

**“ECONOMICS AND PSYCHOLOGY”? THE CASE  
OF HYPERBOLIC DISCOUNTING\***

BY ARIEL RUBINSTEIN<sup>1</sup>

*School of Economics, Tel Aviv University, Department of Economics,  
Princeton University*

The article questions the methodology of “economics and psychology” in its focus on the case of hyperbolic discounting. Using some experimental results, I argue that the same type of evidence, which rejects the standard constant discount utility functions, can just as easily reject hyperbolic discounting as well. Furthermore, a decision-making procedure based on similarity relations better explains the observations and is more intuitive. The article concludes that combining “economics and psychology” requires opening the black box of decision makers instead of modifying functional forms.

1. INTRODUCTION

My interest in this article is to examine how we, theoretical economists, interpret experimental evidence in order to justify our assumptions. Instead of presenting an abstract discussion, I will focus on one currently fashionable topic in the application of economic theory: “hyperbolic discounting.” A recent spate of papers has replaced the standard “constant discount utility function” with a particular form of hyperbolic discounting utility function:  $u(x_0, x_1, \dots, x_t, \dots) = v(x_0) + \beta \sum_{t=1,2,\dots} \delta^t v(x_t)$ . Rewards obtained in period 0, 1, 2, 3, . . . are discounted by 1,  $\beta\delta$ ,  $\beta\delta^2$ ,  $\beta\delta^3$ , . . . , respectively. In this functional form the rate of substitution between today and tomorrow is smaller than that between any other pair of successive periods. The use of this functional form was introduced by Phelps and Pollak (1968) and gained prominence in the wake of the influential work of David Laibson beginning with Laibson (1997).

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Adopting the hyperbolic discounting utility function triggered a revival of the discussion of time inconsistency in the tradition of Strotz (1956). The literature assumed that at each period  $t^*$ , the decision maker uses the utility function  $u(x_{t^*}, x_{t^*+1}, \dots)$  to evaluate the stream of payments from period  $t^*$  onward. This implies time inconsistency since  $\delta$ , the marginal rate of substitution between  $t^*$  and  $t^* + 1$  from the point of view of any previous period, is replaced by  $\beta\delta$  at  $t^*$ . Time inconsistency complicates the modeling of the decision maker since assumptions must be added that specify the decision maker's analysis of his future behavior.

Following Laibson (1997), the hyperbolic discounting utility function has been used in the context of a wide range of issues: growth, self-regulation, information acquisition, job search, choice of retirement age, procrastination, addiction, investment in human capital, etc. Phenomena that "cannot be explained by standard discounting utility functions" appear as equilibrium outcomes once the decision maker is assumed to use hyperbolic discounting. Policy questions were freely discussed in these models even though welfare assessment is particularly tricky in the presence of time inconsistency. The literature often assumed, though with some hesitation, that the welfare criterion is the utility function with stationary discounting rate  $\delta$  (which is independent of  $\beta$ ).

It is interesting how the economic literature has justified the abandonment of constant discounting utility and the adoption of hyperbolic discount functions. The various justifications offered usually have a common structure. In Laibson (1996), for example, the justification goes as follows: (1) Evidence: "Research on animal and human behavior has led psychologists to conclude (see Ainslie, 1992, and Loewenstein and Prelec, 1992) that discount functions are generalized hyperbolas . . .," (2) *Hyperbolic discount functions are then introduced*: "Hyperbolic discount functions generate a preference structure which is a special case of the general class of dynamically inconsistent preferences. . .," and finally (3) Reference to *the response of the economic profession* is mentioned: "Despite the availability of this analytical framework, and the substantial body of evidence supporting hyperbolic discounting, few economists have studied the implications of hyperbolic discount functions."

Over time, the justification became more sweeping: Harris and Laibson (1999) stated: (a) "Laboratory and field studies of time preferences find that discount rates are much greater in the short run than in the long run." (b) "To model this phenomena, psychologists have adopted discount functions from the class of generalized hyperbolas," and (c) "Economists have used the discrete-time quasi-hyperbolic discount function:  $1, \beta\delta, \beta\delta^2, \dots, \beta\delta^t, \dots$ "

Within a few months the "facts" were "established" with even more certainty. Brocas and Carrillo (1999) state in a footnote: "There is well documented literature both in psychology and more recently in economics showing that individuals' discount rates are best approximated by hyperbolic rather than the traditional exponential functions. We refer the reader to Ainslie (1975), Thaler (1981) and Benzion et al. (1989) for empirical support of this theory both in animals and humans. . ."

The literature refers not only to evidence from human beings but also from animals. As Laibson (1996) says in the abstract: "Studies of animal and human

behavior suggest that discount functions are approximately hyperbolic.” Needless to say, the connection between findings on pigeons or even monkeys and the behavior of humans seems rather tenuous. We commonly believe that an animal does not understand the choice it is facing in the same way that a human being does. However, the main justification for the adoption of the hyperbolic discounting utility function is empirical evidence in the cognitive psychology literature, which contradicts the predictions of utility functions with stationary fixed discount rates.

The results of two types of experiments were introduced to support the hyperbolic discounting case: The first type is discussed by Thaler (1981): Some people prefer “one apple today” to “two apples tomorrow” but at the same time they prefer “two apples in one year plus one day” to “one apple in one year.” Ainslie and Haslam (1992) report that “a majority of subjects say they would prefer to have a prize of a \$100 certified check available immediately over a \$200 certified check that could not be cashed before 2 years; the same people do not prefer a \$100 certified check that could be cashed in 6 years to a \$200 certified check that could be cashed in 8 years.” Findings of this type have been replicated with choices involving a wide range of goods (e.g., real cash, hypothetical cash, food, and access to video games) and a wide range of subject populations. Most importantly, the results seem to be confirmed by our intuition.

The second class of experiments is discussed in Thaler (1981) and Ben Zion et al. (1989). Subjects were asked to imagine that they had won a sum of money in a lottery and that they could either take the money now or wait for an increased amount later. They were presented with several variations of the amount and date. For each pair they had to specify the minimal amount of money they would settle for in return for the delay. If a subject was indifferent between the amount  $\$x$  at time  $t$  and  $\$y$  immediately, then it was said that the subject’s choice exhibits the discount rate  $\delta(x, t)$  defined by the equation  $y = \delta(x, t)^t x$ . The results show that the average discount rate is decreasing in  $t$ . However, it was also found that  $\delta(x, t)$  is not constant but is an increasing function of  $x$ . The larger the sum of money at stake, the higher (closer to 1) the discount factor.

The experiments cited above are quite persuasive. The hyperbolic discounting functional form is only marginally different from the standard utility function and seems to provide an “explanation” of the evidence. The economic paradigm of optimizing a simple functional form is “safe” and we are tempted to declare the establishment of a new discipline: “psychology and economics.”

## 2. AN ALTERNATIVE PROCEDURAL APPROACH

Recall that there are an infinite number of functional forms consistent with the psychological findings that support the hyperbolic discounting utility functions. Therefore, it would not be a bad idea to pause and examine the experimental justification for hyperbolic discounting. How exactly does the experimental evidence support hyperbolic discounting? Is the choice of this form just a matter of convenience or does it capture certain psychological processes? How does this functional form stand up in other tests?

My own reading of the experimental results relies on ideas presented in Rubinstein (1988) within the context of decision making under uncertainty. This approach holds that the decision maker uses a procedure that aims at simplifying the choice by applying similarity relations. (The important role of similarity in decision making was emphasized by Tversky, 1977). The first formalization of a notion of similarity is due to Luce (1956). The objects of choice are of the form  $(x, p)$ , which is interpreted as a lottery yielding  $\$x$  with probability  $p$  and  $\$0$  with probability  $1 - p$ . I believe that when comparing the lottery  $(\$3,000, 0.25)$  with the lottery  $(\$4000, 0.2)$ , many subjects consider the two probability numbers 0.2 and 0.25 to be similar; this is not the case for the dollar amounts  $\$3000$  and  $\$4000$ . Thus, in the choice between  $(\$3000, 0.25)$  and  $(\$4000, 0.2)$ , the money dimension is the decisive factor. On the other hand, when comparing the degenerate lottery  $(\$3,000, 1)$  and the lottery  $(\$4000, 0.8)$  almost all decision makers do not consider the dollar amounts or the probability numbers to be similar and they apply different criteria (such as maximizing expected payoff or risk aversion).

In the context of intertemporal choices, the objects of choice are of the form  $(x, t)$  where  $\$x$  is received with a delay of  $t$  units of time. I think that when comparing two pairs  $(x, t)$  and  $(y, s)$ , many decision makers go through the following three-stage procedure using two similarity relations (one in the money dimension and one in the time dimension): (a) The decision maker looks for dominance: If  $x > y$  and  $t < s$  then there is no dilemma and the pair  $(x, t)$  is determined to be preferred over  $(y, s)$ . (b) The decision maker looks for similarities between  $x$  and  $y$  and between  $t$  and  $s$ . If he finds similarity in one dimension only, he determines his preference between the two pairs using the dimension in which there is no similarity. For example, if  $t$  is similar to  $s$  but  $x$  is not similar to  $y$ , and  $x > y$ , then  $(x, t)$  is preferred over  $(y, s)$ . (c) If the first two stages were not decisive, the choice is made using a different criterion.

Much of the analysis in Rubinstein (1988) can be applied here. However, note that the role of probability 0 is replaced here with time equal to infinity and the role of probability 1 is replaced with time equal to 0.

The experimental findings of time inconsistency described in the previous section are compatible with the application of this procedure. Consider, for example, a decision maker who is applying the above procedure and determines that "today" and "a year from now" are not similar whereas "10 years" and "11 years" are. Then, if the decision maker is indifferent between  $\$x$  today and  $\$y$  in a year from now, it must be that  $x < y$  and that  $x$  and  $y$  are not similar: If he considers  $x$  and  $y$  to be similar, then he would prefer " $\$x$  today" over " $\$y$  in a year from now." On the other hand, if a decision maker is indifferent between  $\$x$  in 10 years and  $\$z$  in 11 years, then it must be that  $x < z$  and that  $x$  and  $z$  are similar. If he does not consider  $x$  and  $z$  to be similar, then since we presume that he does find "10 years" and "11 years" to be similar, by applying the above procedure he would find " $\$z$  in 11 years" to be preferred over " $\$x$  in 10 years." If  $y > x$  and  $z > x$  and  $y$  is not similar to  $x$  whereas  $z$  is, then one would expect  $z$  to be smaller than  $y$ .

Both the hyperbolic discounting approach and the above procedural approach are consistent with the evidence. However, in the next section I will try to persuade the reader that the procedural approach is in fact superior to hyperbolic discounting in explaining the experimental results.

### 3. EXPERIMENTS

The results of the following three experiments are incompatible with the hyperbolic discounting hypothesis yet are consistent with a plausible application of the above procedural approach.

3.1. *Experiment I.* Experiment I was conducted in the Fall of 2002. Following a pilot experiment conducted on graduate students at Princeton, I contacted several hundred students at Tel Aviv University (TAU) and Princeton University by e-mail. They were asked to go to a web site prepared for the experiment (by Eli Zvuluny) and were promised that a small fraction of the participants (approximately 5–10%) would be randomly selected to receive \$20 each independently of their answers. A total of 456 students responded: 220 undergraduates from the TAU School of Economics, 163 undergraduates from TAU Law School, and 73 undergraduates in Political Science at Princeton. Students were allocated randomly to answer either question 1 or 2:

Q1 Imagine that you have to choose between the following two options:

- A) Receiving \$467.00 on June 17th 2004.
- B) Receiving \$607.07 on June 17th 2005.

What will be your choice?

Q2 Imagine that you have to choose between the following two options:

- A) Receiving \$467.00 on June 16th 2005.
- B) Receiving \$467.39 on June 17th 2005.

What will be your choice?

The results are presented in the following table:

	A: No Delay	B: Delay	
<b>Q1</b>	100 45%	122 55%	<b>222</b>
<b>Q2</b>	126 54%	108 46%	<b>234</b>
	<b>226</b>	<b>230</b>	456

Two hundred forty-eight out of the 456 subjects chose delay in question 1 and no delay in questions 2; if their choices were random, then such a result would be

obtained with a probability of about 3%. (Stronger results were obtained among the Princeton and the TAU economics students.)

The results demonstrate a tendency among the subjects to consider 39 cents to be sufficiently meaningless to justify a one-day delay even when it is a few years hence. The sums involved in question 2 (\$467.00 and \$467.39) are so similar that receiving this significant amount of money a day earlier is justified. On the other hand, the trade-off between “amount ” and “delay” is stronger in question 1.

Choosing delay in Q1 and no delay in Q2 is inconsistent even according to the more general hyperbolic discounting utility function  $v(x_0) + \sum_{t=1,2,\dots} (\prod_{s=1,\dots,t} \delta_s)v(x_t)$ , where  $(\delta_s)$  is being a (weakly) increasing sequence and  $v$  is increasing and concave. We can see this form from the fact that the choice of no delay in Q2 implies that

$$\delta_{t^*}v(467.39) - v(467.39 - 0.39) < 0 \quad \text{where } t^* = 17.6.2005$$

The concavity of  $v$  and the monotonicity of the discounting rates imply that for any sum of money  $x > 467.00$  and  $s > 17.6.2005$ , we have  $\delta_s v(x) - v(x - 0.39) < 0$ .

A straightforward calculation shows that this implies that

$$\left( \prod_{s=1,\dots,t^*-365} \delta_s \right) v(607.07) - \left( \prod_{s=1,\dots,t^*} \delta_s \right) v(607.07 - 365(0.39)) < 0$$

The fact that 365 times 39 cents exceeds \$140.07 completes the proof. Thus, the above results are inconsistent not only with the constant discounting utility function but with the hyperbolic discounting approach as well.

3.2. *Experiment II.* Following some pilot experiments done primarily at Tel Aviv University, a two-stage experiment was conducted. The subjects were students in a Political Science class at Princeton University (39% freshmen, 27% sophomores, 19% juniors, and 15% seniors). The teacher estimated that 10–15% of the subjects were majoring in economics. The students were approached twice within an interval of 14 days. In each round, the students were asked to go to a web site designed for the experiment and to respond online to several questions. (See <http://www.princeton.edu/~ariel/discounting1> and <http://www.princeton.edu/~ariel/discounting2> for the original forms.) A prize of \$100 was to be randomly awarded to one of the participants in each round. In the first round, the students answered questions Q3 and Q5 below, whereas in the second round they answered questions Q4 and Q6. A total of 165 students responded to the first message and 145 to the second. Of the 145 students in the second round, 45% had not participated in the first round, thus making it possible

to check whether participation in the first round made any difference. Results show that it did not.

Q3 You can receive the amounts of money indicated according to one of the two following schedules:

A	April 1	July 1	Oct 1	Dec 1
	\$1000	\$1000	\$1000	\$1000
B	March 1	June 1	Sept 1	Nov 1
	\$997	\$997	\$997	\$997

Which do you prefer?

Q4 You have to choose between:

- A Receiving \$1000 on Dec 1st.
- B Receiving \$997 on Nov 1st.

Your choice is:

The hyperbolic discounting approach predicts that every subject choosing B in Q4 will choose B in Q3. Consider the more general form of the hyperbolic discount utility function,  $v(x_0) + \sum_{t=1,2,\dots} (\prod_{s=1,\dots,t} \delta_s)v(x_t)$ , where  $(\delta_s)$  is a weakly increasing sequence. If a subject chooses B in Q4, then he is ready to sacrifice \$3 in order to advance the payment due in December by one month. The hyperbolic discounting theory predicts that he would find the three dollar sacrifice worthwhile in order to advance any of the other three scheduled payments by one month.

The results contradict this prediction: 54% of the subjects chose B in question 4, whereas only 34% of the subjects chose schedule B in question 3. In particular, 22% of the 81 subjects who answered both questions chose 3A and 4B whereas only 6% chose 3B and 4A. The following table presents the number of students that gave each possible combination of answers:

		<b>Q4</b>				
		A 1 × 1000	B 1 × 997	Did not participate		
<b>Q3</b>	A 4 × 1000	33	19	56	<b>108</b>	66%
	B 4 × 997	6	23	27	<b>56</b>	34%
	Did not participate	28	36		64	
		<b>67</b>	<b>78</b>	83	228	
		46%	54%			

The explanation of the results according to the procedural approach is as follows: In Q3 many subjects viewed the alternative as a pair, a sequence of dates and a sequence of \$ amounts. The sequence of dates (April 1, July 1, Oct 1, Dec 1) was considered similar to the sequence (March 1, June 1, Sept 1, Nov 1) whereas the sequence of payments (\$1000, \$1000, \$1000, \$1000) was considered less similar to (\$997, \$997, \$997, \$997) than \$1000 was to \$997. These subjects chose A over B

in Q3; however, responding to Q4, they found both the dates and the amounts similar and activated the third stage of the decision procedure.

3.3. *Experiment III*

Q5 In 60 days you are supposed to receive a new stereo system to replace your current one. Upon receipt of the system, you will have to pay \$960. Are you willing to delay the transaction by one day for a discount of \$2?

Q6 Tomorrow you are supposed to receive a new stereo system to replace your current one. Upon receipt of the system, you will have to pay \$1,080.

Are you willing to delay the delivery and the payment by 60 days for a discount of \$120?

According to the hyperbolic discount approach, whoever is not willing to accept \$2 for a one-day delay in delivery 60 days from now should not be willing to accept \$2 for a one-day delay in delivery  $t$  days from now, for any  $t < 60$ . Therefore, by transitivity, they should not agree to a postponement of 60 days from the present in exchange for \$120. The results did not confirm this prediction: 43% of the subjects rejected the delay in Q5 whereas only 31% rejected the delay in Q6. Of the 84 participants who answered both questions, almost a quarter made a switch, which contradicted the prediction of the hyperbolic discounting approach.

		<b>Q6 (\$120)</b>				
		Delay	No Delay	Did not participate		
<b>Q5 (\$2)</b>	Delay	37	10	46	<b>93</b>	57%
	No Delay	20	14	37	<b>71</b>	43%
	Did not participate	43	21		64	
		<b>100</b>	<b>45</b>	83	228	
		69%	31%			

The manipulation of subjects' behavior in this experiment, as in Experiments I and II, was accomplished by triggering the similarity relation with regard to the *money* dimension. In Q5, I believe that some subjects stopped in the second phase of the procedure after judging that a two dollar difference leaves the payments in the two alternatives quite similar whereas a delay in payment of 60 days is much less similar to one of 61 days. In Q4, I think that the subjects found both the payments and the delays not to be similar.

4. DISCUSSION

The main justification for the use of hyperbolic discount utility functions is based on experimental observations that reject constant discount utility functions. In



this article, I have argued that the same sort of evidence can just as easily reject hyperbolic discounting as well. Furthermore, the procedure based on similarity explains the observations even better than the hyperbolic discounting formula and, in my opinion, is more intuitive as a description of the decision makers' process of reasoning.

In defense of hyperbolic discounting one might mention its analytical convenience. I agree that the hyperbolic discounting approach does capture the psychological phenomenon that the present is given special treatment and as such it is a very interesting exercise. However, the approach goes much farther than simply assigning a special role for the present. It assumes the maximization of a utility function with a specific structure and as such misses the core of the psychological decision-making process. Thus, I find it to be no more than a minor modification of the standard discounting approach.

It is interesting to contrast the warmth of the “profession” toward the hyperbolic discount literature with the more hostile response to nonexpected utility theories. Observations such as the Allais paradox motivated many economists and decision theorists in the 1950s and 1960s to search for alternative functional forms for expected utility that would be consistent with the psychological evidence regarding decision making under uncertainty. The functional forms suggested were not less empirically motivated than the hyperbolic discount utility functions and were better established axiomatically (see Fishburn and Rubinstein, 1982). However, we have seen only a few relatively recent applications of nonexpected utility theories to economic problems, whereas the hyperbolic discounting approach immediately entered the mainstream of economics. This I find puzzling.

To conclude, adopting the similarity-based procedural approach may require revolutionary changes in our theories. As discussed in Rubinstein (1988), the application of similarity-based procedures may result in conflicts with transitivity since the transitive closure of the partial relation as determined in the first two stages of the procedure is not likely to be consistent with the third stage. Doing “economics and psychology” requires much more than citing experimental results and marginally modifying our models. We need to open the black box of decision making, and come up with some completely new and fresh modeling devices.

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