Theory and the Real World

The issue of interpreting economic theory is, in my opinion, the most serious problem now facing economic theorists. The feeling among many of us can be summarized as follows. Economic theory should deal with the real world. It is not a branch of abstract mathematics even though it utilizes mathematical tools. Since it is about the real world, people expect the theory to prove useful in achieving practical goals. But economic theory has not delivered the goods. Predictions from economic theory are not nearly as accurate as those offered by the natural sciences, and the link between economic theory and practical problems, such as how to bargain, is tenuous at best. Although I have never heard an economist seriously claim that the Nash bargaining solution is a good predictor of bargaining in real markets, this solution is a standard tool in modeling interactions among negotiators. Economic theory lacks a consensus as to its purpose and interpretation. Again and again, we find ourselves asking the question "where does it lead?"

Nash is very much aware of the difficulty in interpreting the symbols he manipulates. In each of his papers he starts with a short but very clear verbal description of the situation he will discuss. Consider, for example, his definition of a bargaining situation: "A two-person bargaining situation involves two individuals who have the opportunity to collaborate for mutual benefit in more than one way." He goes on to say that the "economic situations of monopoly versus monopsony, of state trading between two nations and of negotiation between employer and labor union can be regarded as bargaining problems". Thus, Nash offers several contexts in which his theory can be applied. But this is the limit of Nash's pretension to relate to the world. Nash does not test the theory, nor does he pretend that it is useful to the bargainer.

I admire Nash's approach as it makes clear that what theoretical economists do is model the concepts which are used in natural reasoning. An economic theoretical model is not required to be tested except in our own brain. The art of economic modeling requires the avoidance of issues which are certainly connected to the main topic but whose inclusion in the analysis would prevent clear-cut results. In his first bargaining paper in *Econometrica* (1950), Nash states that "... we idealize the bargaining problem by assuming that the two individuals are highly rational, that each can accurately compare his desires for various things, that they are equal in bargaining skill...". Certain factors which are relevant to a resolution of a bargaining problem and which are omitted by Nash are probably more important than the only element (attitude towards risk) which is included in his model. Nash chose to ignore the other factors in spite their importance, in order to achieve simple and clear-cut analysis.

V. Beauty

If I had to select a paper for presentation to a group of intelligent students who wish to know what economic theory is about, I would select Nash's 1950 *Econometrica* paper without hesitation. This is my ideal paper in almost all respects but above all, it is just ... beautiful. Every sentence is measured and appropriate. The construction of the model is so logical. The result is surprising. There are plenty of leftover issues. The paper constitutes economic theory in its purest form.

VI. A Non-Academic Reason for My Excitement

Finally, I come to the second reason for my excitement over Nash's Nobel Prize: the prize is being given to John Nash the *human being*. I wish I could write more about John Forbes Nash, Jr. but I know so little about him. Until I met Nash, only a few years ago, I thought of him as an ancient mythological figure. This Nobel Prize has not been given to a professor in economics. This time, the prize is given to an individual who is living his own private life, far from any department of economics. The second message being sent to the world with this Nobel Prize is, in my opinion, not less important than any other. The profession should be both pleased and proud that after a lag of several decades, John Nash is receiving the personal attention and recognition he so rightly deserves.