Comments on the Risk and Time Preferences in Economics*

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Rabin (2000)'s Argument

I heard about Rabin's calibration result from Daniel Kahneman and, like him, I was impressed. Part of the reason is that it does not rely on experiments. It relies instead on a mental experiment and rings true without the need to be verified. I rushed to add the material to the lecture notes for my graduate micro-economics course. My comments there were in the spirit of Rabin's criticism of expected utility theory: "What conclusion should we derive from this observation? Do we economists take our own findings seriously?" However, unlike Rabin, I wrote the comments with question marks and the conclusion I drew was quite different from that of Rabin (2001) and Rabin and Thaler (2001). I still include Rabin's material in my lectures but not as a criticism of expected utility theory but as a demonstration of the problems associated with applying expected utility theory in mainstream economics.

Let us review Rabin's argument: A decision-maker rejects, at all levels of wealth (or at a certain interval, such as $[0, \$4000]$), the lottery $0.5[-10] \oplus 0.5[+11]$. The existence of such decision makers is plausible. If a decision maker behaves according to expected utility theory and is risk averse, his vNM utility function, u, satisfies $u(w+11) - u(w) < u(w) - u(w-10)$ and thus the marginal utility function $mu(w)$ satisfies $mu(w+11) \leq [u(w+11) - u(w)]/11 < (10/11)[u(w) - u(w-10)]/10 \leq (10/11)mu(w-10)$ in that domain. Thus, marginal utility falls at a faster rate than that of a geometrical sequence and therefore one can easily find levels of wealth at which people must reject an equal chance to lose a moderate amount like $\$100$ and to make a large gain like $\$64000$. Such behavior is absurd.

Assessment

This section is built on ideas from my lecture notes which are close to those in Cox and Sadiraj (2001). There are two key components in Rabin's argument: First, Rabin attacks the assumption that decision makers maximize expected utility. Second, he makes an implicit assumption that in expected utility calculations prizes are taken to be the balances following the lottery draw, independent of starting wealth. In other words, the expected utility calculations which lead us to absurd conclusions, presume that there is a single preference relation, $\succsim$, over the set of lotteries with prizes being the "final wealth levels" such that a decision maker at any wealth w who has a vNM preference relation, $\succsim_w$ over the set of "wealth changes" derives that preference from $\succsim$ by $L_1 \succsim_w L_2$ iff $w + L_1 \succsim w + L_2$.

This is indeed the practice of most economists in applying expected utility theory. However, Rabin's criticism is directed towards expected utility itself. Nothing in the vNM axioms dictates use of final wealth levels rather than wealth changes. vNM are silent about the definition of prizes. Standard textbooks are vague on the interpretation of "w"- usually they state that the decision maker derives utility from "money", with no discussion of whether "money" is a flow or a final stock.

The definition of prizes as final wealth levels is no less crucial to Rabin's argument than the expected utility assumption. It might be plausible that for a wide range of w a decision maker rejects the small lottery $0.5[-10] \oplus 0.5[+11]$ given that he starts with w and were he to start from wealth 0, for example, he would prefer the sure amount $[w]$ over the lottery

$0.5[w-10] \oplus 0.5[w+11]$. If w is not small, the decision maker might reason that the uncertainty has little effect on the consequence and thus should be completely ignored.

Needless to say, the criticism of the application of EU to "final wealth levels" as a descriptive theory is not new. Kahneman and Tversky (1979) already pointed out the dramatic difference in our attitudes towards gains and losses . In this respect Rabin is only adding some insight into the implausibility of the combination of assumptions even when no losses are involved.

By the way, this is an opportunity to express my confusion concerning the meaning of the word "consequence" in economics. Is there anything like a "terminal consequence" when it appears that most of us care about events after our death? Shouldn't the term "consequence" be subjective (at least when we seek for a descriptive model), corresponding to what the decision maker considers "final" in a particular context?

Note that modeling a decision maker as a collection of preference relations $(\succsim_w)$ is not a complete way to describe behavior since it allows "time inconsistency". Consider a decision maker who at the level of wealth $w = 1000$ rejects the lottery $0.5[-10] \oplus 0.5[+11]$ but at the level of wealth 0 prefers the lottery $0.5[w-10] \oplus 0.5[w+11]$ to the certain $[w]$. What will this decision maker do when he faced the lottery $0.5[w-10] \oplus 0.5[w+11]$ broken down into a two-stage process in which he receives first $\$1000$ and then receives the outcome of the lottery $0.5[-10] \oplus 0.5[+11]$ but not before he has a final chance to replace it with 0? The content of the collection of preference relations $(\succsim_w)$ does not provide an answer to this question. We require further assumptions about the way in which the decision maker resolves the conflict between his "two selves" at the levels of wealth 0 and 1000.

To summarize, Rabin's criticism of expected utility has validity only if the decision maker's preferences depend on the final level of wealth. This leads me to compare the choice of consequences in the literature of decision making under uncertainty with the choice of consequences regarding intertemporal choice.

Time Preferences

The idea that a person who does not have an apple today is ready to surrender two apples tomorrow in order to get one today is not implausible. Indeed, the hyperbolic discounting literature uses as one of its primary motivating examples the fact that there are people who prefer one apple today over two apples tomorrow and at the same time prefer two apples in 31 days to one apple in 30 days. What is a "final consequence" in the world of apples and time? It has to be a list of dates and quantities of apples to be consumed on those dates. Eating one apple on 13.4.2051 is a final consequence (not only for the apple) independent of the date on which I made the decision to eat it. It is no less a final consequence, and probably more so, than having $1000 deposited in my banking account following the flip of a coin.

So here is Adam, an apple man and... a calibration "theorem" for "time preferences":

Observation: Assume that Adam:

▸ Has a preference relation $\succsim$ over the set of streams of apple consumption.

▸ Hates to eat 3 apples a day.
▸ Is time impatient.
▸ At every period t, and given that he has a consumption stream c, he holds preferences $\succsim_{t,c}$ over changes in apple consumption in periods following t satisfying $D_1 \succsim_{t,c} D_2$ iff $c + D_1 \succsim c + D_2$.
▸ Whenever (or at least within some time period) he lacks an apple he prefers to get it right away in exchange for two apples tomorrow. That is, if $c_t = 0$ he is ready to exchange 2 apples from period $t+1$ for 1 apple at period t.
Then, if (poor) Adam is offered a stream of 2 apples starting in period 18 for the rest of his life (assuming he does not expect to live more than 120 years), he will be willing to exchange it for 2 apples right away!

The essence of the proof can be seen from the following observations: Adam would find the stream of two apples per period for of 2^1 periods, namely $(0,2,2)$, inferior to $(1,0,2)$, and therefore to $(1,1,0)$ and also to $(2,0,0)$. Similarly, Adam would find the stream of two apples per day for 2^2 periods in a delay of 2 periods, namely, the stream $(0,0,2,2,2,2)$, inferior to $(0,2,0,2,0,0)$ and therefore to $(1,0,1,0,0,0)$ and to $(2,0,0,0,0,0)$. By induction we obtain that he must find the stream of 2^{17} periods of two apples per day with a delay of 17 periods, inferior to receiving 2 apples right away. It is left to notice that in 120 years there are no more than 2^{17} days...

This "argument" does not "put an end" to any theory of time preference, just as Rabin's argument does not deal a fatal blow to expected utility theory. In both cases, the message is that we should be sensible in selecting the set of consequences when analyzing a situation especially when we approach the issue from a behavioral, rather than normative, point of view.

Adam, Eve and the Snake in the Garden of Eden:

Indeed, when discussing time preference in the world of pairs (x,t) (interpreted as getting x apples in period t and assuming $c \equiv 0$), Rabin himself does not assume a stream of preferences $(\succsim_t)$ satisfying that $(+1 apple, right\ away) \succsim_t (+2 apples,\ in\ one\ period\ delay)$ iff $(1 apple, t) \succsim (2 apples, t+1)$ for some $\succsim$. Like many others, he attaches to a decision maker a sequence of preference relations $(\succsim_t)$. This sounds alright from a descriptive point of view but here, once again, we find a problem that has been understood at least since Strotz, half a century ago. Once we do not assume consumption schedules to be the consequences in intertemporal choice problems, (just as when we depart from the practice of defining prizes to be final levels of wealth) we enter the jungle of "time-inconsistencies", "multiple-selves", "money pumps" and other "wild ideas".

We are forced to enrich the model with assumptions about the way in which decision makers compromise between their different selves. The common assumption is that the decision maker's behavior is consistent with a "perfect equilibrium procedure" ("sophisticated behavior" as it is called in the behavioral economics literature). If you seek paradoxical results you have found your garden of Eden.

How about another "Adam the apple man" story based on Rubinstein (1998):

When Adam has to choose between eating an apple today or an apple tomorrow he prefers

to postpone the pleasure. He is also short-sighted. Essentially he is an extreme example of hyperbolic discounter who cares only about what happens in the upcoming two periods.

In the garden, Adam finds Eve, a tempting lady, and the snake, a sophisticated consultant. Eve offers Adam one apple. Whenever he is about to eat the apple she tells him, "Why don't you give me the apple and get an additional one tomorrow? Naive Adam will take the bait and never eat the apple. Were he "sophisticated", Adam would eat the apple on the first or second day. Wonderful!

Frustrated by Eve and desiring to be a more sophisticated decision maker, Adam goes to his consultant. The snake tells Adam he can pick one free apple every day. What could be simpler than that? Adam feels he should just pick the apple every day. However, the snake has other plans for Adam. The snake recommends to Adam a "perfect equilibrium": Adam should take an apple only after an odd number of consecutive periods in which he has not done so. Adam, falls into the trap. He never finds a reason not to follow the snake's advice. In a period in which he is not supposed to pick an apple, he expects to get an apple a day after which is better for him than eating the apple now and not eating it (as the equilibrium suggests) in the second period (since this will follow 0 periods of not eating apples). In a period in which he is supposed to eat the apple, not eating the apple will cause him not eat it in the following period as well (as it will be odd number of periods since he ate an apple) and this is worse for Adam than eating the apple right away. Thus, a consistent prediction of the model is that Adam eats apples only once in two days. This behavior is at least as bizarre as that of Rabin's decision maker.

Conclusion

Rabin and Thaler should be praised for challenging the fundamental assumptions of the profession which is often overprotective of its models. However, strong criticism (mine included...) invites strong counter-criticism. If Rabin and Thaler feel "much like the customer in the pet shop, beating at a dead parrot", it is no wonder that Paleacios-Huerta, Serrano and Volij (2001) quote the "expected utility parrot" saying that "the reports of my death have been highly exaggerated". I bring these quotes with a heavy dose of cynicism. As a skeptic I doubt that the parrot was even born, at least in the behavioral sense in which Rabin mourns the refusal to recognize its death.

Define a model, spell out a combination of assumptions and someone will find a paradoxical conclusion. Why? Because any model is no more than a combination of partial considerations which decision makers may or may not use, under certain circumstances. All we do in economic theory is a **calculus of considerations**. A good model identifies one or two considerations and calculates the implications of their use. Experimental economics does something related. What makes experiments like those of Allais or Kahneman and Tversky so ingenious is not the discovery that a certain consideration enters into people's reasoning process but their ability to uncover it in isolation from other conflicting considerations which almost always exist.

Unlike parrots, human beings also have the ability to invent new ways of reasoning that will confound any theory. Therefore I believe that any set of assumptions will produce

paradoxical results in certain circumstances when the omitted assumptions become crucial. A paradoxical conclusion by itself does not signal the end of a theory. At most, it means that we should be careful in applying the theory.

References

Cox, J.C. and V. Sadiraj (2001) ”Risk Aversion and Expeted-Utility Theory: Coherence for Small- and Large- Stakes Gambles”, (mimeo),

Kahneman, D. and A.Tversky (1979) “Prospect Theory: An Analysis of Decision under Risk”, *Econometrica*, 47, 263-292.

Palacios-Huerta, I., Serrano, R. and O.Volij (2001) ”Rejecting Small Gambles Under Expected Utility: A Comment on Rabin” (mimeo).

Rabin, M. (2000) ”Risk Aversion and Expected Utility Theory: A Calibration Theorem”, *Econometrica*, 68, 1281-1290.

Rabin, M. and R. Thaler (2001), ”Anomalies: Risk Aversion”, *Journal of Economic Perspectives*, 15, 219-232.

Rubinstein, A. (1998), *Modeling Bounded Rationality*, MIT Press.