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DEFINABLE PREFERENCE RELATIONS - THREE EXAMPLES

by

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ABSTRACT

Three different topics are treated: Arrow's impossibility theorem, Rawls' difference principle and a choice problem for "the youth in a strange town".

Three axiom systems on the order relations, which include the requirement that the relation be definable in the pertaining language, are correspondingly given. The order relations satisfying the axioms in certain models are characterized.

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1. Introduction.

The three sections deal with three different topics. Beyond that, this paper aims to exemplify the importance of including the element of "language" in discussions within the theories of social choice, utility and measurement.

In each example we shall describe a language containing relation symbols which may be used in addition to the other logical signs - the connectives, the quantifiers and the variables. For each language we will provide a model and particularly an interpretation for the relation symbols in it. We will examine the question of what order relations which are definable in the language and satisfy several further axioms exist for this model.

In section 2 we will consider Arrow's problem of social choice. The language will contain n relation symbols which "represent" the n preference relations of the individuals. We will construct a model in which every order relation satisfying "unanimity" and which can be formulated in terms of these n relations is a dictatorial order relation, in the sense that it coincides with exactly one of the individuals' preference relations is identical to it. We will discuss the question of to what extent the requirement of definability replaces the requirement of independence of irrelevant alternatives.

In section 3 we will deal with Rawls' maximin principle. The language contains a single binary relation. We will consider a set of the form $X \times N$ with a binary relation $\succ$, where $(y,j) \succ (x,i)$ means that the social possibility y is more acceptable to j than the possibility x is to i. We will give an axiom system which will include the definability requirement and prove that in this model the only order relation satisfying the axioms is the maximin.

In section 4 we will discuss "the youth who arrives at a strange town and has to choose one of two girls. The information given him is the names of the girls' male companions over the last n days. Assuming that the youth is "rational", we may take it that his choice is determined by an order relation on n-tuples of boys' names. We will characterize the order relations expressible in the youth's language.

Definitions and Notation.

Background material in logic may be found in [3] and [8].

We will employ the formal language of the predicate calculus. The language has the following atomic symbols: 1. object symbols 2. variables 3. relative symbols 4. connectives ($\neg$ - negation, $\vee$ - disjunction, $\wedge$ - conjunction, $\rightarrow$ - implication and $\leftrightarrow$ - equivalence) 5. quantifiers ($\forall$ - for all, $\exists$ - there exists). Atomic formulae are relative symbols in whose empty positions are inserted object symbols and variables. A well formed formula (wff.) is defined inductively by:

1) Atomic formulae are wffs.

2) If ψ and φ are wffs., so are $\neg\psi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, $\varphi \rightarrow \psi$, $\varphi \leftrightarrow \psi$.

3) If ψ is a wff., and does not contain the signs $\exists y$, $\forall y$, then $\forall y\psi$, $\exists y\psi$ are wffs.

Let A be a set. An n-place relation on A is a subset of A^n . For a 2n-place relation R we write $(x_1,\ldots,x_n)R(y_1,\ldots,y_n)$ in place of

$(x_1,\dots,x_n,y_1,\dots,y_n) \in R$. A binary relation on A is an order relation if it is reflexive, transitive and complete. R is a strong order if it is asymmetric $(aRb \Rightarrow b\not{R}a)$, transitive and connected. If $\succsim$ is an order relation we will as usual write " $a \sim b$ " in place of " $a \succsim b$ and $b \succsim a$ ", and " $a \succ b$ " in place of " $a \succsim b$ and not $b \succsim a$ ". $\geq^L$ denotes the lexicographic order on R^n , that is $(b_1,\dots,b_n) \geq^L (a_1,\dots,a_n)$ if there exists k such that for all $k > i$, $a_i = b_i$, and $b_k > a_k$, or else for all $1 \leq i \leq n$ $a_i = b_i$. If $x \in A^n$, we will write $x = (x_1,\dots,x_n)$.

If φ is a wff., then $1 \circ \varphi = \varphi$, $(-1) \circ \varphi = \neg\varphi$. $|A|$ is the order of the set A .

Let L be a language containing n relation symbols with $i_1,\dots,i_n$ places. A model is an (n+1)-tuple $\langle A,R_1,\dots,R_n\rangle$, where A is a set (the universe of the model) and the R_j are i_j-place relations on A . The notion of a wff. φ being "satisfied in M under the substitution of $t_1,\dots,t_n$ " is defined naturally, and is denoted by $M \models \varphi(t_1,\dots,t_n)$. Let Σ be a set of wffs. in the language L , and let M be a model for L . A model M admits elimination of quantifiers relative to Σ if for every formula $\varphi(v_1,\dots,v_n)$ there exists a Boolean combination of formulae in Σ , $\psi(v_1,\dots,v_n)$, such that $M \models (\forall v_1,\dots,v_n)\varphi(v_1,\dots,v_n) \leftrightarrow \psi(v_1,\dots,v_n)$. If Σ is the set of atomic formulae, we will say that M admits elimination of quantifiers.

A formula $\varphi(x_1,\dots,x_n, y_1,\dots,y_n)$ explicitly induces an order on M if the relation φ^M defined by

$(x_1,\dots,x_n)\varphi^M(y_1,\dots,y_n) \Leftrightarrow M \models \varphi(x_1,\dots,x_n, y_1,\dots,y_n)$ is an order relation on $|M|^n$. A relation R will be called explicitly definable in M if there exists φ such that $\varphi^M = R$.

2. Arrow's Impossibility Theorem

Theorem 2.1. Let L be a language containing n binary relation symbols $\succ_1, \ldots, \succ_n$. There exists a model M for L where the binary relations are interpreted as complete and strong order relations and such that if $\succ$ is a strong order relation on the universe of M, definable in L and satisfying the axiom of unanimity (for all i $b \succ_i a \Rightarrow b \succ a$) then there exists a unique i such that $\succ = \succ_i$.

Proof. Let Q be the rationals. It is a simple matter to construct a set $A \subset Q^n$ such that $A \cap \overset{n}{\underset{i=1}{X}} I_i \neq \emptyset$ for all intervals $I_i, \ldots, I_n$ and such that for all $a, b \in A$, $a \neq b \Rightarrow a_i \neq b_i$ for all $i = 1, \ldots, n$. Define $\succ_i$ $b \succ_i a$ if and only if $b_i > a_i$.

Let $M = \langle A, \succ_1, \ldots, \succ_n \rangle$.

Clearly $\succ_i$ are strong complete order relations on A, and it is easily checked that M admits elimination of quantifiers. Let $\varphi(y,x)$ be the formula in L satisfying $y \succ x$ if $M \models \varphi(y,x)$. φ is equivalent to a disjunction of "orderings" where an "ordering" is a conjunction of the form

$$(\delta_i \in \{1,-1\}) \ , \quad \bigwedge_{i=1}^{n} \delta_i \circ (y \succ_i x)$$

Suppose $\succ \neq \succ_i$ for all i. Let $\psi_i(y,x)$ be the ordering in φ containg $x \succ_i y$. Let $b^0 \in A$; we will define $b^1, b^2, \ldots, b^n$ inductively so that

$$M \models \psi_i(b^i, b^{i+1}) \text{ and for all } 1 \leq j \leq i \ , \quad b_j^i > b_j^0 \ .$$

Suppose we have chosen b^{m-1}; b^m is chosen so that for $1 \leq j \leq m-1$

$$b_j^m \in \begin{cases} (b_j^0, b_j^{m-1}) & \text{if } \psi_m(y,x) \to y \succ_j x \\ (b_j^{m-1}, \infty) & \text{if } \psi_m(y,x) \to x \succ_j y \end{cases}$$

$b_m^m \in (\max \{b_m^0, b_m^{m-1}\}, \infty)$, and for $j > m$

$$b_j^m \in \begin{cases} (b_j^{m-1}, \infty) & \text{if } \psi_m(y,x) \to x \succ_j y \\ (-\infty, b_j^{m-1}) & \text{if } \psi_m(y,x) \to y \succ_j x \ . \end{cases}$$

and finally $b_i^n > b_i^0$ for all i so $b^n \succ_i b^0$ for all i .

Because of the unanimity, $b^n \succ b^0$, in contradiction to the transitivity and inflexivity of $\succsim$.

Two differences between this result and Arrow's impossiblity theorem are worthy of mention. Arrow considers the collection of models of the form $\langle X, R_1,\ldots,R_n\rangle$ where X is a given set and the R_i are strong order relations on X . Theorem 2.1 considers a single model.

Secondly, Arrow uses an independence axiom, here replaced by the "definability" requirement. Had we not in addition restricted our discussion to a single model the theorem would have been invalid. In fact it is easily verified that the following preference relation is definable in the language L .

$$\succ(R_1,\ldots,R_n) = \begin{cases} \text{majority rule} \\ R_1 \end{cases} \quad \text{according as}$$

$$\begin{cases} \text{majority rule induces an order on } X \\ \text{otherwise} \end{cases} .$$

3. Rawls' Maximin Principle

The main theorem here is Theorem 3.1 which characterizes the order relations $\succsim$ on R^n satisfying the following three axioms:

Axiom A. For all $(a_1,\ldots,a_n) \in R^n$ and for all $\sigma \in S_n$

$$(a_1,\ldots,a_n) \sim (a_{\sigma(1)},\ldots,a_{\sigma(n)}) .$$

Axiom P. If $a_i \geq b_i$ for all i and there exists i_0 such that $a_{i_0} > b_{i_0}$, then $(a_1,\ldots,a_n) \succ (b_1,\ldots,b_n)$.

Axiom D. There exists a formula $\varphi(y_1,\ldots,y_n,x_1,\ldots,x_n)$ in the language containing a single binary relation such that in the model of the reals, $M = \langle R,> \rangle$,

$$M \models \varphi(y_1\ldots y_n\ x_1\ldots x_n) \quad \text{iff} \quad (y_1\ldots y_n) \succsim (x_1\ldots x_n) .$$

For $a \in R^n$, we will denote the components of a in decreasing order by $a^1,\dots,a^n$ $(a^1 \ge a^2 \ge \dots \ge a^n)$.

Theorem 3.1. The order relations on R^n satisfying axioms A, P, and D are the $n!$ order relations for which there exists a permutation $i_1 \dots i_n$ of $1,\dots,n$ such that $y \succsim x$ iff $(y^{i_1} \dots y^{i_n}) \ge_L (x^{i_1} \dots x^{i_n})$.

Proof. The theory of dense order with no initial and final elements is a typical example of a theory admitting elimination of quantifiers ([2]); we may therefore assume that given $\varphi(y_1 \dots y_n, x_1 \dots x_n)$, the formula explicity defining $\succsim$, φ is equivalent to a disjunction of orderings where an ordering is a conjunction which "orders" $\{y_1,\dots,y_n, x_1,\dots,x_n\}$, that is, contains $v_i > v_j$ or $\neg\, v_i > v_j$ as formulae in the conjunction for all $v_i, v_j \in \{y_1,\dots,y_n, x_1,\dots,x_n\}$.

Axiom A allows us to restrict ourselves to the order induced by φ on non-increasing sequences. Suppose that for all $1 \le i \le n$ there exists an ordering ψ_i such that $x_i > y_i$ appears in the ordering. Axiom P enables us to assume that ψ_i orders $\{y_1,\dots,y_n, x_1,\dots,x_n\}$ with strong inequalities. Let $a^0 \in R^n$. Choose $a^1 \in R^n$ as follows:

Let j_1 be the largest number j for which $x_j > y_j$ appearing in ψ_1 $(1 \le j_1)$.

Choose $a^1_1 > a^1_2 > \dots > a^1_{j_1} > a^0_1$.

Choose $a^1_{j_1+1} \dots a^1_n$ from $(-\infty, a^0_1)$ so that they satisfy the relations in ψ_1 relative to $a^0_1 \dots a^0_n$.

Suppose we have chosen $a^1 \dots a^k \in R^n$ and $a^0_1 \ge a^k_n$. Let j_{k+1} be the maximal j for which $x_j > y_{j_k+1}$ appears in the conjunction ψ_{j_k+1} $(j_k+1 \le j_{k+1})$.

We will choose $a_1^{k+1} > \dots > a_{j_{k+1}}^{k+1}$ from the interval (a_1^0, ∞) so that they satisfy the relations in ψ_{j_k+1} relative to $a_1^k,\dots,a_{j_k}^k$, and we will choose $a_{j_{k+1}+1}^{k+1},\dots,a_n^{k+1}$ from the interval $(-\infty, a_1^0)$ so that they satisfy the relations in ψ_{j_k+1} relative to $a_{j_k+1}^k,\dots,a_n^k$. The denseness and non-existence of initial and final elements guarantee the existence of a^{k+1}. Thus $\psi_{j_k+1}(a^k,a^{k+1})$ and hence $a^k \succsim a^{k+1}$. We will finally obtain $a^0 \succsim a^1 \succsim \dots \succsim a^m$, $a_n^m > a_1^0$ from P $a^m \succ a^0$, and from the transitivity - $a^0 \succsim a^m$, a contradiction.

So far we have proved the existence of i_1, such that $y_{i_1} > x_{i_1} \Rightarrow y \succ x$.

Let us examine the ordering disjunction in ψ which contain $y_{i_1} = x_{i_1}$.

This disjunction defines an order on decreasing sequences of length n whose i_1'th element is 0. Similar considerations lead to the existence of i_2 such that $y_{i_1} = x_{i_1}$, $y_{i_2} > x_{i_2} \Rightarrow y \succ x$, and generally to the existence of $i_1,\dots,i_n$ such that for all $1 \le k \le n$,

$$\bigwedge_{j=1}^{k-1} y_{i_j} = x_{i_j} \wedge y_{i_k} > x_{i_k} \Rightarrow y \succ x .$$

"Isolation" of the maximin principle from the n! relations of theorem 3.2 may be achieved by any one of the following axioms:

<u>Axiom CON</u> $\succsim$ is convex.

<u>Axiom ME</u> For all i,j there exist $a,b \in R^n$ such that

$b_1 = a_1 > b_2 = a_2 > \dots a_i > b_i > \dots b_j > a_j > \dots b_n = a_n$

and $b \succ a$.

Proposition 3.2. i) The only order relation on R^n satisfying axioms C, P, D, A is the maximin.

ii) The only order relation on R^n satisfying axioms ME, P, D, A is the maximin.

Proof. i) By 3.1 there exists a permutation $i_1 \dots i_n$ such that $y \succsim x$ iff $(y^{i_1} \dots y^{i_n}) \geq^L (x^{i_1} \dots x^{i_n})$. Suppose $\succsim$ is not the maximum order. Let k be the smallest integer such that for $i_k \neq n - k + 1 = N$

$$(-N^2, \dots, -N^2, N-N^2, N, \dots) \succ (-N^2, \dots, -N^2, 0, \dots, 0)$$

$$\vdots$$

$$(-N^2, \dots, -N^2, N, N, \dots N-N^2) \succ (-N^2, \dots, -N^2, 0, \dots, 0)$$

Thus by axiom C,

$$(-N^2, \dots, -N^2, -1, \dots, -1) \succ (-N^2, \dots, -N^2, 0, \dots, 0)$$

which contradicts P .

ii) By 3.1, there exists a permutation $i_1 \dots i_n$ such that $y \succsim x$ iff $(y^{i_1} \dots y^{i_n}) \geq^L (x^{i_1} \dots x^{i_n})$. Let k be the samllest integer for which $i_k \neq n-k+1$ $(i_k < n-k)$. There exist, by Axiom ME, a,b such that $b \succ a$ but for all $j < k$, $a^{i_j} = b^{i_j}$ and $a^{i_k} \succ b^{i_k}$, thus $a \succ b$, a contradiction!

Definition. The relation $\succsim$ on R^n which satisfies $y \succsim x$ iff $(y^n \dots y^1) \geq^L (x^n \dots x^1)$ is called the maximin relation.

Axiom P* For all $(b_1 \dots b_n)(a_1 \dots a_n) \in R^n$, $a_i > b_i$ for all i implies $(a_1 \dots a_n) \succ (b_1 \dots b_n)$.

Axiom C $\succsim$ is a continuous relation (for all a $\{x \mid x \succsim a\}$ is closed) .

Proposition 3.3. If $\succsim$ is an order relation on R^n satisfying axioms A, D, P*, C, then there exists i such that $b \succsim a$ iff $t^i \geq a^i$.

Proof. The existence of i such that $y^i > x^i \Rightarrow y \succ x$ follows from the proof of 3.1. Let $a,b \in R^n$, $a^1 = b^1$. We can easily construct sequences $x_k \to a$, $y_k \to b$ such that $y_k^i > x_k^i$, and so $y^k \succsim x^k$ for all k . From axiom C, $b \succsim a$. Thus $b^i \geq a^i \Rightarrow b \succsim a$.

Axiom ME* for all $1 \leq i \leq n-1$ there exist $a,b \in R^n$ such that $b_1 = a_1 > b_2 = a_2 > \ldots > a_i > b_i > \ldots b_{n-1} = a_{n-1} > b_n > a_n$ and $b \succsim a$.

Corollary 3.4. If $\succsim$ is an order relation on R^n which satisfies axioms A, D, P*, C, and CON (or ME*) then $b^n \geq a^n$ iff $b \succsim a$.

Several axiomatic characterizations of Rawls' maximin principle have been published recently ([2], [4], [6], [10]). In the notation of [6], we have dealt with a ~~certain universe~~ choice problem where X , the set of social possibilities was identified with R^n (in fact we could have generalized to A^n where A is a densely ordered set with no initial and final elements), the set of individuals was $N = \{1,\ldots,n\}$ and the relation $\tilde{R}$ on $X \times N$, interpreted by " $(x,i)\tilde{R}(y,j)$ " iff "the utility i obtains in situation x is greater than the utility j obtains in situation y " , is $(x,i)\tilde{R}(y,j) \Leftrightarrow x_i \geq y_j$. In this lies the first main difference between this paper and the others.

The second difference is in the omission of the axiom of Binary Relevance and the including of the requirement of definability. It may be easily verified that in this case too, mere replacement of binary relevance by the requirement of definability without restricting the domain would not have given the maximin characterization. For example, the following criterion is definable: " xSy if the majority rule defines an order relation on the model, and also a majority prefer x to y, and also xSy, if the majority rule does not define an order relation, but x is preferred to y under the maximin criterion."

4. A Youth Arrives in a Strange Town

A youth arrives in a strange town. He has to choose between two blind dates. ~~is about to receive two offers of friendship.~~ The information given him will be the list of boys who went out with the girls over the last n days. Being a stranger in town all he can tell about the boys in the list is whether they are different or not. Our youth's language, L, will be "the pure language with equality" (without relations). The model M is the set of boys in town. Assuming the youth is "rational", we may take that there exists an order relation on a list of boys of length n which determines the youth's choice.

We will characterize (by proposition 4.1 and 4.2) the order relations expressible in L, i.e., the order relation which fulfill the following requirement: There exists in L a formula $\varphi(x_1,\ldots,x_n, y_1,\ldots,y_n)$ such that $(x_1,\ldots,x_n) \succsim (y_1,\ldots,y_n)$ iff $M \models \varphi(x_1,\ldots,x_n, y_1,\ldots,y_n)$.

Given an n-tuple $x = (x_1,\ldots,x_n)$ we will denote by E_x the partition of $\{1,\ldots,n\}$ into equivalence classes under the equivalence relation

$=_x$ defined by

$$i =_x j \quad \text{iff} \quad x_i = x_j$$

(for example, for $x = (a,b,b,a,c,a)$ the equivalence partition $\{1,\ldots,6\}$ is $\{ \{1,4,6\},\{2,3\},\{5\} \}$) . We will denote the set of equivalence classes of $\{1,\ldots,n\}$ by E_n .

Proposition 4.1. Let R be a relation on E_n . There exists $\varphi(x_1,\ldots,x_n, y_1,\ldots,y_n)$ is the language L such that $M \models \varphi(x_1,\ldots,x_n, y_1,\ldots,y_n)$ iff $E_x \; R \; E_y$.

Proof. For all $e \in E_n$ we denote the equivalence relation on $\{1,\ldots,n\}$ which induces e by e' , and define $\varphi_e(v_1,\ldots,v_n) = \bigwedge_{ie'j} v_i = v_j \wedge \bigwedge_{\neg ie'j} v_i \neq v_j$. Define

$\varphi(x_1,\ldots,x_n, y_1,\ldots,y_n) = \bigvee_{sRt} \varphi_s(x_1,\ldots,x_n) \wedge \varphi_t(y_1,\ldots,y_n)$. It is immediately clear that φ satisfies the requirements.

The main proposition in this section is the following:

Proposition 4.2. Let $\varphi(x_1,\ldots,x_n,y_1,\ldots,y_n)$ be a formula in L which defines explicitly an order relation in M . Then there exists an order relation R on E_n such that

$$M \models \varphi(x_1,\ldots,x_n,y_1,\ldots,y_n) \quad \text{iff} \quad E_x \; R \; E_y \quad .$$

Proof. φ may contain predicates. But from a theorem of Tarski (see [2]) it follows immediately that there is a formula ψ in the same free variables $x_1,\ldots,x_n,y_1,\ldots,y_n$ such that

$$M = \forall x_1\ldots x_n \; y_1\ldots y_n[\varphi(x_1\ldots y_n) \leftrightarrow \psi(x_1\ldots y_n)] \quad .$$

Further, we may assume that ψ is the disjunction of formulae which are conjunctions in which for all $v_i, v_j \in \{x_1,\ldots,x_n,y_1,\ldots,y_n\}$ either $v_i = v_j$ or $\neg v_i = v_j$ appear. Such a conjunction will be called an "ordering" of $x_1\ldots x_n\ y_1\ldots y_n$. Clearly all that is required is to show that given two n-tuples a and b with identical equivalence structures $(E_a = E_b)$, we have

$$M \models \varphi(a,b) \leftrightarrow \varphi(b,a) .$$

Before the proof it may help to examine an example. Let $n = 3$ and let the universe of M be $\{a,b,c,d\}$. (a,b,c) and (d,a,b) have identical equivalence structures $(\{\{1\},\{2\},\{3\}\})$. Suppose $(d,a,b)\varphi^M(a,b,c)$. Then $(a,b,c)\varphi^M(b,c,d)\varphi^M(c,d,a)\varphi^M(d,a,b)$, and thus $(a,b,c)\varphi^M(d,a,b)$.

Let $a = (a_1,\ldots,a_n)$ and $b = (b_1,\ldots,b_n)$ be two n-tuples of the elements of the model such that $E_a = E_b = \{I_j\}_{j=1}^k$.

Suppose $b\ \varphi^M\ a$. Let $i_1,\ldots,i_k$ be representatives from the equivalence classes. We denote $c = (a_{i_1}, a_{i_2},\ldots,a_{i_k},b_{j_1},\ldots,b_{j_\ell})$ where

$$\{b_{j_1},\ldots,b_{j_\ell}\} = \{b_{i_1},\ldots,b_{i_k}\} \setminus \{a_{i_1},\ldots,a_{i_k}\} .$$

Let τ be a permutation of $\{1,\ldots,k+\ell\}$ such that $c_i = b_{i_j} \Rightarrow \tau(j) = i$.

For all natural numbers t we define $d^t, d^t \in |M|^n$, and

$$d_i^t = c_{\tau^t(j)} \quad \text{where } j \text{ satisfies } i \in I_j .$$

$d^0 = a$ and $d^1 = b$. For all m, $E_{d^m} = E_a$, and $d_i^m = d_j^{m-1}$ iff $b_i = a_j$. Thus $d^m\ \varphi^M\ d^{m-1}$. There exists r such that $\tau^r = e$ (e the identity permutation); thus

$$a = d^r \varphi^M d^{r-1} \varphi^M \ldots \varphi^M d^1 = b \varphi^M d^0 = a$$

hence

$$a \varphi^M b \quad \text{and} \quad b \varphi^M a \quad .$$

Remark. "I will check who is the most popular boy in town. Lacking other information I will make do with the two candidates' lists and designate the boy appearing in the two lists most often as the most popular. I will choose the girl who went out with him most."

In accordance with the previous discussion such a procedure is irrational - in the sense this term has been used here - since a girl with the list (b,a,b) is preferred to a girl with (c,b,c) in spite of the fact that their equivalence structures are identical.

Conclusion

What is the meaning of the requirement that preference relations be definable in a given language?

It appears to me that for many problems in the theories of measurement, utility and social choice, it would be natural to investigate the preference relations in conjunction with the relevant language.

The following is from Shelley and Bryan [6] :

> The importance of judgements and the importance of the human being in making optimal decisions rests ultimately on the fact that the problems men address themselves to are those they have chosen to state. If an unstated problem is 'solved' there is no awareness of a 'solution'. Therefore the relations between

men and their problems are those of the languages used to state a problem and to reason out a solution (select a course of action) and those of the languages used to produce a solution and to express what is considered to be good, best, or optimal."

I feel that if every one of us tried to analyse the way he makes his decisions and evaluations, he would find it difficult to separate them from the 'language' he uses, 'thinks' and 'decides' in.

We instinctively justify to ourselves any decision and try to rationalize it. We are inclined to formulate our judgements in words and to justify them in words.

If this applies to the behavior of an individual, the more so does it apply to a collective. A social body cannot but formulate its preferences and decisions verbally, if only because the conveyance of information from one individual to another within the body is impossible without language.

I would like to mention in passing a basic difficulty we encouter. The scope of the mathematical treatment is confined to formal languages, whereas decision problems involve natural languages, and the relationship between natural and formal languages is not a clear one.

R E F E R E N C E S

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