Equilibrium in a Civilized Jungle

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ABSTRACT: The *jungle model* with an equal number of agents and objects is enriched by adding a set of orderings over the set of agents. The orderings provide potential criteria for determining the justifiability of an assignment of an agent to an object. A *civilized equilibrium* is an assignment such that every agent is the *strongest* within the set of agents who are *justifiable* in the group consisting of himself and agents who envy him.

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1. Introduction

Consider a society consisting of an equal number of agents and objects. Each agent has preferences over the objects and there are no externalities. Each agent must be assigned to only one object. The agents are objectively ranked by a *power relation*. We think of power not necessarily as physical but instead it can reflect, for example, social status or seniority. Up to this point, what we have is the *jungle model* à la Piccione and Rubinstein (2007) adapted to the object assignment model of Shapley and Scarf (1974). We enrich the model by including a *language* consisting of a set of orderings over the set of agents. The orderings can be thought of as potential criteria for arguing that an agent is the best-suited to be assigned to an object. For example, the criteria might rank the agents according to their economic status, intelligence, or level of education.

The phenomenon that we aim to formulate is that the assignment of objects is not entirely based on who is stronger, but also requires some socially legitimate justification. The language specifies the legitimate criteria that can be used to justify a claim. As is often the case in real life, an agent may cynically adapt the criteria that justify assignment of himself to his favorite object. In one case an agent might justify his claim by being the wealthier and in another by being the most intelligent.

A candidate for equilibrium in our model is an assignment that assigns each agent to a single object. Given an assignment, the claim of a particular agent to be assigned to an object is *justifiable* if the agent is the best-suited according to an ordering in the language from among the set of candidates who wish to be assigned to the object. Thus, when an agent claims that he should have been assigned to an object, he needs to justify his claim using a criterion according to which he is not only better-suited than the person who is assigned to the object but also better-suited than any other agent who wishes to be assigned to the object.

The solution concept we propose is *civilized equilibrium* (C-equilibrium). For an assignment to be a C-equilibrium, each agent must be justifiable within the group consisting of agents who envy him and must be stronger than any other agent who is justifiable within the same group.

The C-equilibrium is related to the Jungle Equilibrium, in which the power relation determines the equilibrium, for given preferences of the agents. However, in a civilized jungle, power is restricted such that when an agent wishes to be assigned to an object that another agent is assigned to, even if he is stronger, he needs to come up with a justifiable claim. For an agent, a justifiable claim is a specific formula in the language of the form: "I am the best-suited agent from among those who wish to be assigned to the object." Thus, the language restricts the use of power by determining what can be viewed as a valid criterion for being assigned to an object. For that reason, we refer to our equilibrium notion as "civilized equilibrium".

As mentioned, any justification can be used by an agent wishing to be assigned to an object. Therefore, the scope of the model is limited to situations where the objects are different but of the same type (such as office spaces, similar positions in an organization, time slots for lectures) and therefore it is reasonable to justify a claim using any relevant criterion.

2. The civilized jungle and the civilized equilibrium

A **civilized jungle** is a tuple $\langle N, X, (\succsim^i)_{i\in N}, \trianglerighteq, \mathscr{L}\rangle$. The set of *agents* is $N = \{1, \ldots, n\}$ and the set X consists of n *objects*. Each agent i has a strict preference relation $\succsim^i$, which is a complete, transitive, and anti-symmetric binary relation over X. The **power relation** $\trianglerighteq$ is a strict ordering over N. The statement $i \rhd j$ means that agent i is **stronger than** agent j. The **language** $\mathscr{L}$ is a set of complete and transitive (but not necessarily antisymmetric) binary relations over the set of agents N. We represent $\mathscr{L} = \{\geq_\lambda\}_{\lambda \in \Lambda}$ where Λ is the index set of $\mathscr{L}$'s members. The set $\mathscr{L}$ is the stock of criteria that can be used to justify the choice of an agent from within a **group** that is a nonempty subset of agents. We refer to a civilized jungle without a language as a **jungle**.

Unlike the model of the jungle, the use of the power relation $\trianglerighteq$ in the civilized jungle is restricted such that a stronger agent can exercise his power in order to be assigned to an object only if he can justify being assigned to it by one of the criteria recognized as legitimate in the civilized jungle. An agent i is **justifiable by** $\geq_\lambda$ from within the group I, if i is the *unique* maximizer of $\geq_\lambda$ from within I. An agent i is **justifiable** in group I if i is **justifiable by** $\geq_\lambda$ from within the group I for some $\geq_\lambda \in \mathscr{L}$. Let $J_{\mathscr{L}}(I)$ denote the set of agents who are justifiable in group I. That is, $J_{\mathscr{L}}(I) = \{i \in I \mid i \text{ is the unique } \geq_\lambda\text{-maximal agent in } I \text{ for some } \geq_\lambda \in \mathscr{L}\}$. By definition, $J_{\mathscr{L}}(\{i\}) = \{i\}$.

A candidate for our solution concept of a *civilized equilibrium* is an **assignment** $(x^i)_{i \in N}$ that maps each agent to an exclusive object. For brevity, we write (x^i) instead of $(x^i)_{i \in N}$. For an assignment (x^i), an agent j **envies** agent i if $x^i \succ^j x^j$. For an assignment (x^i) and an agent j, we denote the group consisting of agent j and all the agents who envy him by $E((x^i), j)$. An assignment (x^i) is a *civilized equilibrium* if each agent j is the $\trianglerighteq$-strongest agent in the set of agents that are justifiable in the group $E((x^i), j)$.

Definition 1 An assignment (x^i) is a **civilized equilibrium** (C-equilibrium) if each agent j is the $\trianglerighteq$-strongest agent in $J_{\mathscr{L}}(E((x^i), j))$.

2.1 Dichotomous languages

A dichotomous language consists of properties (unary relations) that an agent may or may not satisfy. Formally, a **dichotomous language** consists of orderings with two indifference sets, a top one and a bottom one, where every agent in the top set is superior to every agent in the bottom set. For each $\lambda \in \Lambda$, we identify the ordering $\geq_\lambda$ by means of a proposition λ in the sense that agent i satisfies λ if i is in the top set of $\geq_\lambda$ and fails to satisfy λ if he is in the bottom set of $\geq_\lambda$. We treat total indifference in two ways: If all agents are in the top set of $\geq_\lambda$, then they all satisfy λ, and if all agents are in the bottom set of $\geq_\lambda$, then none do. A dichotomous language can be represented as a profile $(\phi^i)_{i\in N}$ where ϕ^i is a nonempty subset of propositions in Λ that are valid for agent i.

Note that in the case of a dichotomous language $(\phi^i)_{i\in N}$, "an agent i is justified by λ in the group I" means that i is the unique agent in I for whom $\lambda \in \phi^i$. Therefore, in a civilized jungle with a dichotomous language $\langle N, X, (\succsim^i), \trianglerighteq, (\phi^i)_{i\in N}\rangle$, an assignment (x^i) is a C-equilibrium if and only if for every $j \in N$ either no one envies j or j is the $\trianglerighteq$-strongest from among the justifiable agents who envy him, that is the set of agents who can find a proposition that he uniquely satisfies (that "makes him special") within the group $E((x^i), j)$.

2.2 Examples

Example A (Restrictive languages) Consider a civilized jungle with a restrictive language $\mathscr{L}$ consisting of a single strict ordering $\geq$ over N. Then, the unique C-equilibrium is obtained by running the serial dictatorship according to $\geq$, independently of $\trianglerighteq$.

Example B (Justification by "I am who I am") Consider a civilized jungle with the dichotomous language $\phi^i = \{m^i\}$ for every $i \in N$. The statement m^i stands for "my name is i". Such a civilized jungle is extremely permissive in the sense that every agent can justify being assigned to any object by arguing that he is the unique agent who deserves to be assigned to it. Since every agent is justifiable in every group of agents, the unique C-equilibrium is obtained by running serial dictatorship according to the power relation $\trianglerighteq$, and therefore is Pareto efficient.

Example C (Identical preferences) Assume that all agents share the same preferences $a_1 \succ a_2 \succ \cdots \succ a_n$. Suppose that $\mathscr{L}$ is a language that contains at least one strict ordering, which guarantees that there is always a justifiable agent in a group. Then, inductively choose the sequence of agents such that i_l is the $\trianglerighteq$-strongest agent in $J_{\mathscr{L}}(N \setminus \{i_1, \ldots, i_{l-1}\})$. Then, the assignment of i_l to a_l is the unique C-equilibrium.

Example D (Nested dichotomous languages) Consider a dichotomous language where the sets of orderings in which agents in the top sets are *nested*, that is $\phi^{i_n} \subset \cdots \subset \phi^{i_2} \subset \phi^{i_1}$. For each preference profile and independently of $\trianglerighteq$, the associated civilized jungle has a unique C-equilibrium obtained by running the serial dictatorship according to the ordering $i_1, \ldots, i_n$.

3. Existence and efficiency of the C-equilibrium

In this section we explore the existence and efficiency of the C-equilibrium. The following example demonstrates that not every civilized jungle has a C-equilibrium.

Example E Let $N = \{1,2,3\}$ and $X = \{a,b,c\}$. The preference profile $(\succsim^i)$, the language $\{\geq_\alpha, \geq_\beta\}$ and the power relation $\trianglerighteq$ are specified as follows:

$\succ^1$	$\succ^2$	$\succ^3$
a	b	a
b	a	c
c	c	b

$\geq_\alpha$	$\geq_\beta$	$\trianglerighteq$
1	2	3
3	3	1
2	1	2

Assume that (x^i) is a C-equilibrium. Agent 1 does not envy agent 2, since otherwise $1 \in J_{\mathscr{L}}(E((x^i),2))$ (by $\geq_\alpha$) and $1 \triangleright 2$. Then, 3 does not envy 2, since otherwise $3 \in J_{\mathscr{L}}(E((x^i),2))$ (by $\geq_\alpha$) and $3 \triangleright 2$. This leaves the assignments $[b,c,a]$ and $[a,b,c]$. The former is not a C-equilibrium since 1 and 2 envy 3 who is not justifiable in N. The latter is not a C-equilibrium since only 3 envies 1, 3 is justifiable in $\{1,3\}$ (by $\geq_\beta$) and $3 \triangleright 1$. Thus, there is no C-equilibrium.

The existence of a C-equilibrium requires some type of dependence of the power relation and the criteria used for justification. One type of dependence which comes to mind is that the power relation is **$\mathscr{L}$-reflective**, in the sense that if an agent i is better-suited than agent j according to all the criteria in $\mathscr{L}$, then i must be stronger than j, i.e. for every $i,j \in N$, if $i \geq_\lambda j$ for every $\geq_\lambda \in \mathscr{L}$ with at least one strict inequality, then $i \triangleright j$. However, the power relation in Example E is $\mathscr{L}$-reflective and thus we need an additional condition. The *weak $\mathscr{L}$ - concavity of the power relation* (a weaker version of the *strong $\mathscr{L}$ - concavity* defined by (Richter and Rubinstein, 2019)) turns out to be the additional condition that we need.

Definition 2 A power relation $\trianglerighteq$ is **weakly** (**strongly**) **$\mathscr{L}$-concave** if the following condition holds: If for every $i,j \in N$ and $\geq_\lambda \in \mathscr{L}$ there exists $i_\lambda \in N$ such that $i_\lambda \geq_\lambda j$ and $i \triangleright i_\lambda$ $(i \trianglerighteq i_\lambda)$, then $i \triangleright j$.

A power relation is weakly $\mathscr{L}$-concave means that an agent i can argue that he is stronger than another agent j if for each criterion he can point to an agent who is as suited as j according the criterion and is recognized by the power relation to be weaker than i.

We will show that for every civilized jungle with a weakly $\mathscr{L}$-concave power relation, the associated Jungle Equilibrium that is obtained by running serial dictatorship according to the power relation is a C-equilibrium. Note that this assignment always exists and is Pareto efficient.

Proposition 1 *Let $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$ be a civilized jungle with a weakly $\mathscr{L}$-concave power relation.*

(i) The Jungle Equilibrium of $\langle N, X, (\succsim^i), \trianglerighteq\rangle$ is a C-equilibrium.

(ii) If $\mathscr{L}$ is a language of strict orderings and $\trianglerighteq$ is a strongly $\mathscr{L}$-concave power relation, then the C-equilibrium is unique.

Proof. To prove (i), let (x^i) be the assignment obtained by running the serial dictatorship according to $\trianglerighteq$. Then, for each $j \in N$, $E((x^i), j) \subseteq \{i | j \trianglerighteq i\}$. To conclude that (x^i) is a C-equilibrium, we need to verify that $j \in J_{\mathscr{L}}(E((x^i), j))$ for every agent j. By contradiction, if there is an agent j such that $j \notin J_{\mathscr{L}}(E((x^i), j))$, then for each $\lambda \in \Lambda$, there exists $j_\lambda \in E((x^i), j) \setminus \{j\}$ such that $j_\lambda \geq_\lambda j$ and $j \triangleright j_\lambda$. Therefore, by the weak $\mathscr{L}$-concavity of $\trianglerighteq$, we get $j \triangleright j$.

To prove (ii), suppose there exists another C-equilibrium (y^i). Then, there exists $i, j \in N$ such that $i \triangleright j$ and i envies j in (y^i). Since (y^i) is a C-equilibrium, $i \notin J_{\mathscr{L}}(E((y^i), j))$. Therefore, for each $\lambda \in \Lambda$, there exists $j_\lambda \in J_{\mathscr{L}}(E((y^i), j))$ such that $j_\lambda \geq_\lambda i$. Since (y^i) is a C-equilibrium, then $j \trianglerighteq j_\lambda$ for every $\lambda \in \Lambda$. However, in that case, the strong $\mathscr{L}$-concavity of $\trianglerighteq$ implies $j \triangleright i$, a contradiction. □

The following example demonstrates that, in a civilized jungle, if the power relation is not weakly $\mathscr{L}$-concave, then the jungle equilibrium may not be a C-equilibrium, even though there exists a unique C-equilibrium.

Example F Let $N = \{1,2,3,4\}$ and $X = \{a,b,c,d\}$. The preference profile $(\succsim^i)$, the language $\{\geq_\alpha, \geq_\beta\}$ and the power relation $\trianglerighteq$ are specified as follows:

$\succsim^1$	$\succsim^2$	$\succsim^3$	$\succsim^4$
a	b	a	a
b	a	c	d
c	c	b	b
d	d	d	c

$\geq_\alpha$	$\geq_\beta$	$\trianglerighteq$
2	3	1
4	1	2
1	4	3
3	2	4

The Jungle Equilibrium associated with this example is $[a,b,c,d]$. It is easy to see that $[b,a,c,d]$ is a C-equilibrium, which is Pareto dominated by the Jungle Equilibrium. To see that there is no other C-equilibrium, let (y^i) be a C-equilibrium. Then, it cannot be that $y^1 = a$, since otherwise both 3 and 4 would envy 1 and thus he would not be justifiable. It cannot be that $y^4 = a$, since then 3 envies 4, is justifiable in $E((y^i),4)$, and is stronger than 4. If $y^3 = a$, then $y^2 = b$ (otherwise 2 envies 3, is justifiable in $E((y^i),3)$, and is stronger than 3); however then 1 envies 2, is justifiable in $E((y^i),2)$ (because 3 is absent), and has higher priority than 2. Thus, $y^2 = a$ in a C-equilibrium. Then, it is straightforward to show that $(y^i) = [b,a,c,d]$.

The following proposition shows that in a civilized jungle with a language of strict orderings, if the power relation is $\mathscr{L}$-reflective but not weakly $\mathscr{L}$-concave, then we can find a preference profile for which there is no Pareto efficient C-equilibrium. Thus, combined with Proposition 1, this implies that in a civilized jungle with a $\mathscr{L}$-reflective power relation and a language of strict orderings, $\mathscr{L}$-concavity of the priority relation is the precise requirement that guarantees the existence of a Pareto efficient C-equilibrium for every preference profile.

Proposition 2 *Let $\langle N, X, (\succsim^i), \trianglerighteq \mathscr{L}\rangle$ be a civilized jungle with a language $\mathscr{L}$ of strict orderings. If the power relation $\trianglerighteq$ is $\mathscr{L}$-reflective but not weakly $\mathscr{L}$-concave, then there is a preference profile $(\succsim^i)$ such that there is no Pareto efficient C-equilibrium.*

Proof. Since $\trianglerighteq$ is not weakly $\mathscr{L}$-concave, there exist $i, j \in N$ such that for every $\lambda \in \Lambda$, there exists $j_\lambda \in N \setminus \{i\}$ such that $j_\lambda >_\lambda i$ and $j \triangleright j_\lambda$, but $i \trianglerighteq j$. Let $I = \{i_\lambda\}_{\lambda \in \Lambda} \cup \{i\}$. The set $J_{\mathscr{L}}(I)$ consists of agents in I who are the maximizers of $\geq_\lambda$ for some $\lambda \in \Lambda$. Since $\geq_\lambda$ is a strict ordering for every $\lambda \in \Lambda$, for each $j \in I \setminus \{i\}$, if $j \notin J_{\mathscr{L}}(I)$ then $J_{\mathscr{L}}(I) = J_{\mathscr{L}}(I \setminus \{j\})$. Let $I^* = \{i, j_1, \ldots, j_m\}$ be a subset of I such that $J_{\mathscr{L}}(I^*) = I^* \setminus \{i\}$. We assume without loss of generality that $j_1 \triangleright j_2 \triangleright \cdots j_m$.

Let $Z = \{z_0, z_1, \ldots, z_m\}$ be a set of distinct alternatives. Define a preference profile $(\succsim^i)$ such that:

i. Every $j \in I^*$ prefers every alternative in Z to every alternative in $X \setminus Z$ and every $j \in N \setminus I^*$ prefers every alternative in $X \setminus Z$ to every alternative in Z; and

ii. the preferences of every $j \in I^*$ restricted to Z are as follows:

$\succ^i$	$\succ^{j_1}$	$\succ^{j_2}$	$\succ^{j_3}$	$\cdots$	$\succ^{j_m}$
z_0	z_0	z_1	z_2		z_{m-1}
z_m	z_1	z_0	z_0		z_0
$\cdot$	$\cdot$	z_2	z_3		z_m
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
$\cdot$	$\cdot$	$\cdot$	z_m		$\cdot$
$\cdot$	z_m	z_m	z_1		$\cdot$

Let (x^i) be a Pareto efficient C-equilibrium. Then, by Pareto efficiency, $x^j \in Z$ for every $j \in I^*$. For each $j_k, j_l \in I^*$, if $k < l$, then j_k does not envy j_l. Otherwise, since $j_k \in$

$J_{\mathscr{L}}(I^*)$, we have $j_k \in J_{\mathscr{L}}(E((x^i), j_l))$ and $j_k \rhd j_l$, contradicting that (x^i) is a C-equilibrium. Therefore, either i or j_1 must be assigned to z_0 and the agents in $I^* \setminus \{i\}$ must be assigned to objects by the running serial dictatorship according their indices in ascending order. Thus, we are left with two cases, both of which lead to a contradiction of (x^i) being a C-equilibrium.

Case 1: $[x^i, x^{j_1}, \ldots, x^{j_m}] = [z_0, z_1, z_2, \ldots, z_m]$. Then, $x^i = z_0$, even though $E((x^i), i) = I^*$ and $i \notin J_{\mathscr{L}}(I^*)$.

Case 2: $[x^i, x^{j_1}, \ldots, x^{j_m}] = [z_m, z_0, z_1, \ldots, z_{m-1}]$. Then, $E((x^i), j_1) = \{i, j_1\}$. Since $i \rhd j_1$ and $\unrhd$ is *reflective*, there exists $\lambda \in \Lambda$ such that $i >_\lambda j_1$. Thus, we have $x^{j_1} = z_0$, even though $i \in J_{\mathscr{L}}(\{i, j_1\})$ and $i \rhd j_1$. □

4. A counterpart to the second welfare theorem

The second welfare theorem states that for every Pareto efficient allocation the initial allocation can be adjusted such that the Pareto efficient allocation is a competitive equilibrium allocation. In the context of a jungle, Piccione and Rubinstein (2007) suggest an analogous result: For every Pareto efficient allocation, there is a power relation such that the allocation is an equilibrium of the jungle with the power relation. An analogous result in a civilized jungle would have the structure that for a given tuple $\langle N, X, (\succsim^i), \mathscr{L} \rangle$, every assignment that satisfies a certain efficiency requirement is a C-equilibrium. The requirement cannot be Pareto efficiency (see Example A) but it should be one that takes the language into account. Thus, we define that an assignment (x^i) is **justifiable** if each agent j is justifiable in $E((x^i), j)$. An assignment (x^i) is **J-constrained efficient** if (x^i) is justifiable and there is no justifiable assignment (y^i) that Pareto dominates (x^i).

We show that, given a language of strict orderings, for every J-constrained efficient assignment there is a power relation $\trianglerighteq$ such that (x^i) is a C-equilibrium in the civilized jungle with the power relation $\trianglerighteq$.

Proposition 3 *Let $\langle N, X, (\succsim^i), \mathscr{L}\rangle$ be a tuple where $\mathscr{L}$ is a language of strict orderings. Then, for every J-constrained efficient assignment (x^i), there exists a power relation $\trianglerighteq$ such that (x^i) is a C-equilibrium for the civilized jungle $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$.*

Proof. Let (x^i) be a J-constrained efficient assignment. Then, for every distinct $i, j \in N$, define $j\ P\ i$ if i is justifiable in $E((x^i), j)$. We first show that the relation P is acyclic. Suppose by contradiction and without loss of generality that $1\ P\ 2\ P \cdots\ m\ P\ 1$. For $i = 1$, we identify $i-1$ with m. Define (y^i) by $y^i = x^{i-1}$ for every $i \in I$ and $y^j = x^j$ for every agent $j \notin I$. The assignment (y^i) Pareto dominates (x^i) since by the definition of P, for every $i \in \{1, \ldots, m\} = I$, $x^{i-1} \succ^i x^i$.

Now, we show that (y^i) is justifiable. For every $j \in N$, if j envies $i \in I$ in (y^i), then j envies $i-1$ in (x^i). That is, for every $i \in I$, $E((y^i), i) \subseteq E((x^i), i-1)$. Then, since, by the definition of P, i is justifiable in $E((x^i), i-1)$, i is also justifiable in $E((y^i), i)$. For every $j \in N$, if j envies $i \notin I$ in (y^i), then j also envies i in (x^i). That is, for every $i \in I$, $E((y^i), i) \subseteq E((x^i), i)$. Then, since (x^i) is justifiable, i is justifiable in $E((y^i), i)$.

Finally, let $\trianglerighteq$ be a completion of the transitive closure of P. To see that (x^i) is a C-equilibrium for $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$, suppose that an agent i envies an agent j and i is justifiable in $E((x^i), j)$. Then, $j\ P\ i$ and therefore $j \triangleright i$. □

If there exists a justifiable assignment, then obviously there exists a J-constrained efficient assignment. But, so far we have not discussed the existence of a justifiable assignment. Indeed, for a civilized jungle with an arbitrary language $\mathscr{L}$, there may not be any justifiable assignment (e.g. a civilized jungle in which agents have identical preferences and $\mathscr{L}$ is a single ordering that ranks all the agents at the top). However, for every tuple $\langle N, X, (\succsim^i), \mathscr{L}\rangle$ with a language $\mathscr{L}$ of strict orderings, there exists a justifiable assignment (x^i). To see this, consider the *associated marriage problem* à la Gale and Shapley (1962) such that the two sides of the market are N and X such that each $i \in N$ has the preference relation $\succsim^i$ over X and each $x \in X$ has a preference relation $\geq_x$ over N where $\geq_x \in \mathscr{L}$. It follows from Gale and Shapley (1962) that for this marriage problem there exists an assignment (x^i) that is *stable*, in the sense that if there exists an agent i who envies j in (x^i), then $j >_{p(x^j)} i$. This means that each agent j is justifiable in $E((x^i), j)$, and thus we conclude that every stable assignment (x^i) is justifiable.

As pointed out above, in the case of a language of strict orderings the notion of justifiability is closely related to the notion of *stability*. This is no longer true when the language consists of weak orderings (see Example D). In that case, given an assignment (x^i), when an agent i envies agent j, the stability of (x^i) only requires that agent j be ranked weakly above i according to the preference relation attached to object x^j. On the other hand, justifying the assignment of an agent j to x^j by an ordering in $\mathscr{L}$ requires j be ranked strictly above i in the ordering.

Finally, as a corollary to Proposition 3, we show that for every language of strict orderings, there exists a power relation such that the associated civilized jungle has a C-equilibrium that is weakly Pareto efficient, in the sense that there is no assignment such that each agent is assigned to one of his more preferred objects.

Corollary 1 *Let $\langle N, X, (\succsim^i), \mathscr{L}\rangle$ be a tuple where $\mathscr{L}$ is a language of strict orderings. Then, there exists a power relation $\trianglerighteq$ such that the civilized jungle $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$ has a weakly Pareto-efficient C-equilibrium.*

Proof. Consider the marriage problem associated with the given tuple, as described above. Then, by Gale and Shapley (1962), there exists a stable assignment (x^i) such that for every other stable assignment (y^i), we have $x^i \succsim y^i$ for every $i \in N$ (the *proposers-optimal stable assignment*). Theorem 3 by Gale and Sotomayor (1985) shows that (x^i) is weakly Pareto efficient in the sense that there is no assignment (z^i) (stable or not) such that $z^i \succ^i x^i$ for every $i \in N$. Therefore, we get that (x^i) is a justifiable assignment that is weakly Pareto efficient. Then, either (x^i) a J-constrained efficient or there exists a J-constrained efficient assignment (y^i) that Pareto dominates (x^i). If the latter holds, then (y^i) is also weakly Pareto efficient. Thus, the conclusion directly follows from Proposition 3. □

The following example demonstrates that Corollary 1 does not extend to dichotomous languages even if a justifiable assignment exists.

Example G Let $N = \{1,2,3,4\}$ and $X = \{a,b,c,d\}$. The preference profile $(\succsim^i)$ and the language $(\geq_x)_{x\in X}$ are specified below. The assignment $(x^i) = [b,a,d,c]$ is justifiable (i.e. each agent j is justified by $\geq_{x^j}$ in $E((x^i), j)$) but is not weakly Pareto efficient. Moreover, there is no other justifiable assignment.

$\succ^1$	$\succ^2$	$\succ^3$	$\succ^4$
a	b	a	a
c	a	c	d
d	c	d	c
b	d	b	b

$\geq_a$	$\geq_b$	$\geq_c$	$\geq_d$
2	1	4	3
1,3,4	2,3,4	1,2,3	1,2,4

5. Final comments

5.1 Enriching the jungle with a language

A civilized jungle differs from a jungle in that it specifies a set of orderings that can be used to justify the assignments of agents to objects. This approach is consistent with our general view that solution concepts in economic theory (whether they refer to markets, games or decision scenarios) should be expressed using terms that are relevant to the participants. Rubinstein (1978) and Rubinstein (2000, Chapter 4) argue that an agent's preferences in an economic model should be *definable* using a given langauge. In this vein, Richter and Rubinstein (2015) formulate their *primitive equilibrium* by adding to the model an exogenous set of orderings $\mathscr{L}$ (called there the set of primitive orderings) to restrict the set of preferences to be $\mathscr{L}$-convex (defined analogously to $\mathscr{L}$-concavity) and the social ordering (which plays a role analogous to competitive equilibrium prices) to be an element of $\mathscr{L}$.

5.2 Relation to cooperative game theory

A C-equilibrium is an assignment such that every valid objection of the form – "According to a legitimate criterion I am the most suited agent from among the group of agents who wish to be assigned to the object"– can be responded by the agent assigned to object with a valid counter objection of the form – "According to a legitimate criterion I am also the most suited agent in the same group, and furthermore I am stronger than you." As such, the structure of the notion of C-equilibrium is similar to that of many solution concepts in cooperative game theory, where an outcome is a solution if for any valid objection (by some definition), there is a valid counter objection (by some definition). This structure is clear in both von Neumann and Morgenstern (1953)'s *stable set* and Aumann and Maschler (1961)'s *bargaining set*, but it is also consistent with most cooperative solution concepts, including the *Nash Bargaining solution* (Rubinstein, Safra, and Thomson, 1992). Note also that the objections and the counter objections here depend on the set of criteria used to justify an assignment of an an agent to an object, whereas in cooperative solution concepts the justification typically involves an action by a coalition that includes the objecter or the counter objector. Piccione and Razin (2009) applied such a cooperative game theoretic approach to a jungle model in which agents have identical preferences and the power relation is over the coalitions rather than individuals.

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